

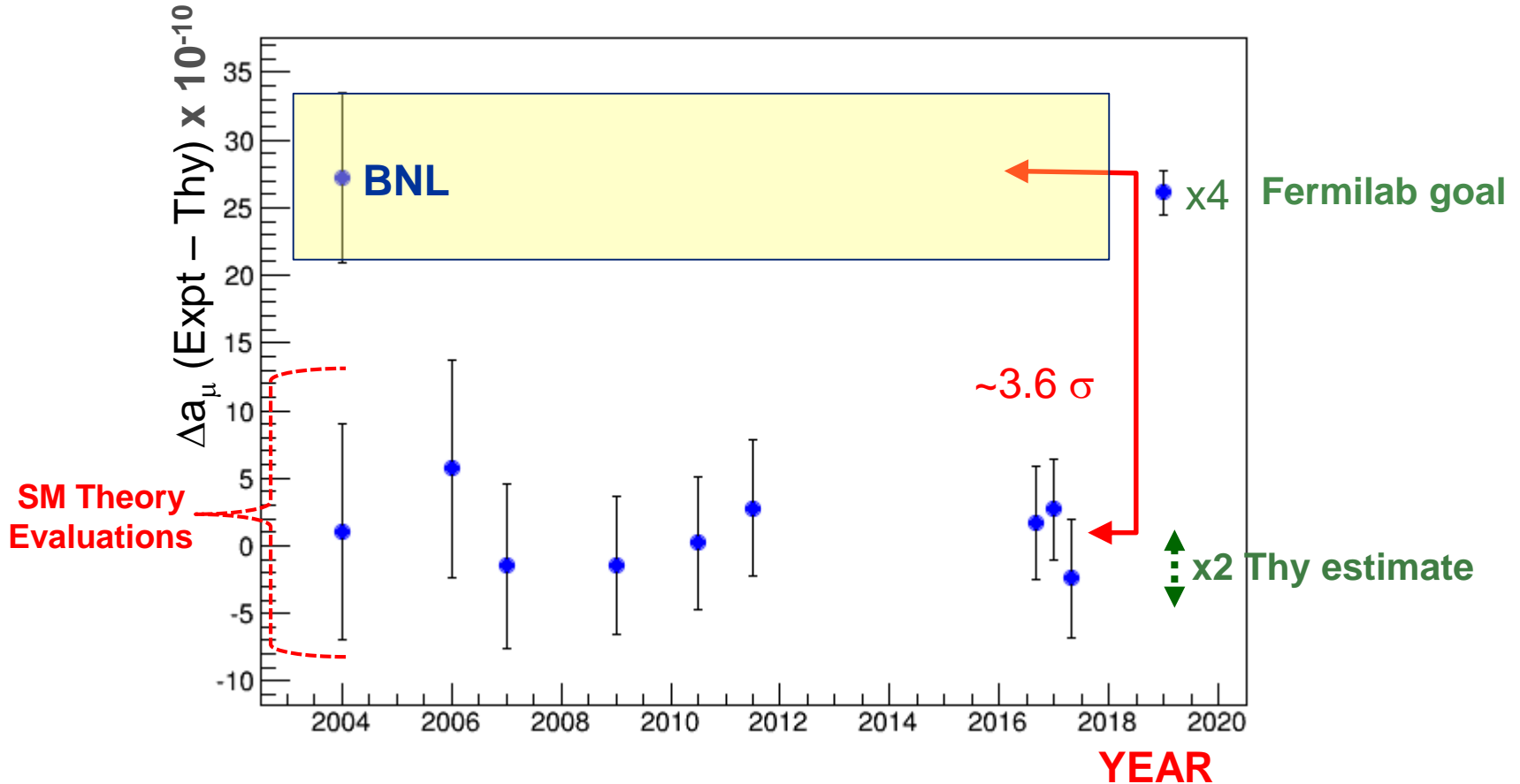
The (anomalous) Magnetic Moment of the Muon

David Hertzog

University of Washington

January 12, 2018

To briefly setup the story for today ...



a_μ is now measured to 540 ppb; Goal is 140 ppb

But first, how I “knew” Sidney Drell ...



Grad school thought: *“Whatever these things were, I wanted to know ...”*

Current thought: *“I need to understand this 700+ page book since I’m making the measurement ...”*

Start with Dirac equation for electron in an EM potential

$$i(\partial_\mu - ieA_\mu(x))\gamma^\mu\psi(x) = m\psi(x)$$

- Anticipates **antiparticles** (which were later found)
- Predicts **$g = 2$** , as observed in atomic fine-structure experiments for the **spin-1/2** electron magnetic moment (whereas an orbital picture $\rightarrow g = 1$)

$$\vec{\mu} = g \left(\frac{Qe}{2m} \right) \vec{s}, \quad e > 0$$

- And, it allows for a so-called **Pauli interaction** term to accommodate possible **deviations of g from 2**

At first, $g \approx 2$ was observed. But later, the proton ...

$$g_p = 5.59$$

and then the neutron

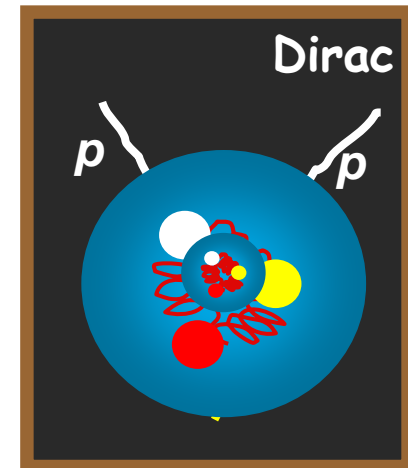
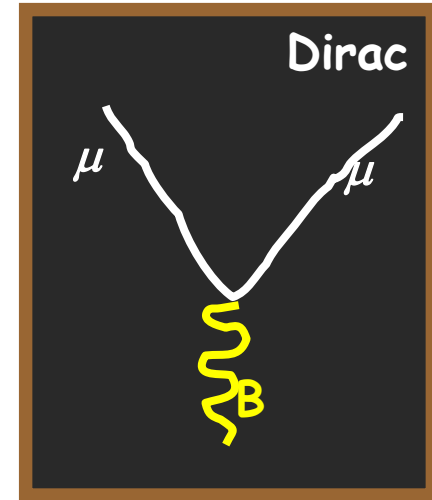
$$g_n = -3.8$$

each showed large magnetic moments ($g \neq 2$ by a lot)

The neutron? It's not even charged!

These are “Anomalous” magnetic moments owing to substructure

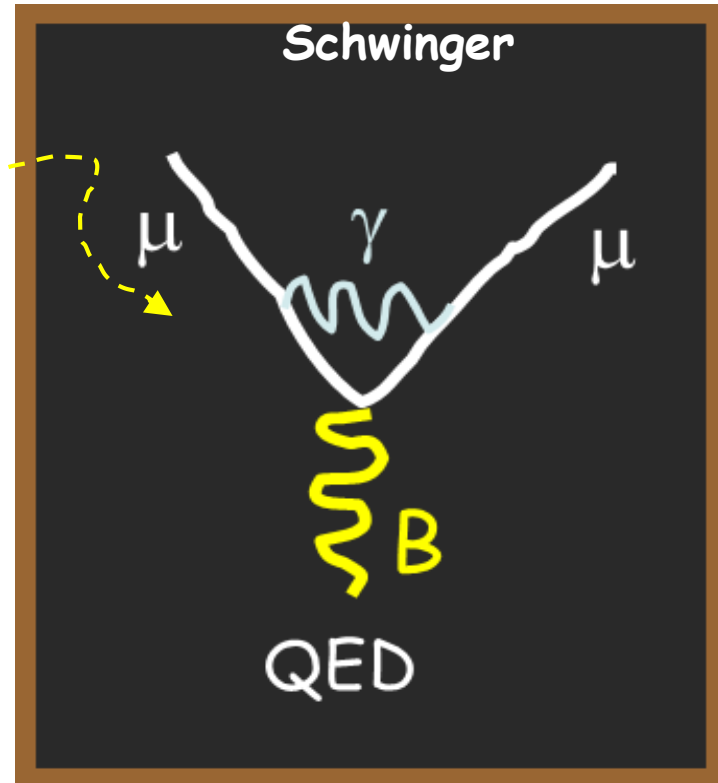
$$g = 2(1 + a) \quad \text{or} \quad a = \frac{(g - 2)}{2}$$



Leading order in QED

$$a_e = \frac{(g - 2)}{2} \approx \frac{1}{2} \frac{\alpha}{\pi} \approx \frac{1}{800}$$

$$\rightarrow 116\,140\,973.301 \times 10^{-11}$$



In 1947, small deviations from $g = 2$ for the “pointlike” electron were observed at about the $\sim 0.1\%$ level

Anomalous Magnetic Moment of the Electron, Muon, and Nucleon*

S. D. DRELL AND H. R. PAGELS†

Stanford Linear Accelerator Center, Stanford University, Stanford, California

(Received 14 June 1965)

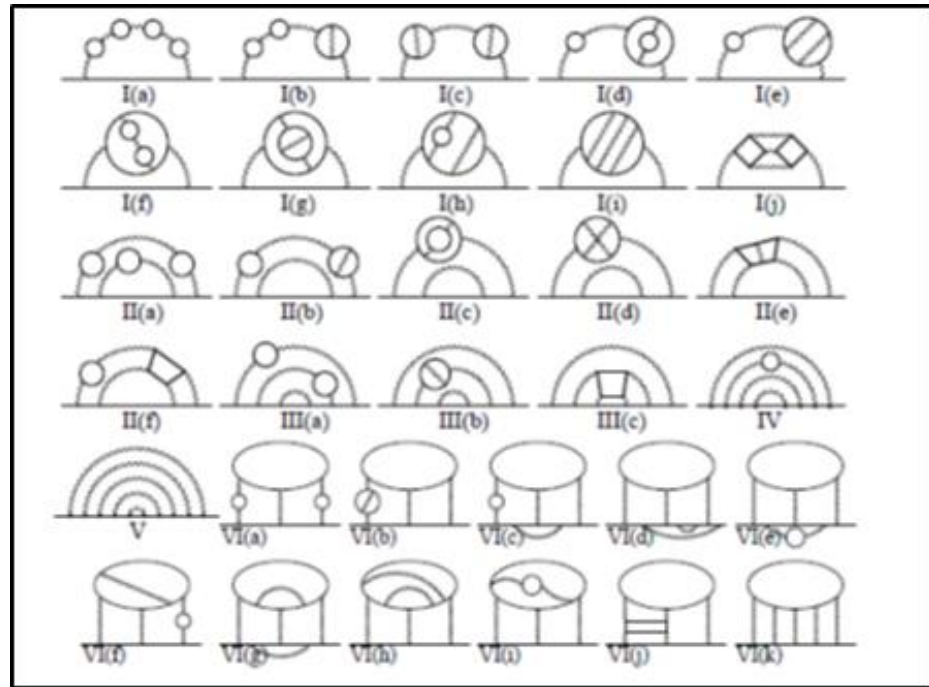
This paper is our answer to this challenge.

■ In response to this unique Feynman challenge:

- ◆ It seems that **very little physical intuition has yet been developed in this subject**. In nearly every case we are reduced to computing exactly the coefficient of some specific term. We have no way to get a general idea of the result to be expected. We have no physical picture by which we can easily see that the correction is roughly $\alpha/2\pi$ in fact, **we do not even know why the sign is positive** (other than by computing it)
- ◆ We have been computing terms like a blind man exploring a new room, but soon we must develop some concept of this room as a whole, and to have some general idea of what is contained in it. As a **specific challenge**, is there any method of computing the anomalous moment of the electron which, on first rough approximation, gives a fair approximation to the α term and a crude one to α^2 ...

Higher order QED !

QED 1st Order	116140973.301
QED 2nd Order	413217.621
QED 3rd Order	30141.902
QED 4th Order	380.807
QED 5th Order	4.483
	$\times 10^{-11}$



Presently: QED thru tenth-order terms (12,672 diagrams)

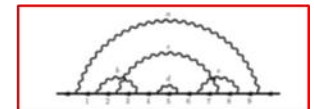
$$a_{\mu}(\text{QED}, \alpha(a_e)) = 1\,165\,847\,188.41\ (7)(17)(6)(28) \times 10^{-12}$$

Revised and Improved Value of the QED Tenth-Order Electron
Anomalous Magnetic Moment

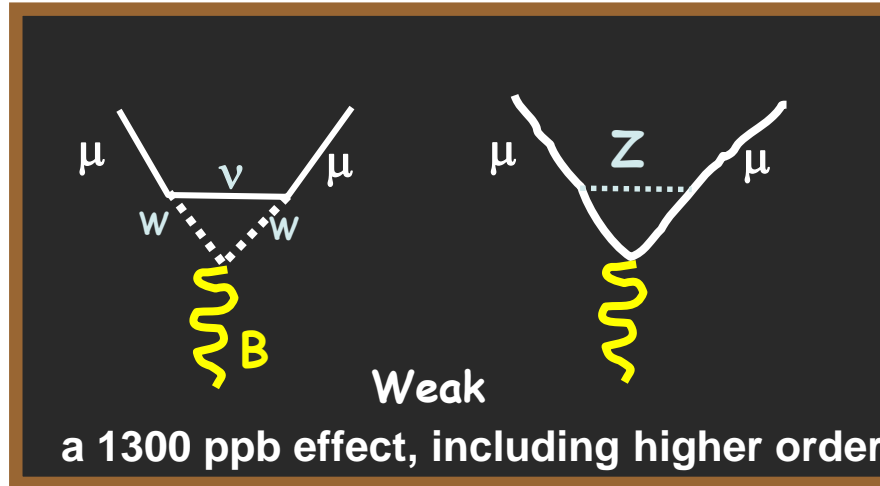
Tatsumi Aoyama,^{1,2} Toichiro Kinoshita,^{3,4} and Makiko Nio²



(Dated: December 19, 2017)



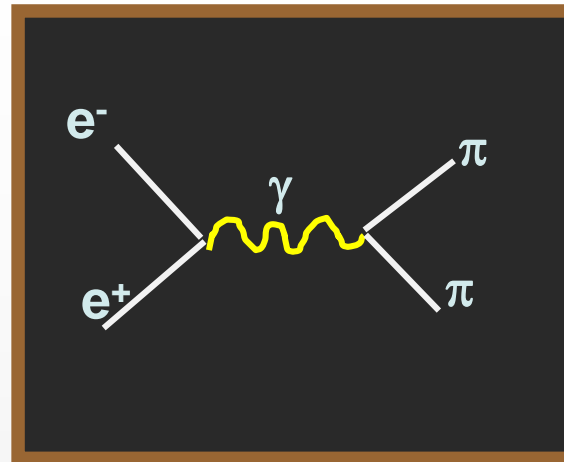
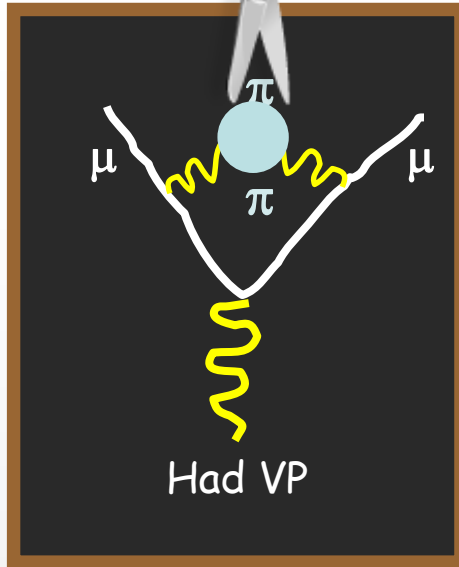
Now add in Electroweak loops (e.g., replace γ with a Z ... etc)



Well known now, but not an easy calculation

Weak 1st Order	194.820
Weak 2nd Order	-41.760
	$\times 10^{-11}$

And now Hadrons ... which cannot be calculated using perturbation theory because of the strong coupling



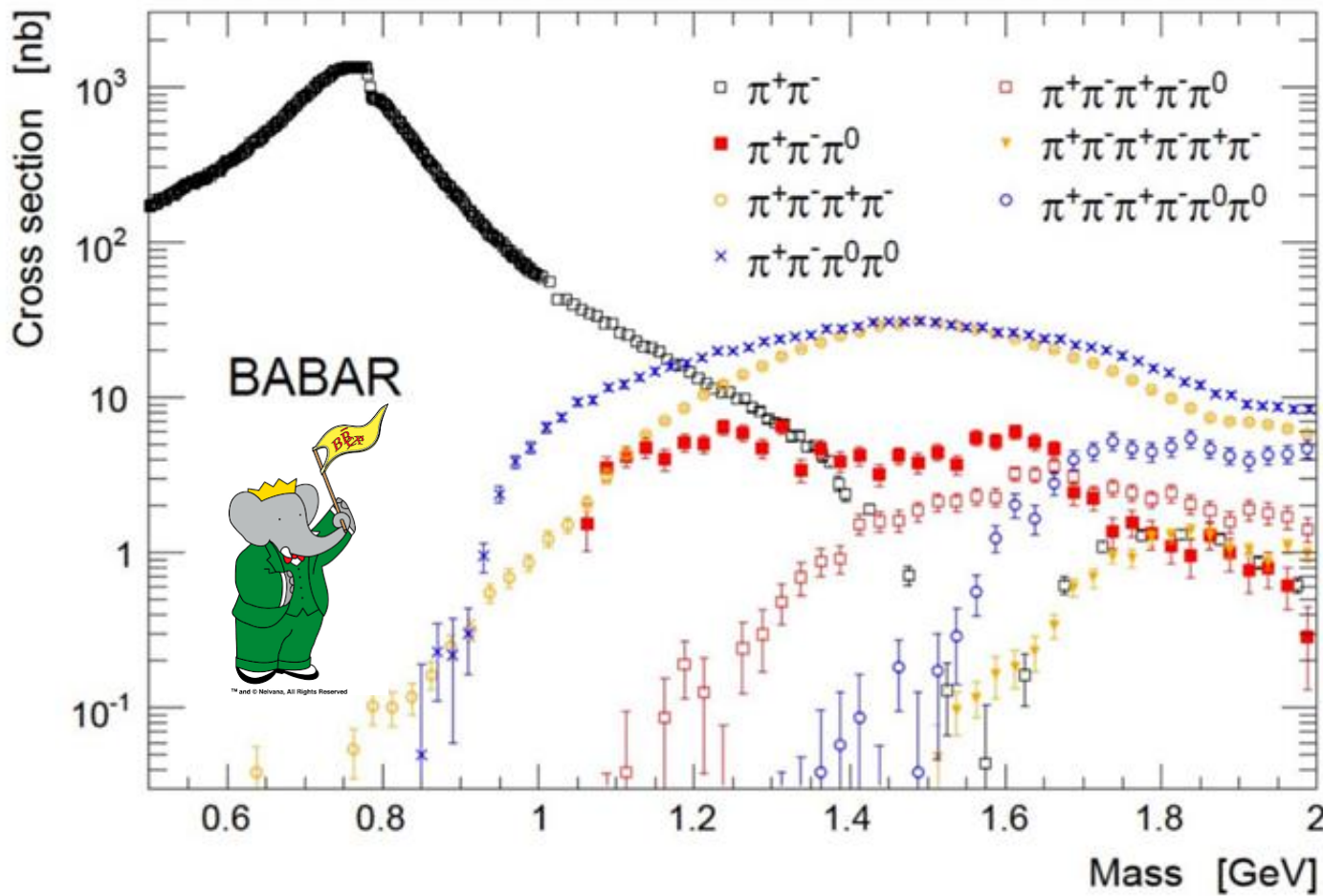
This contribution can be exactly linked to experimental data

1. Cut diagram down middle
2. Looks like $\gamma \rightarrow \pi\pi$
3. Dispersion relation connects $e^+e^- \rightarrow \pi\pi$ cross section measurement to anomaly contribution of 1st-order HVP

$$a_{\mu}^{\text{had,LO}} = \frac{\alpha^2(0)}{3\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \frac{K(s)}{s} R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \text{muons})}$$

The cross sections scan a wide range in energy



Examples above is from BaBar, but many collaborations are involved ...

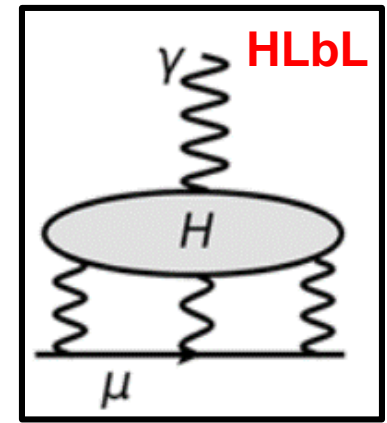
... and even the lattice

- KLOE (Frascati)
- SND and CMD2 (Novosibirsk)
- BES III (Beijing) ... (and very recently CLEO folks posted a contribution)

Higher-order Hadronic, especially HLbL

Modern attack uses the LATTICE (vs previous models approach)

- Physical pion mass and large lattice
- Statistical precision 2 x better already than best model estimates
- Systematics in progress over the next year.
- No big “shift” seen in early results compared to models



PRL 118, 022005 (2017)

PHYSICAL REVIEW LETTERS

13 JANUARY 2017

Connected and Leading Disconnected Hadronic Light-by-Light Contribution to the Muon Anomalous Magnetic Moment with a Physical Pion Mass

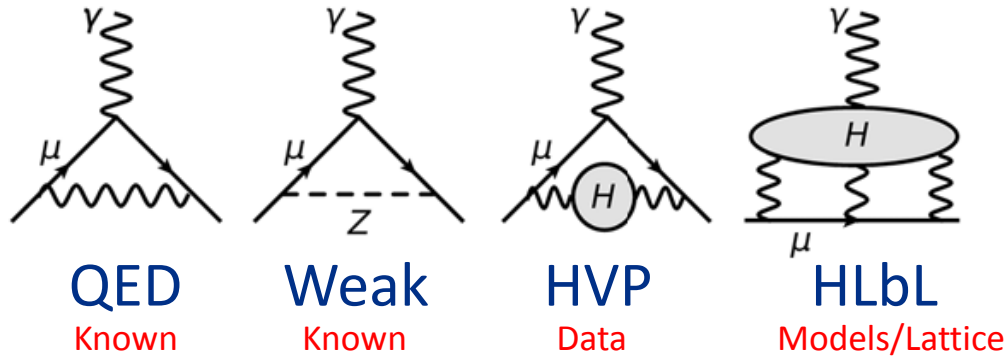
Thomas Blum,^{1,2} Norman Christ,³ Masashi Hayakawa,^{4,5} Taku Izubuchi,^{6,2}
Luchang Jin,^{3,*} Chulwoo Jung,⁶ and Christoph Lehner⁶

We find $a_{\mu}^{\text{HLbL}} = \underline{5.35(1.35)} \times 10^{-10}$, where the error is statistical only. The finite-volume and finite lattice-spacing errors could be quite large and are the subject of ongoing research. The omitted disconnected graphs, while expected to give a correction of order 10%, also need to be computed.

“statistical”

In 10^{-10} units $\rightarrow 5.35 \pm 1.35$ to be compared with 10.5 ± 2.6 usual HLbL from models

Standard Model contributions to a_μ ... updates $\rightarrow 3.6 \sigma$



	VALUE ($\times 10^{-10}$) UNITS
QED ($\gamma + \ell$)	$11\,658\,471.8951 \pm 0.0009 \pm 0.0019 \pm 0.0007 \pm 0.0077_\alpha$
HVP(lo) Davier17	692.6 ± 3.33
HVP(lo)KNT2017	693.9 ± 2.6
HVP(ho) KNT2017	-9.84 ± 0.07
HLbL Glasgow	10.5 ± 2.6
EW	15.4 ± 0.1
Total SM Davier17	$11\,659\,181.7 \pm 4.2$
Total SM KNT17	$11\,659\,182.7 \pm 3.7$

\leftarrow This is a fancy guess; it will improve \rightarrow

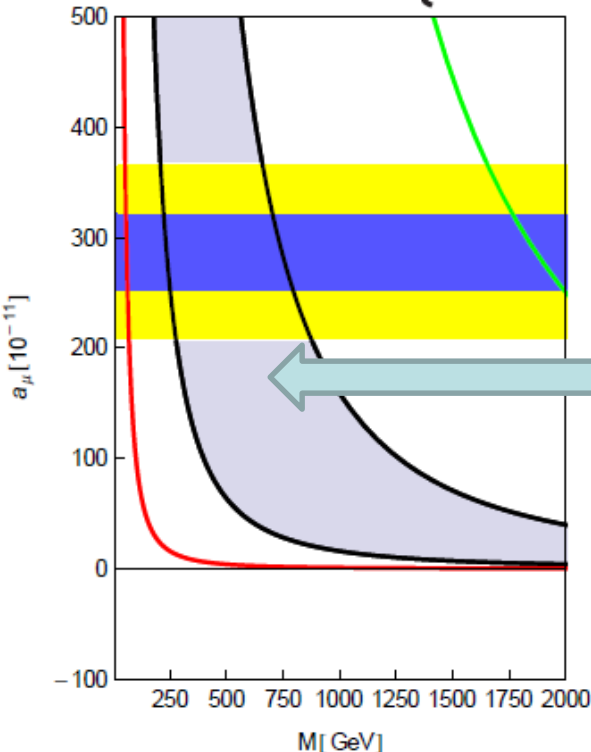
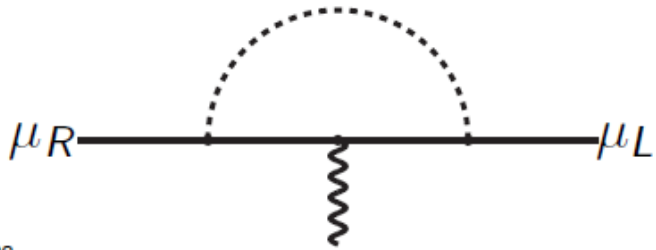
BNL E821 $\delta a_\mu(\text{Expt}) = \pm 6.3$

What could it mean if Expt \neq Theory at $> 5\sigma$?

Generically, “loop effects” couple to the muon **mass** and **moment** in similar fashion, characterized a coupling, $\propto C$

$$\mathcal{O}(C) \left(\frac{m_\mu}{M}\right)^2$$

$$C = \frac{\delta m_\mu(\text{N.P.})}{m_\mu}$$



$\mathcal{O}(1)$

radiative muon mass generation ...

[Czarnecki, Marciano '01]

[Crivellin, Girschbach, Nierste '11][Dobrescu, Fox '10]

$\mathcal{O}\left(\frac{\alpha}{4\pi} \times \text{Factor}\right)$

supersymmetry ($\tan \beta$)

vectorlike fermions ...

$\mathcal{O}\left(\frac{\alpha}{4\pi}\right)$

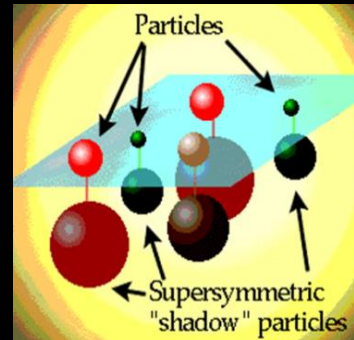
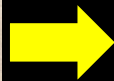
SM: Z, W . New physics: $Z', W' \dots$

$< \frac{\alpha}{4\pi}$

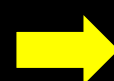
2-Higgs doublet model, dark photon .

And the situation now?

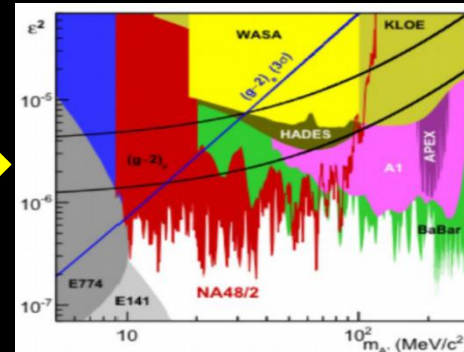
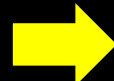
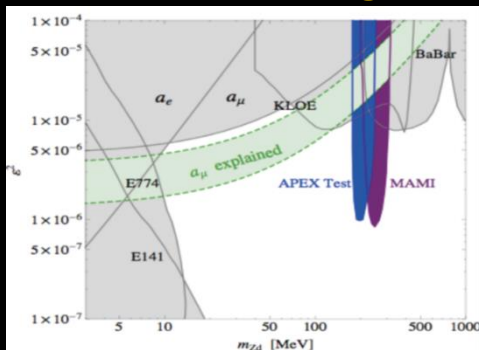
Some things "seen" just wash away ... And some things are under Tension



LHC limits growing, but SUSY, if exists, is hiding well



And some things don't seem to be so ...



g-2: An uncomfortably lonely search for a Crack in the SM

The Measurement

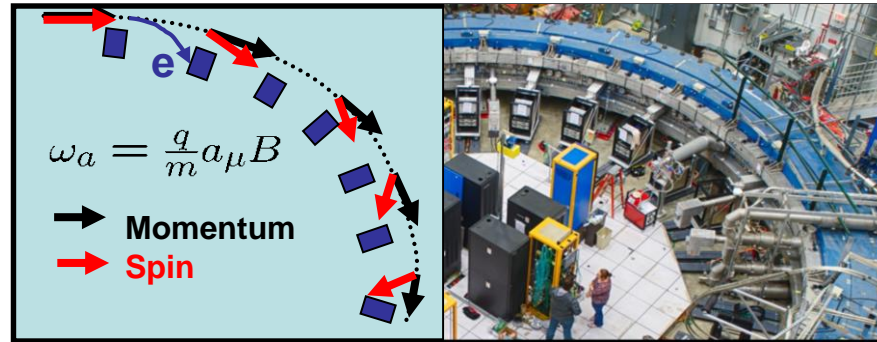


Muon $g-2$ Collaboration
(190 collaborators, 34 institutes, 7 countries)

**We hail from: Particle-, Nuclear-, Atomic-, Optical-, Accelerator-,
and Theory-Physics Communities**

But, we are all measuring $g-2$

The Fundamental Principle



Determine difference between spin precession and cyclotron motion for a muon moving in a magnetic field:

The expression including E -field focusing and possible μ EDM

$$\vec{\omega}_{net} = -\frac{q}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} + \frac{\eta}{2} \left(\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c} \right) \right]$$

↑ Measure these ↑ Get a_μ ↓ Magic γ EDM

$$\vec{\omega}_{net} = \vec{\omega}_a + \vec{\omega}_{EDM}$$

A formal way to write this looks like this:

$$\text{In E821} \equiv \mathcal{R}_\mu(\text{E821}) = 0.003\,707\,206\,4(20) \text{ [0.54 ppm]}$$

$$\bullet \quad a_\mu(\text{Expt}) = \frac{g_e}{2} \frac{\omega_a}{\tilde{\omega}_p} \frac{m_\mu}{m_e} \frac{\mu_p}{\mu_e}$$

-2.002 319 304 361 53(53) [0.26 ppt]
 Electron g-2 + QED

-0.001519270384(12) [8 ppb]
 206.768 2843(52) [25 ppb]

We will measure these two frequencies and report the Ratio

Requirements for a better measurement

1. Store More Muons

- 21 x BNL in statistics ... (100 ppb)

2. Prepare A More Uniform Magnetic Field

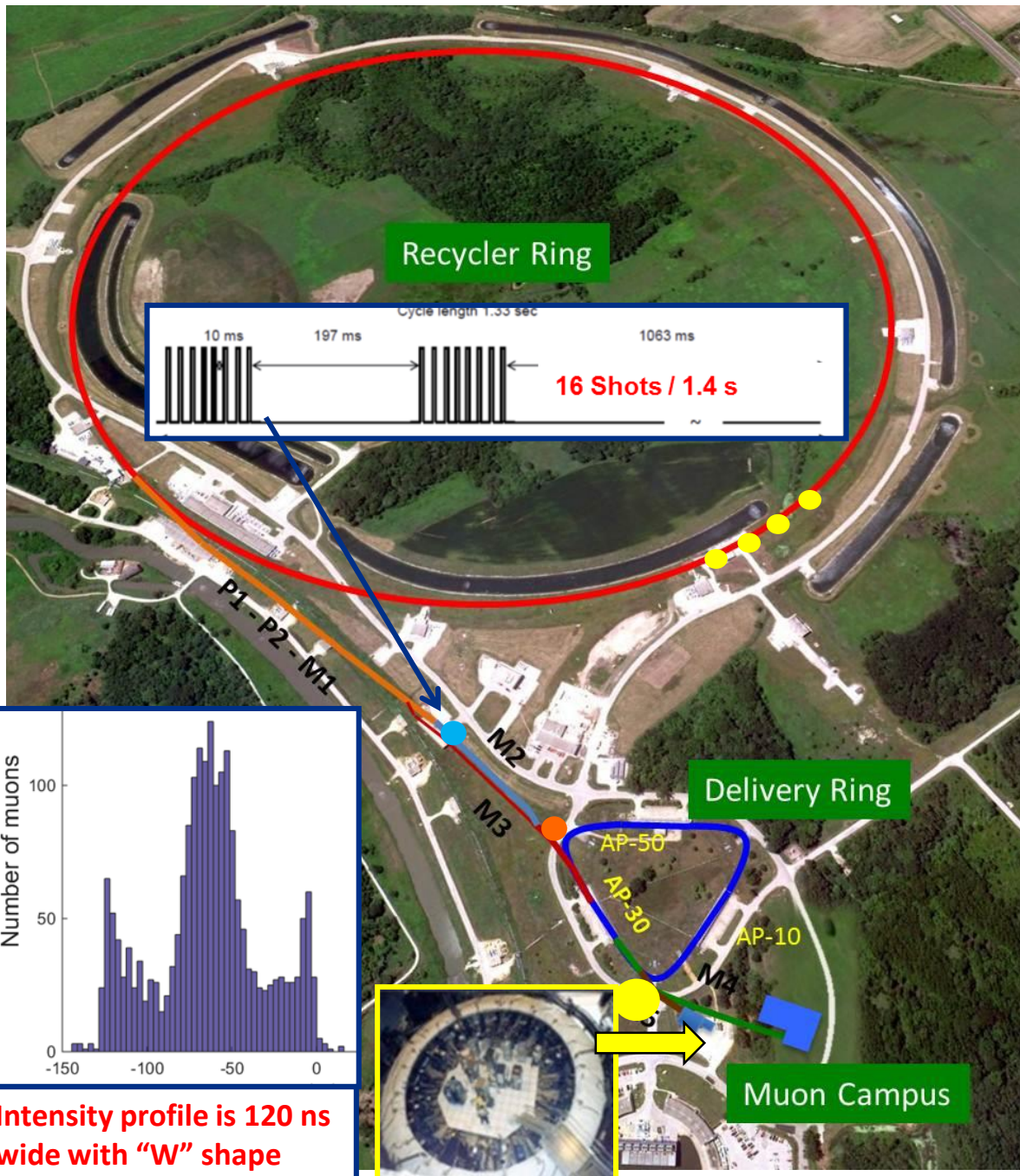
- Goal → 3 x better and more carefully measured (70 ppb)

3. Improve the Precession Frequency Measurement

- All new instrumentation with high-fidelity recording of muon decays by many systems (70 ppb)

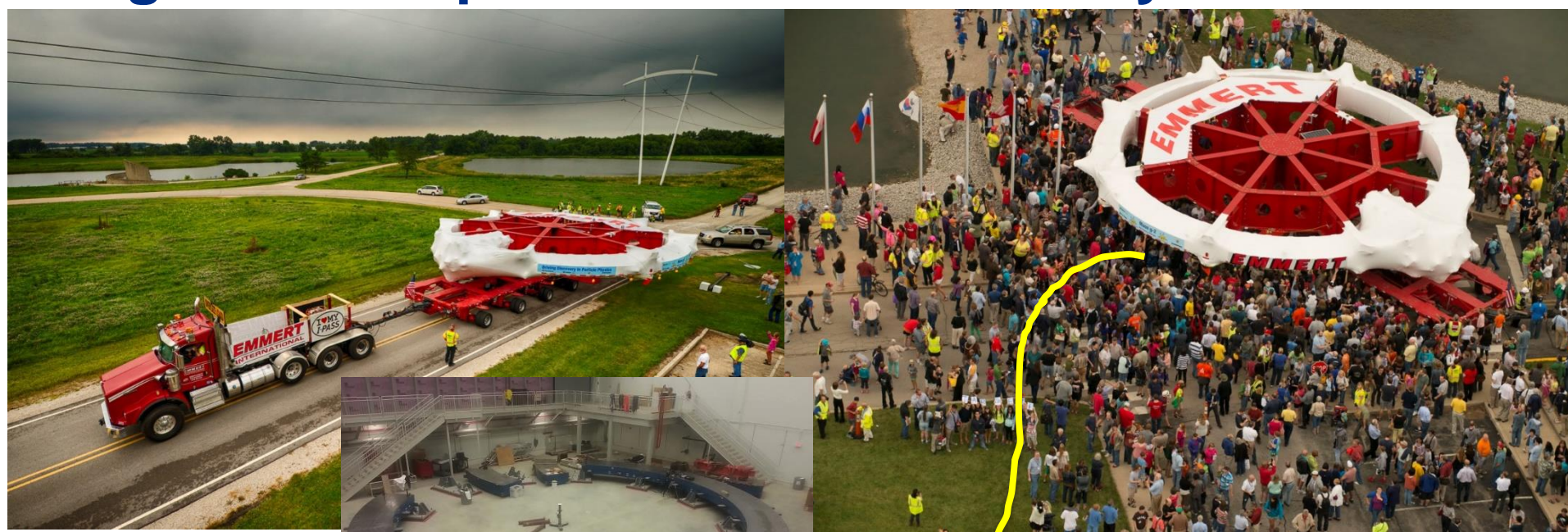
Creating the Muon Beam for g-2

- 8 GeV p batch into Recycler
- Split into 4 bunches
- Extract 1 by 1 to strike target
- Long FODO channel to collect $\pi \rightarrow \mu\nu$
- $\rho/\pi/\mu$ beam enters DR; protons kicked out; π decay away
- μ enter storage ring

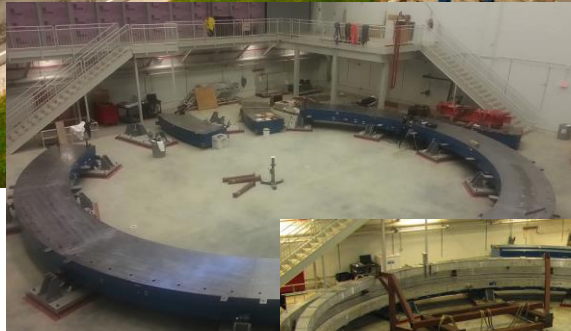


Intensity profile is 120 ns wide with "W" shape

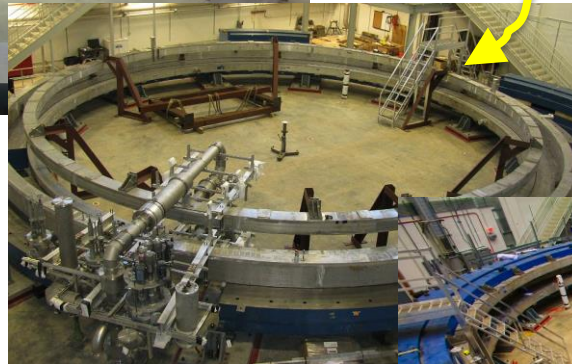
To measuring the Field, ω_p we start with the BNL magnet but improve its field uniformity



Yoke Iron
Aligned to sub-mil
precision



Superconducting coils
And cryostat



Magnet shimming kit

- NMR probes
- Probe Multiplexer
- Pulser-Mixer

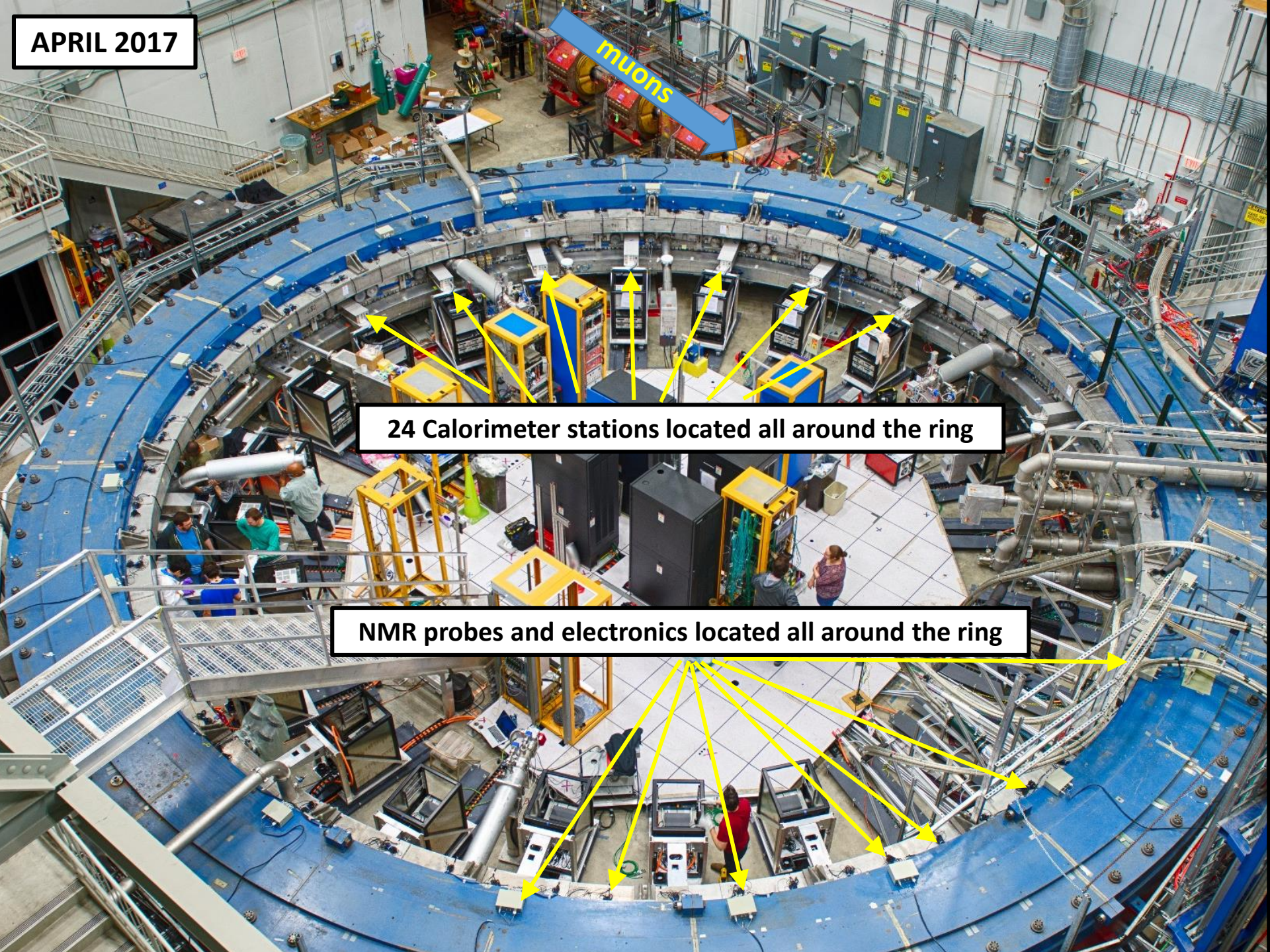


APRIL 2017

muons

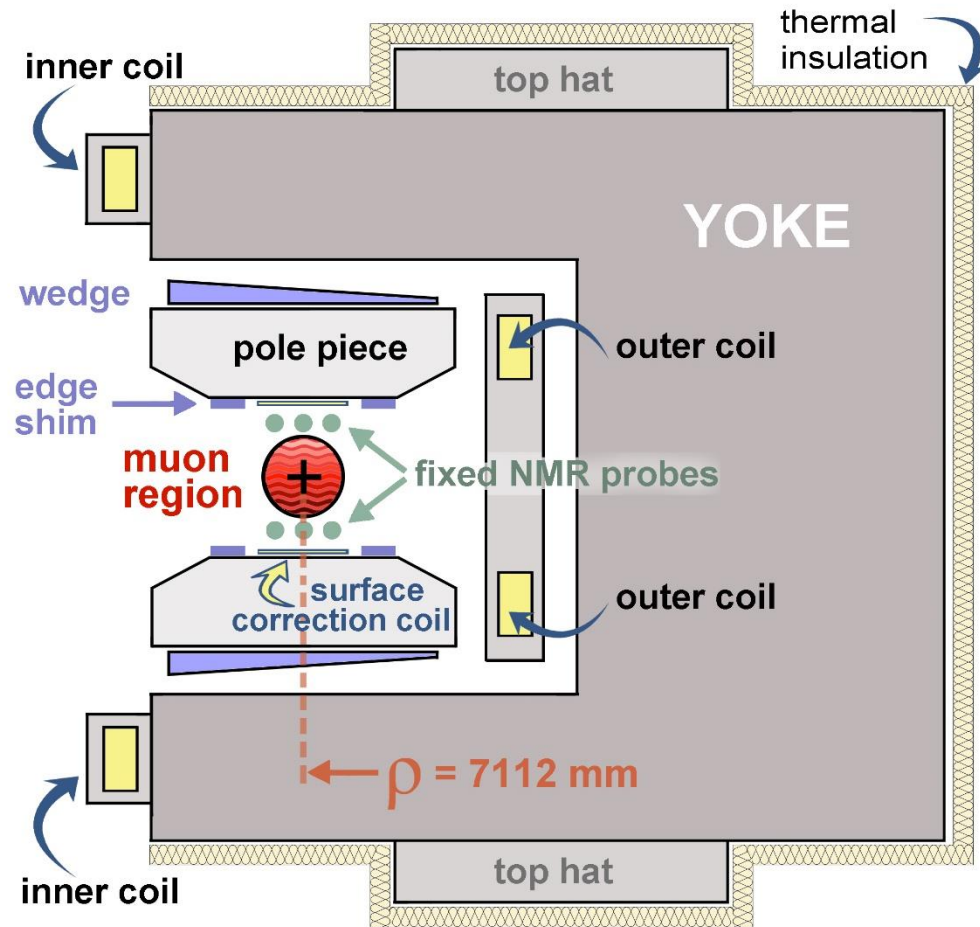
24 Calorimeter stations located all around the ring

NMR probes and electronics located all around the ring

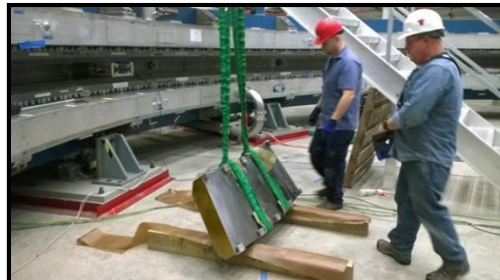


Built-in Shimming Tools provide many knobs

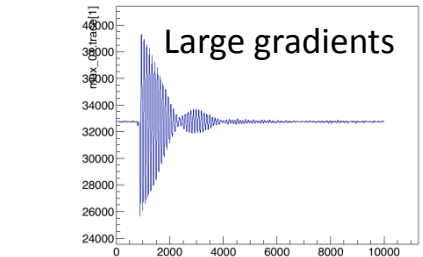
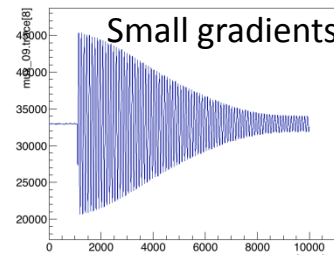
- **B Field 1.45T**
- **12 Yokes:** C shaped flux returns
- **72 Poles:** shape field
- **864 Wedges:** angle - quadrupole (QP)
- **24 Iron Top Hats:** change effective μ
- **Edge Shims:** QP, sextapole (SP)
- **8000 Surface iron foils:** change effective μ locally
- **Surface coils:** will add average field moments (360 deg)



g-2 Magnet in Cross Section



The Field is measured using a proxy: pulsed NMR of protons



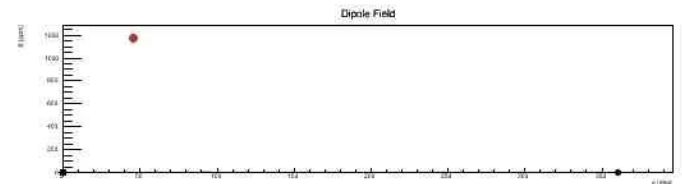
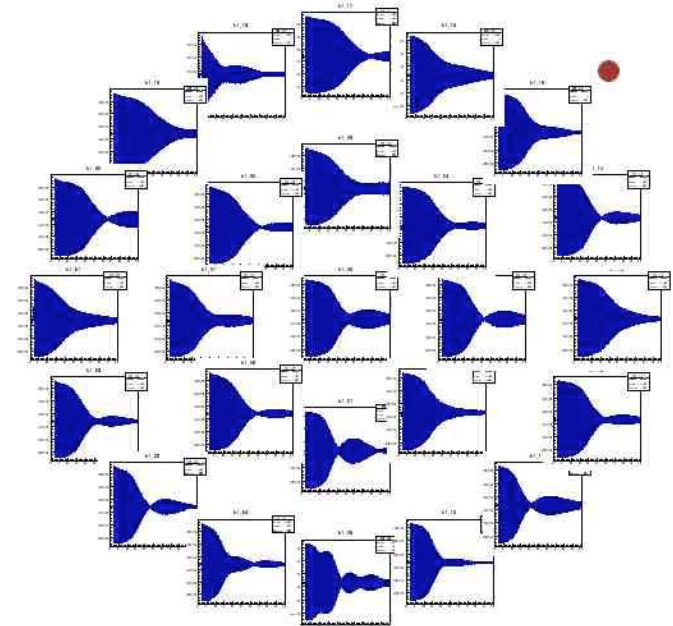
Free Induction Decay (FID) Waveforms

Extracted frequency precision is ~10 ppb per FID

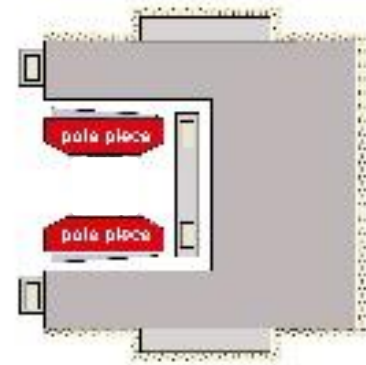
A 25-element pNMR *Trolley* maps the field during shimming.

You see the probes here as the trolley goes around the ring in azimuth

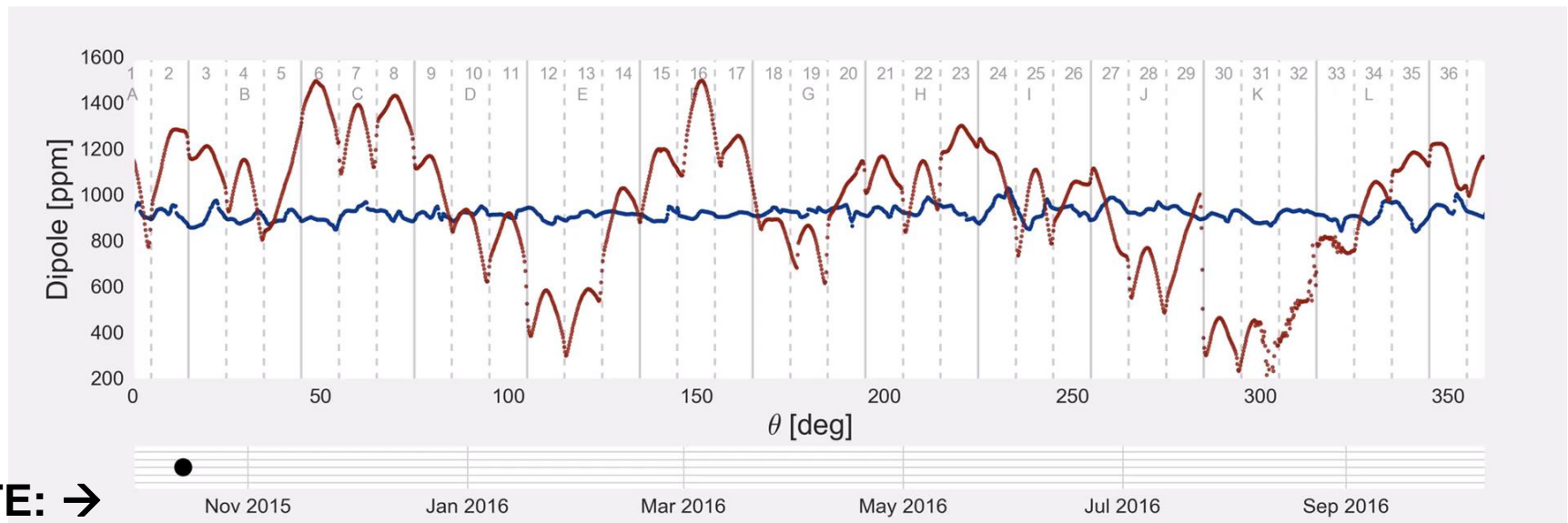
Probe Matrix



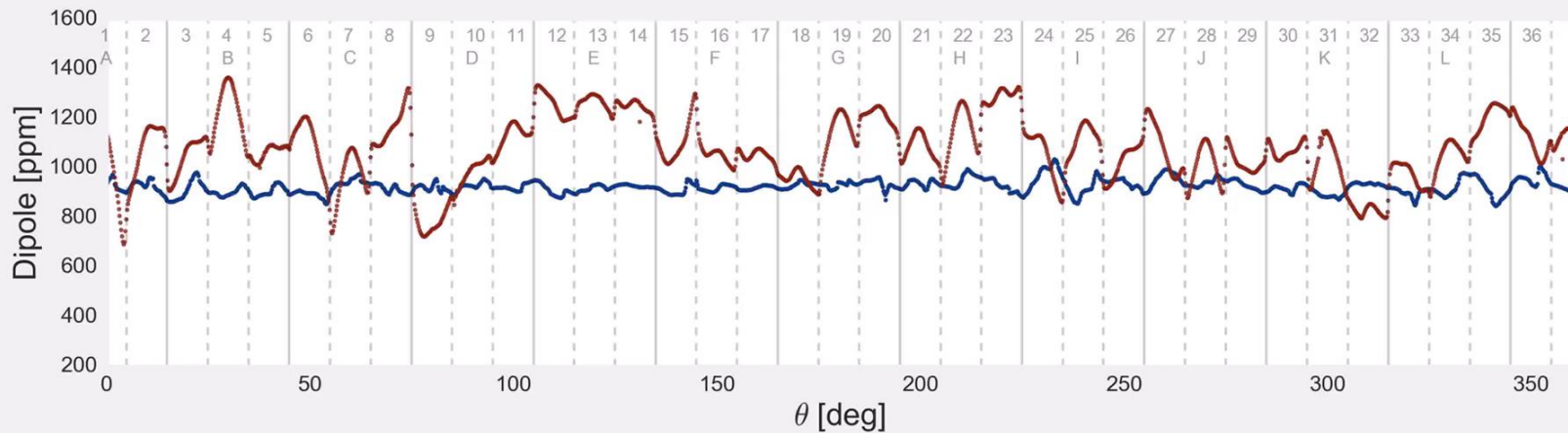
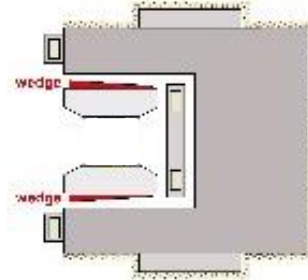
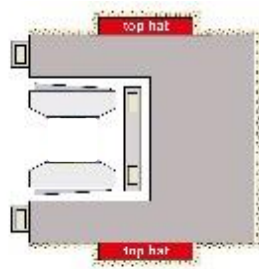
Start by (painfully) aligning **only** pole surface:



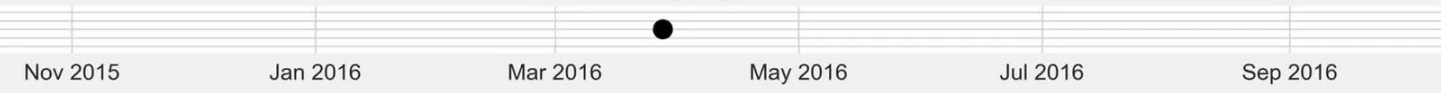
- **Red** is Initial dipole field starting point at Fermilab
- **Blue** is typical BNL final field *after* shimming



Then adjust **only Top Hats and Wedges**

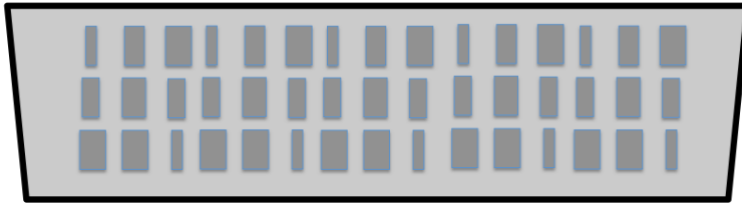


DATE: →

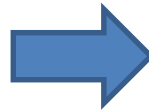
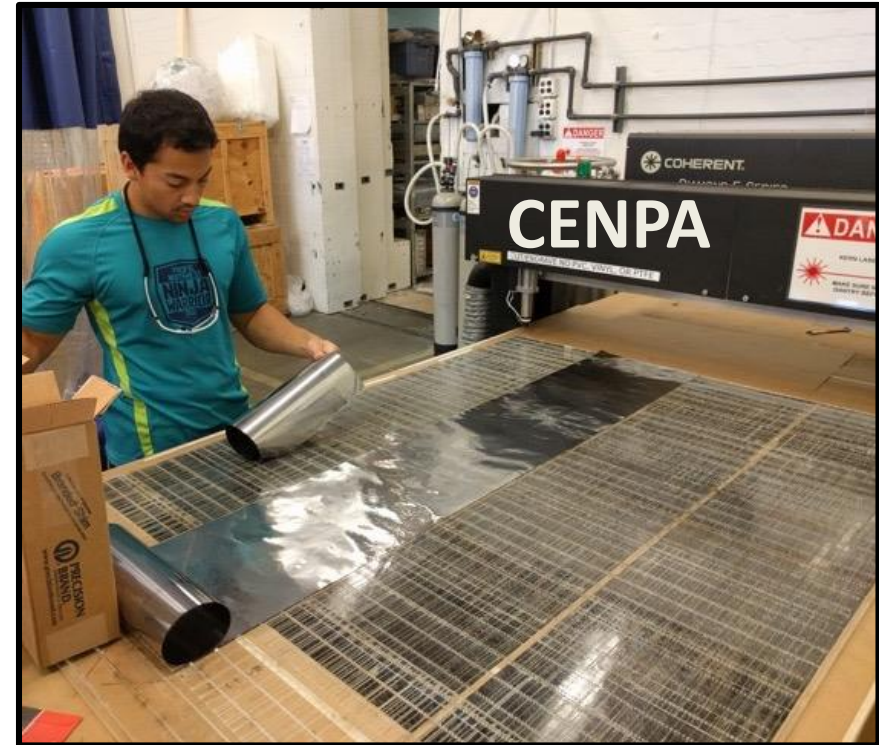
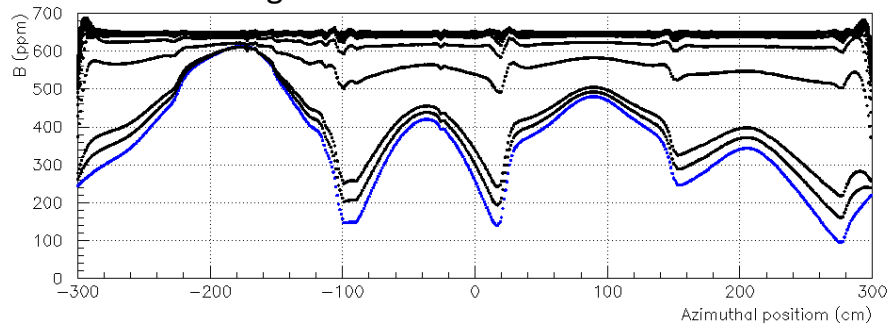


The big and final breakthrough idea: Iron Laminations:

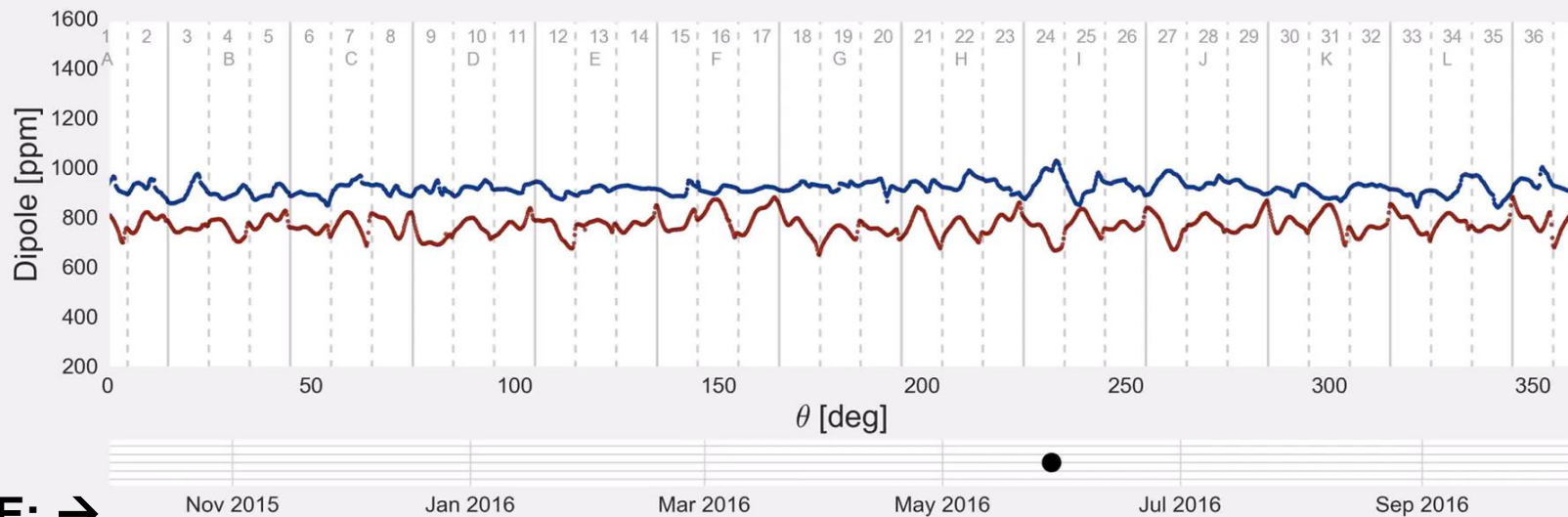
- Cover each pole with a patchwork of foils in 41 azimuthal and 3 radial sections



- Determine optimal foil mass values by iterative procedure that minimizes field inhomogeneity around a target value – Dave Kawall U Mass



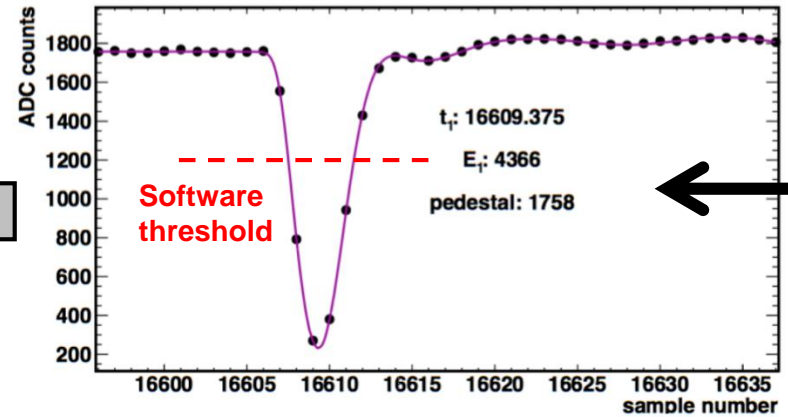
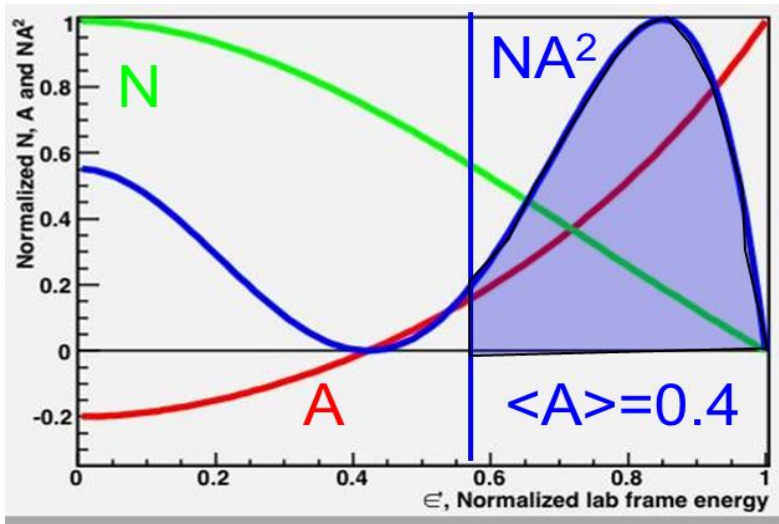
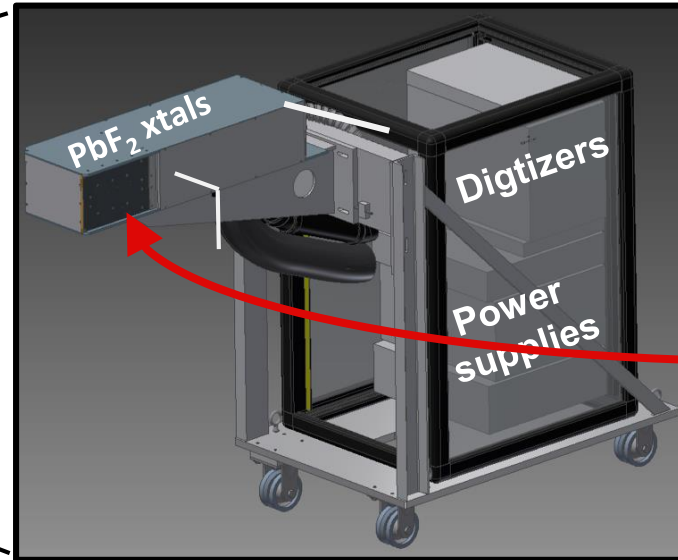
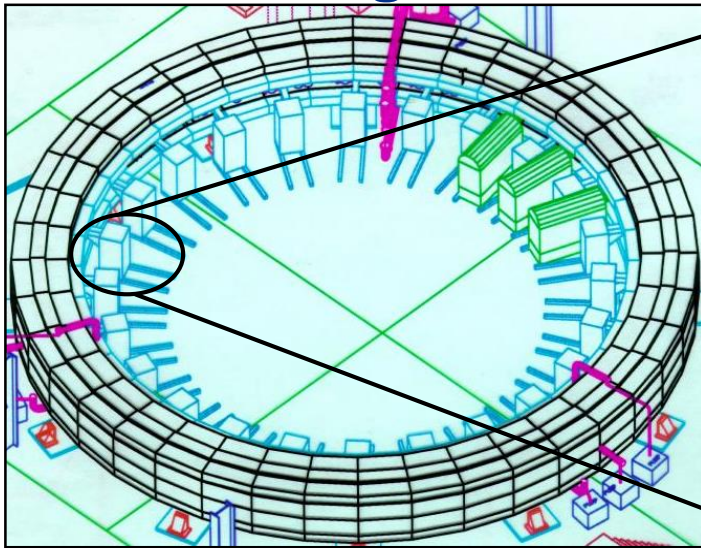
After adding the 8000 **iron foils**, the precision field emerges



The Goal is Achieved !

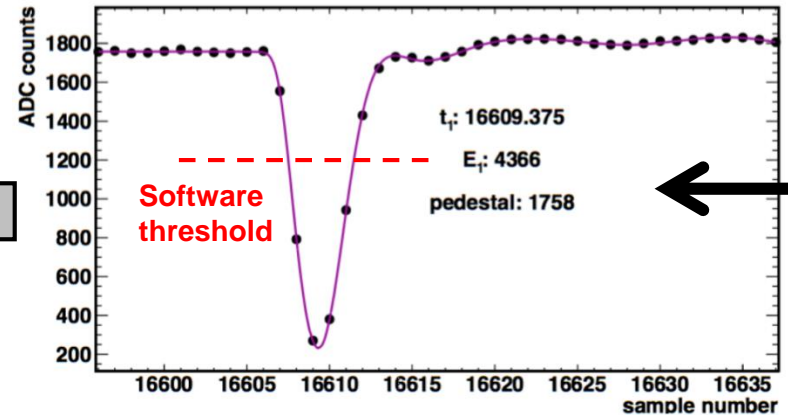
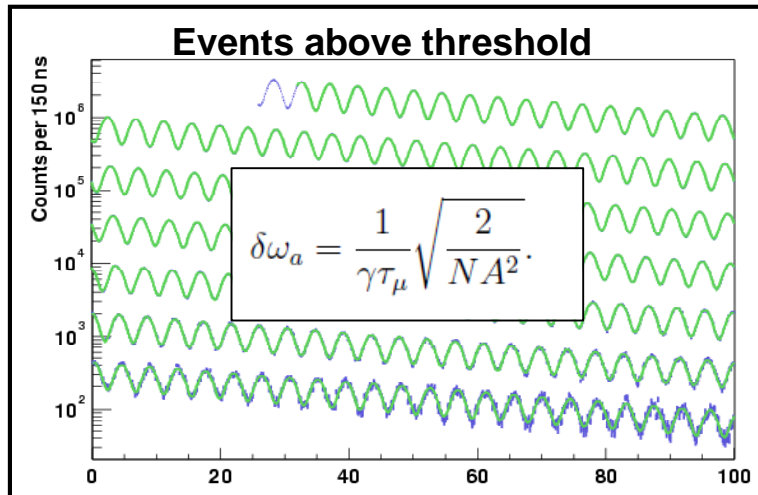
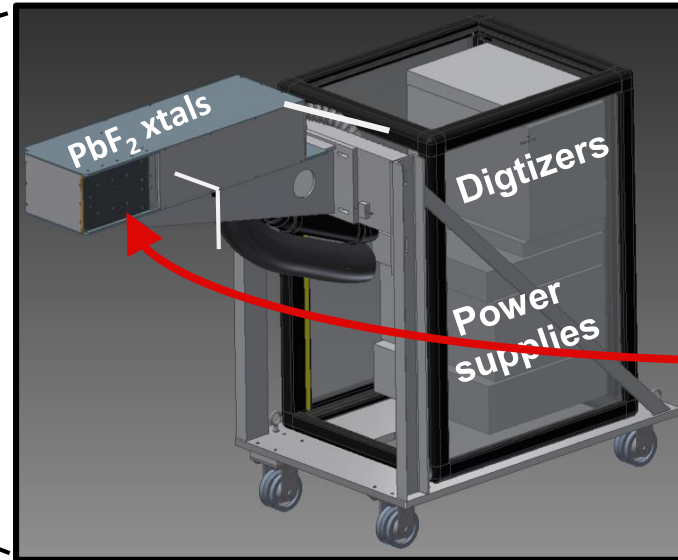
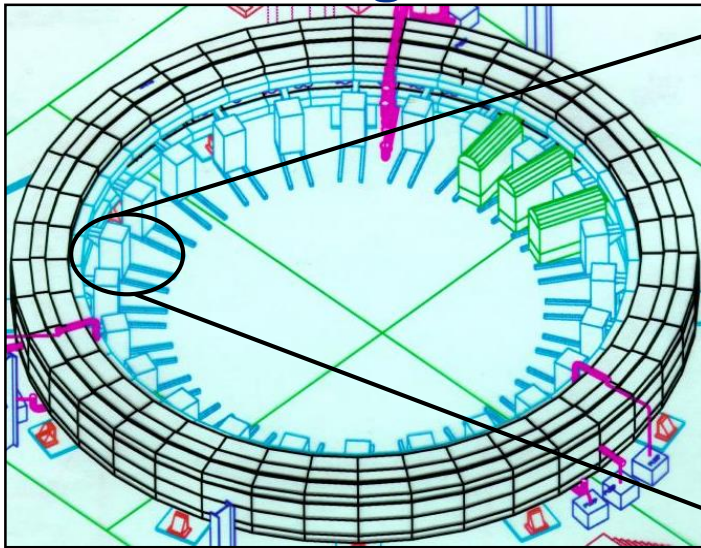
This is only the start of a full story ... it now must be measured precisely and mapped

Measuring the precession frequency, ω_a by sorting e^+ decay Times and Energies.



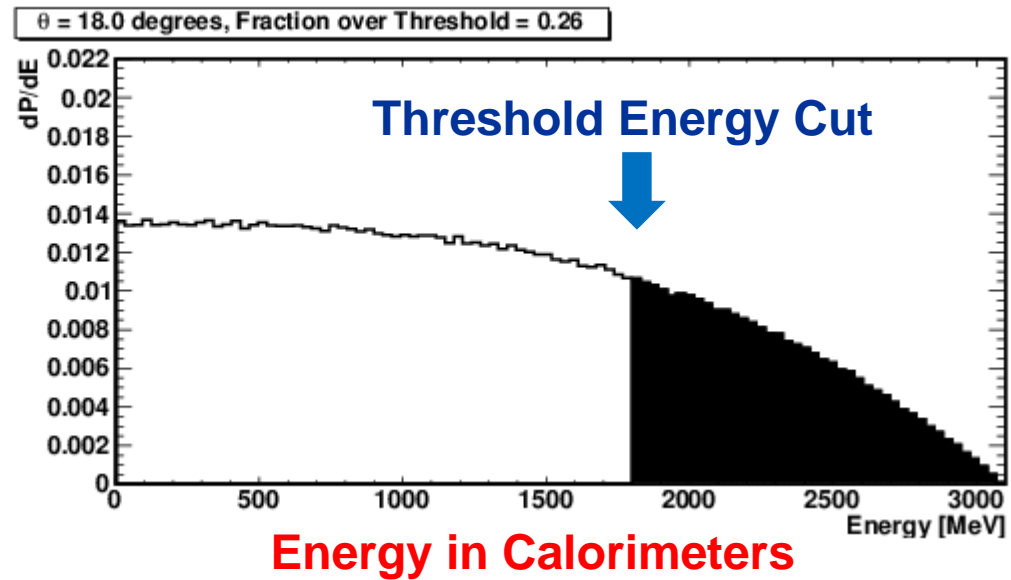
Systematic error total ω_a budget: [210 ppb \rightarrow 70 ppb]

Measuring the precession frequency, ω_a by sorting e^+ decay Times and Energies.

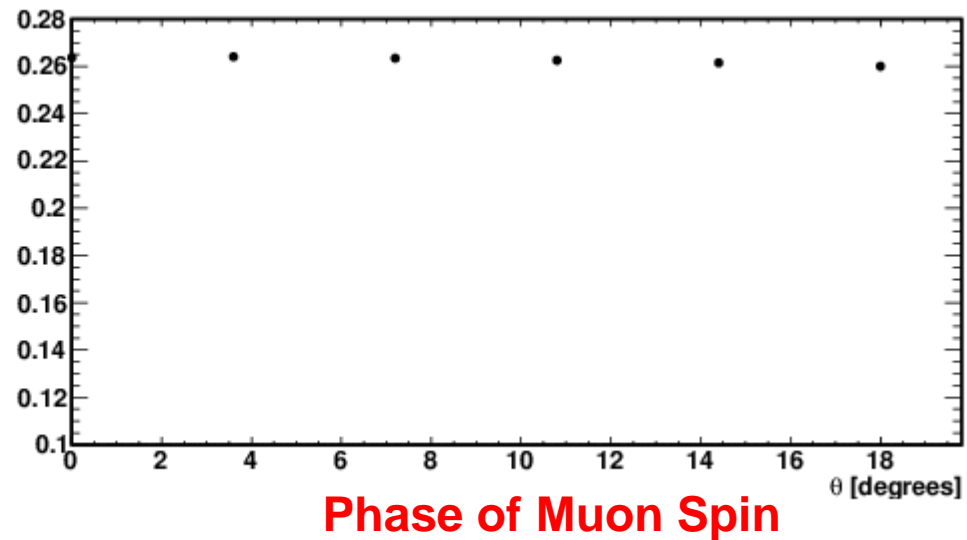


Systematic error total ω_a budget: [210 ppb \rightarrow 70 ppb]

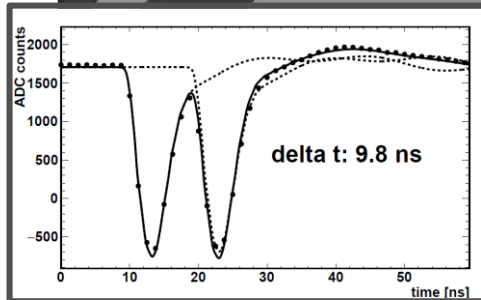
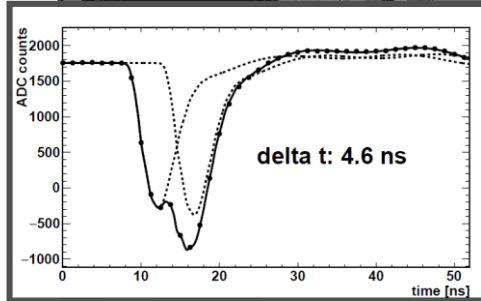
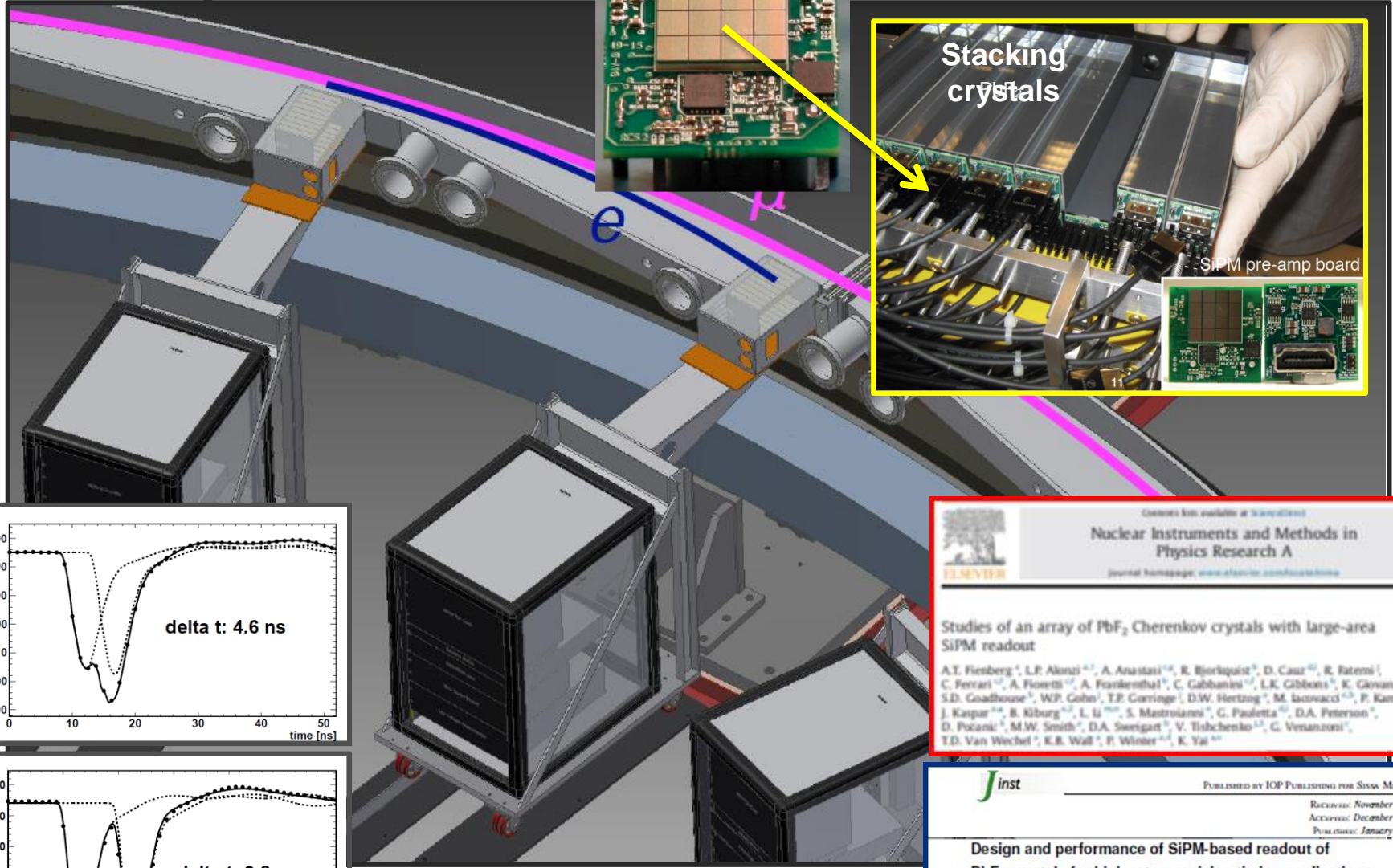
What makes the “wiggle” ?



**Fraction e^+
above Threshold**



24 Calorimeter with 54 PbF₂ Cherenkov crystals and very fast SiPMs



**Tested at SLAC Test Beam
Thanks to Carsten Hast !**

Contents list available at www.iopscience.iop.org

Nuclear Instruments and Methods in Physics Research A
Journal homepage: www.elsevier.com/locate/nucinst

Studies of an array of PbF₂ Cherenkov crystals with large-area SiPM readout

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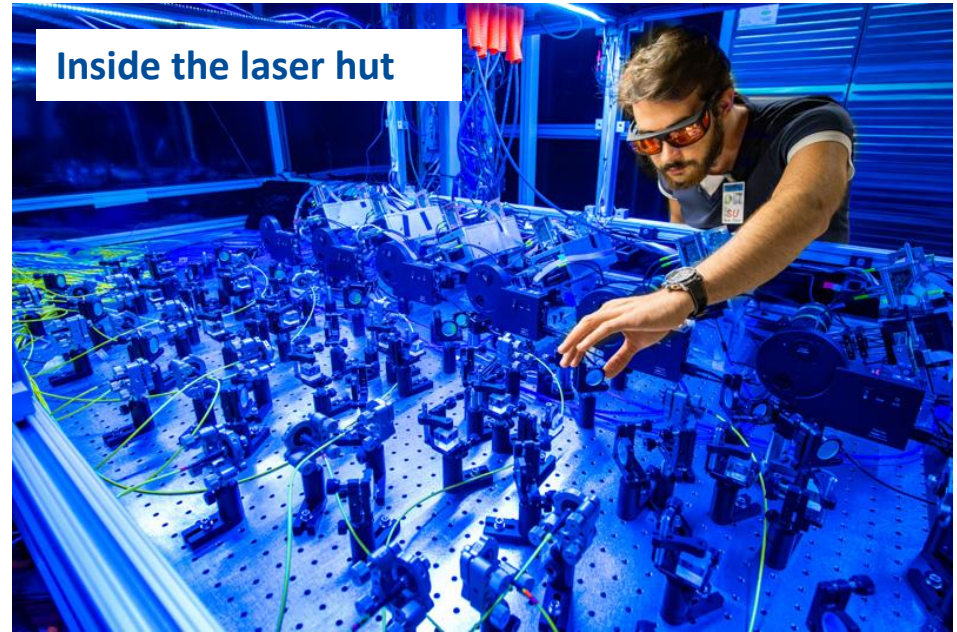
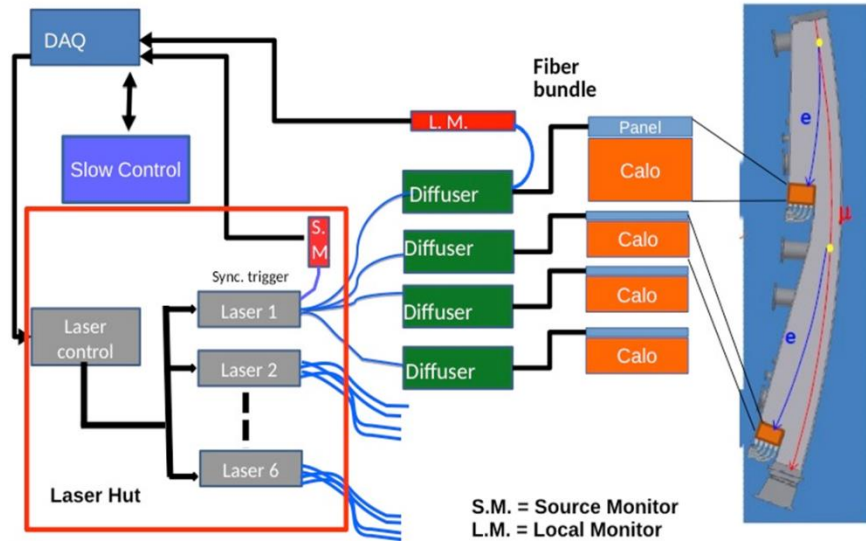
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ACCEPTED: December 28, 2016
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Design and performance of SiPM-based readout of PbF₂ crystals for high-rate, precision timing applications

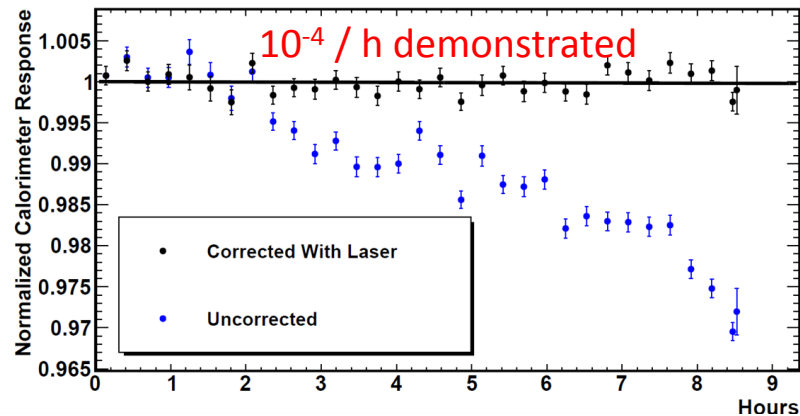
J. Kaspar,^{a,1} A.T. Fienberg,^a D.W. Hertzog,^a M.A. Huehn,^a P. Kammel,^a K.S. Khaw,^a D.A. Peterson,^a M.W. Smith,^a T.D. Van Wechel,^a A. Chapelain,^b L.K. Gibbons,^b D.A. Sweigart,^a C. Ferrari,^{a,2} A. Fioretti,^{a,2} C. Gabbanini,^{a,2} G. Venanzoni,^a M. Iacovacci,^{a,2} S. Mastrolanni,^c K. Giovanetti,^c W. Gohn,^b T. Gorrings,^b and D. Pocar,^a

GAIN stability established to $\sim \text{few} \times 10^{-4}$

State-of-the-art Laser-based calibration system also allows for pseudo data runs for DAQ



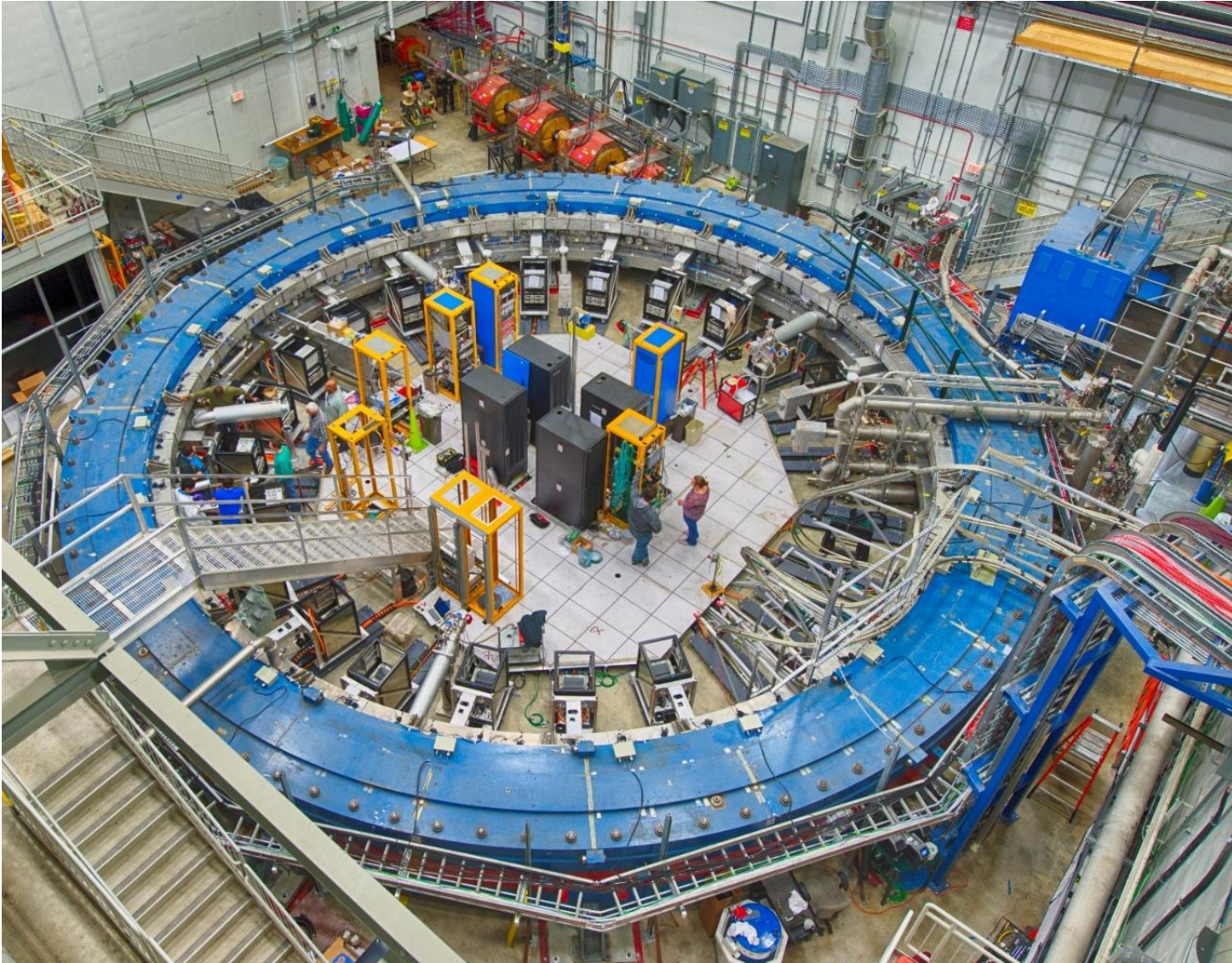
(in Test Beam)



BREAKING NEWS

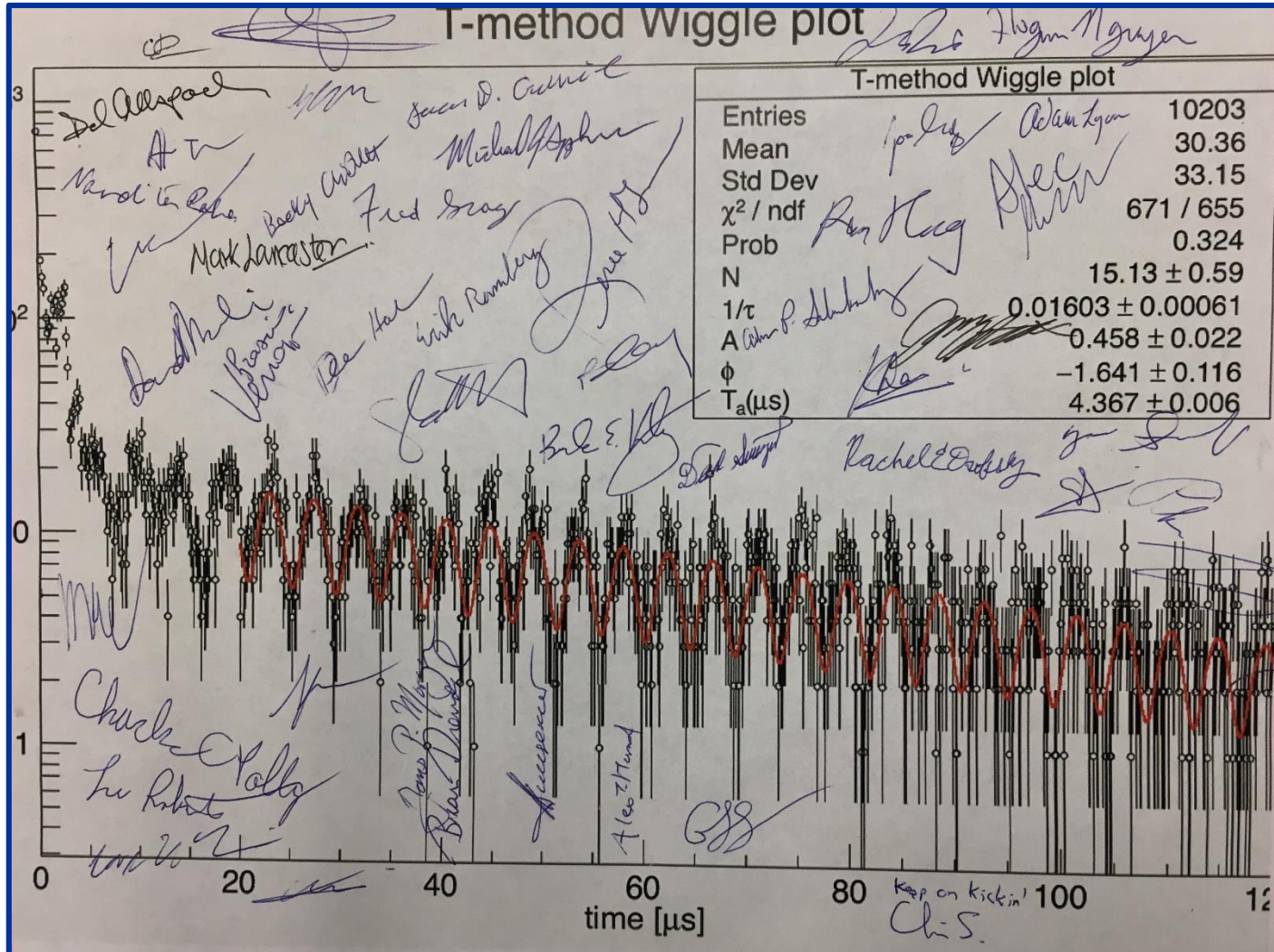
The experiment is completing commissioning phase soon

We are in the rare process of “christening a battleship”



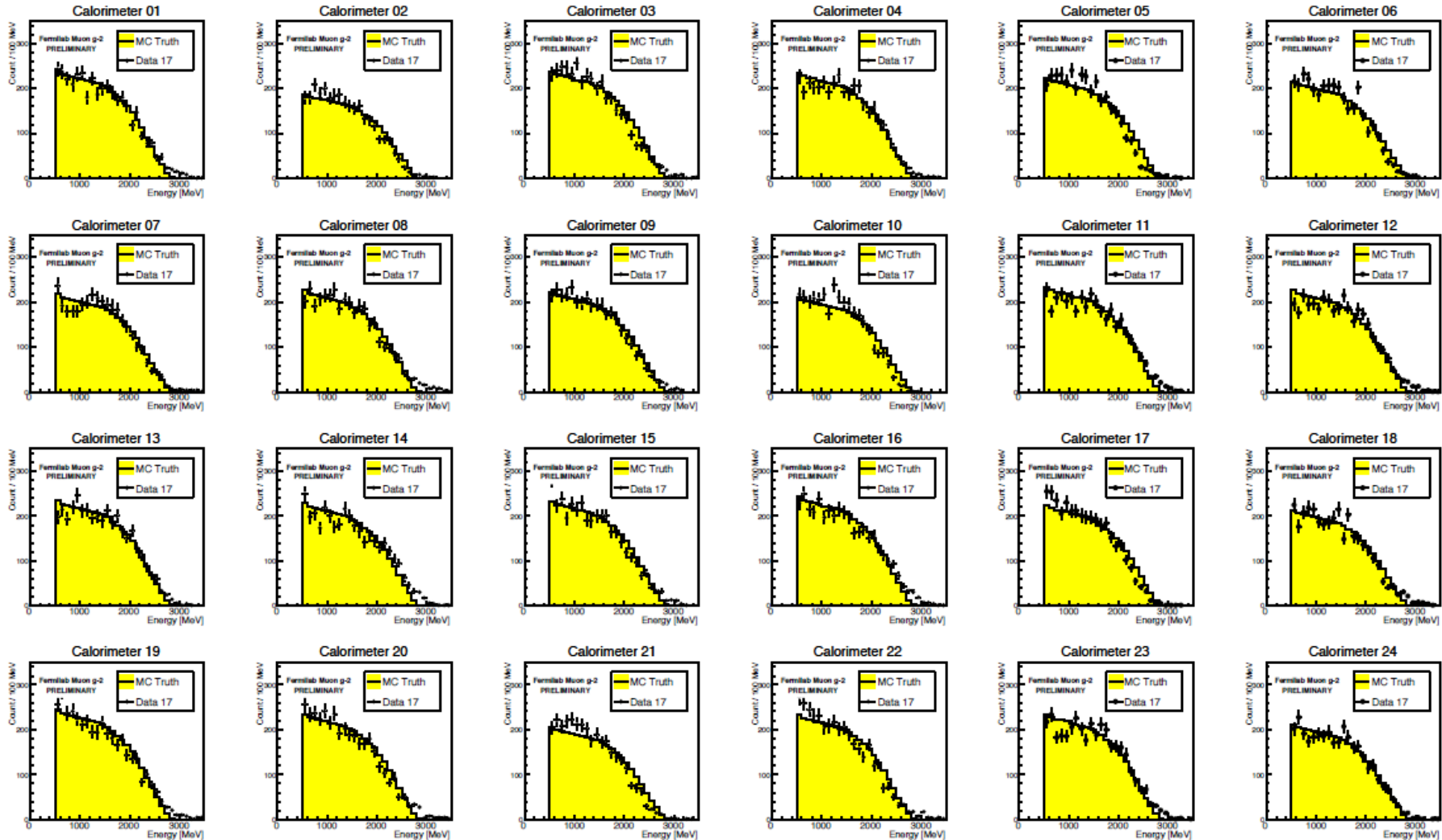
June 2017 .

First evidence of stored muon precession

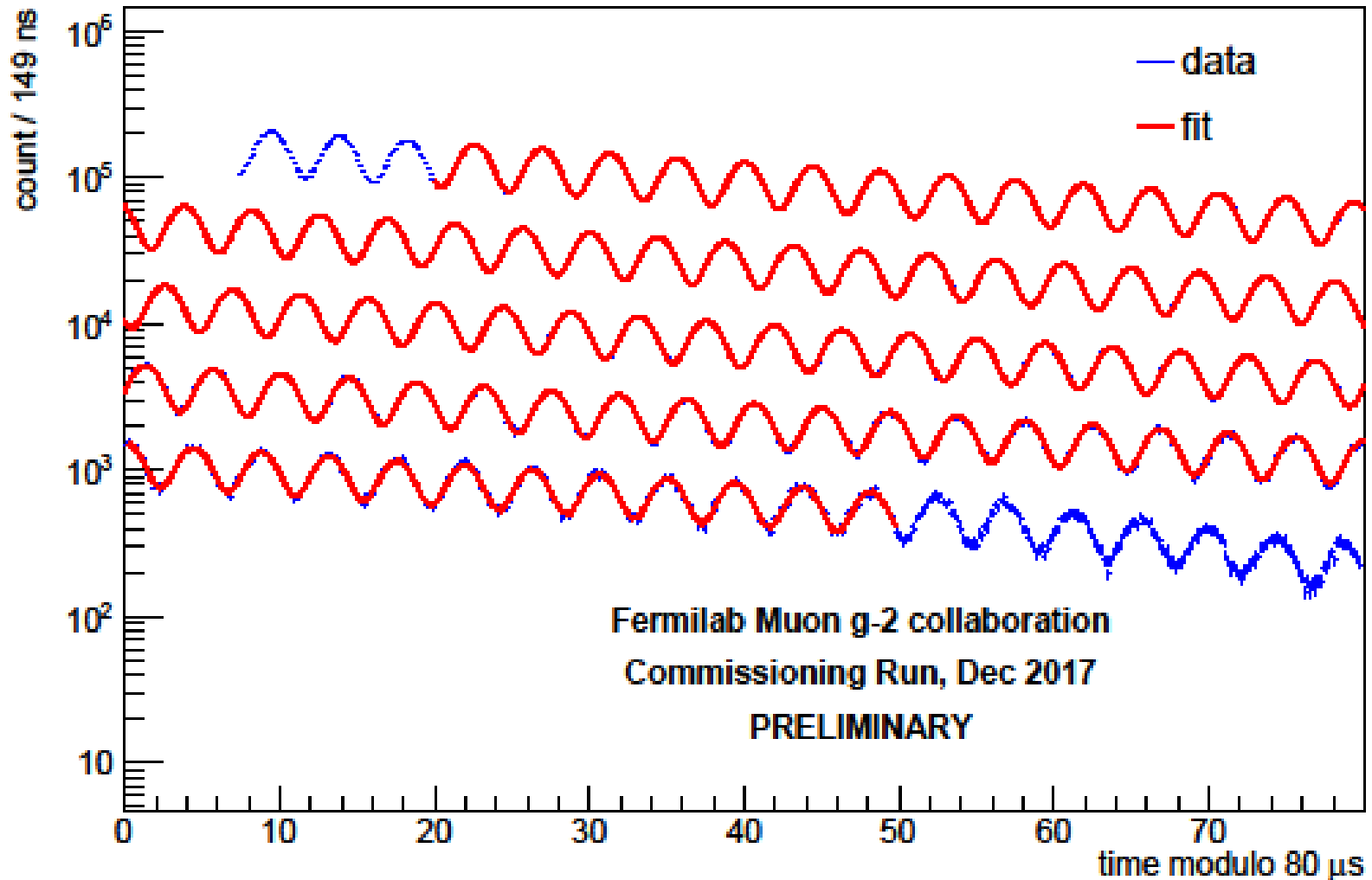


24 Calorimeters Data and Simulation in detail

(and, before any fancy calibration)

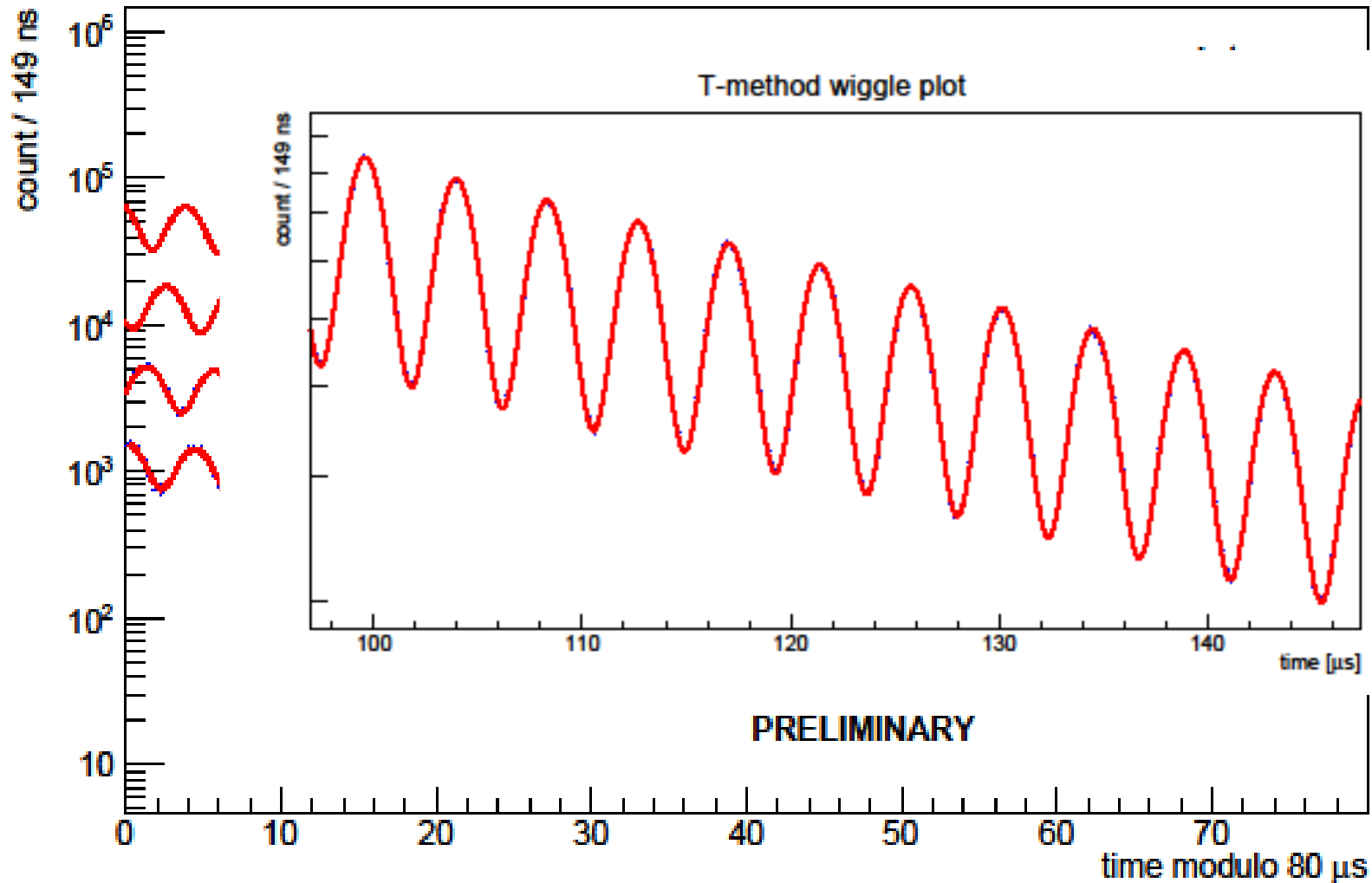


Sample ** from recent days ... with a statistical uncertainty, $\delta a_\mu/a_\mu$ of ~5 ppm



****Warning!!** this is not yet “physics” data. Many calibrations are not completed and magnetic field is not mapped during this period.

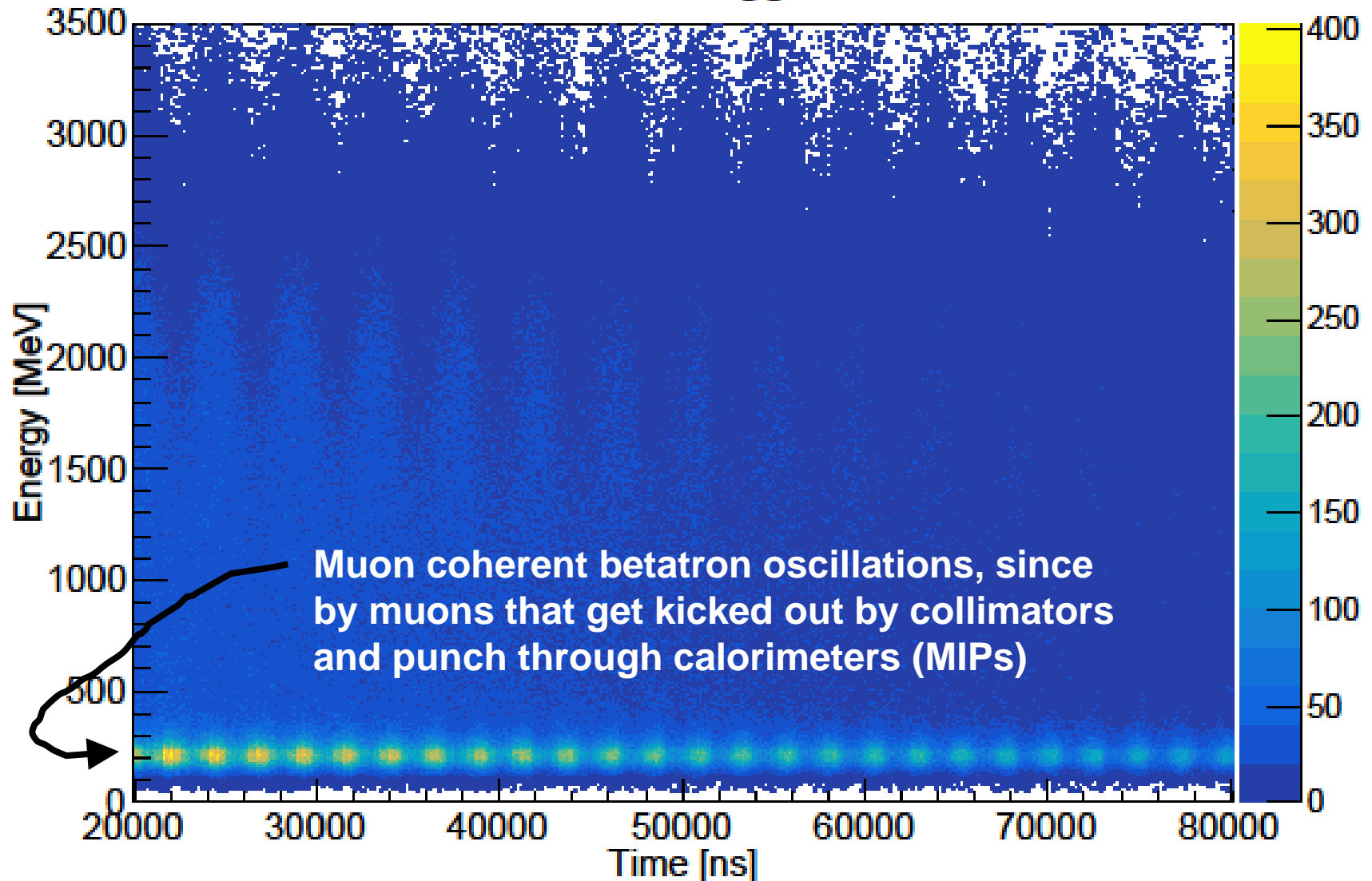
Sample ** from recent days ... with a statistical uncertainty, $\delta a_\mu/a_\mu$ of ~5 ppm



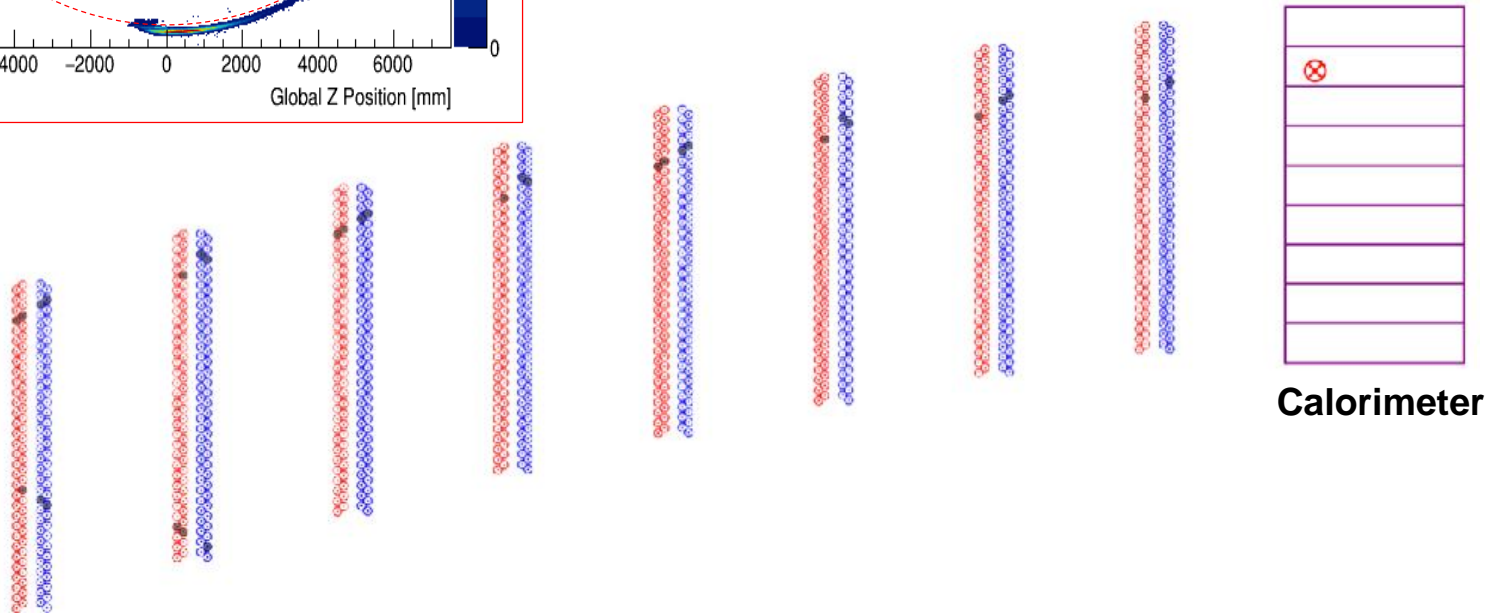
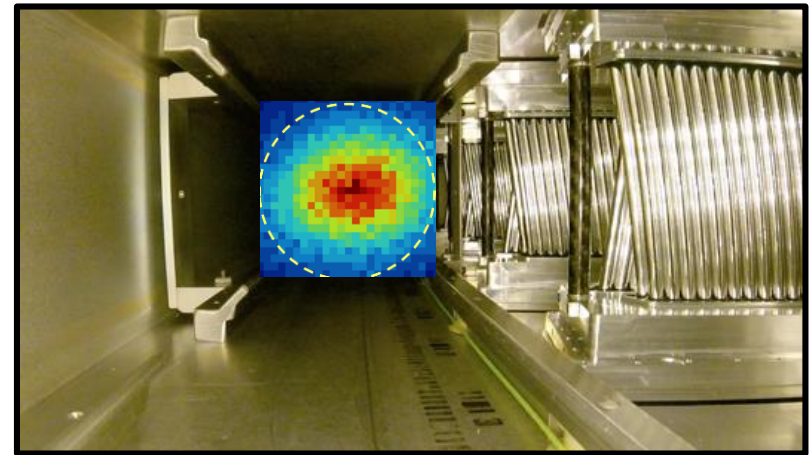
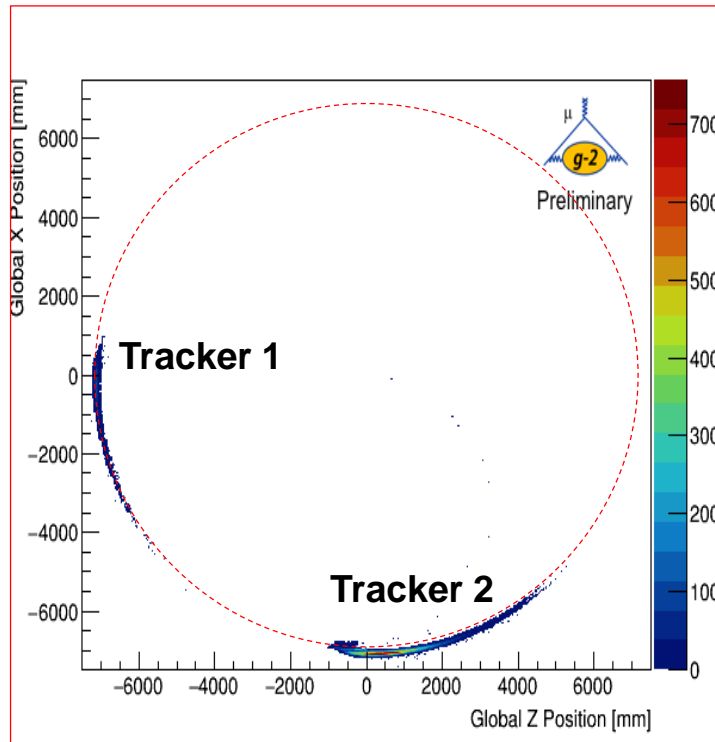
****Warning!!** this is not yet “physics” data. Many calibrations are not completed and magnetic field is not mapped during this period.

Energy vs Time

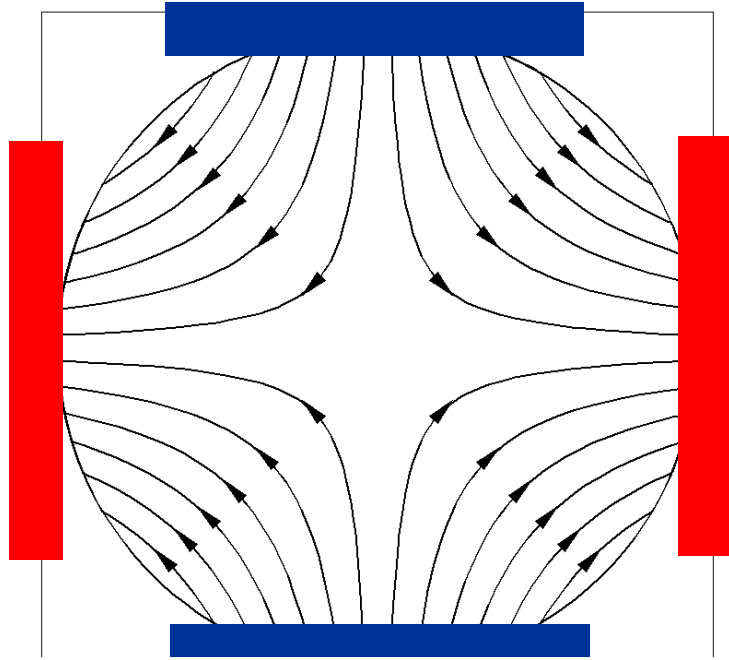
shows muon precession and muon coherent betatron oscillations (from losses)



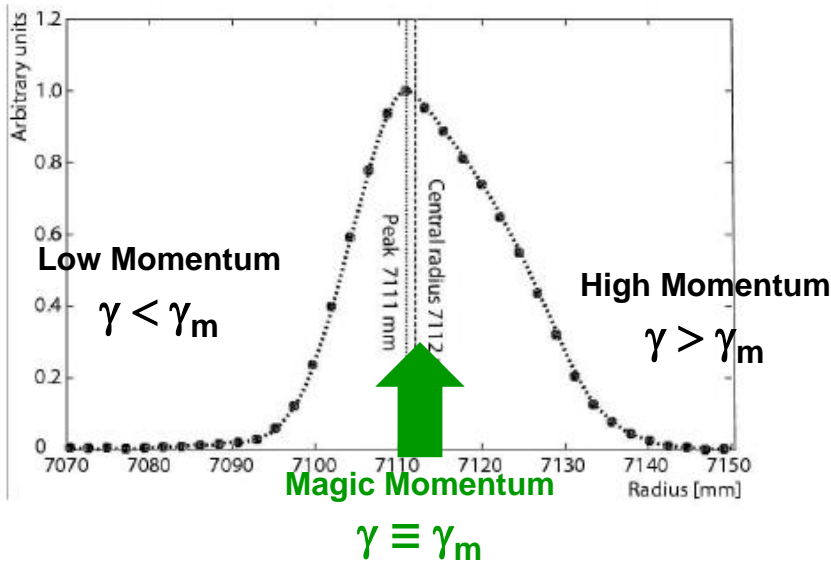
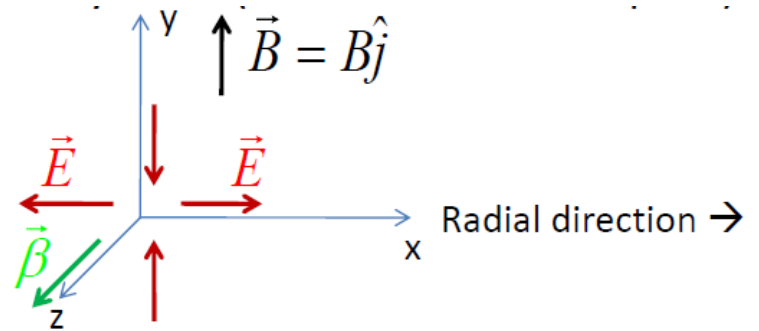
In-vacuum Straw Tracker determines Muon Distribution needed for the “ \sim ” in ω_p formula



The off-momentum muon spins are slightly affected by the radial E field



$$\vec{\omega} = -\frac{q}{m} \left[a_\mu \vec{B} - a_\mu \frac{\gamma}{\gamma+1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} + \left(-a_\mu + \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$



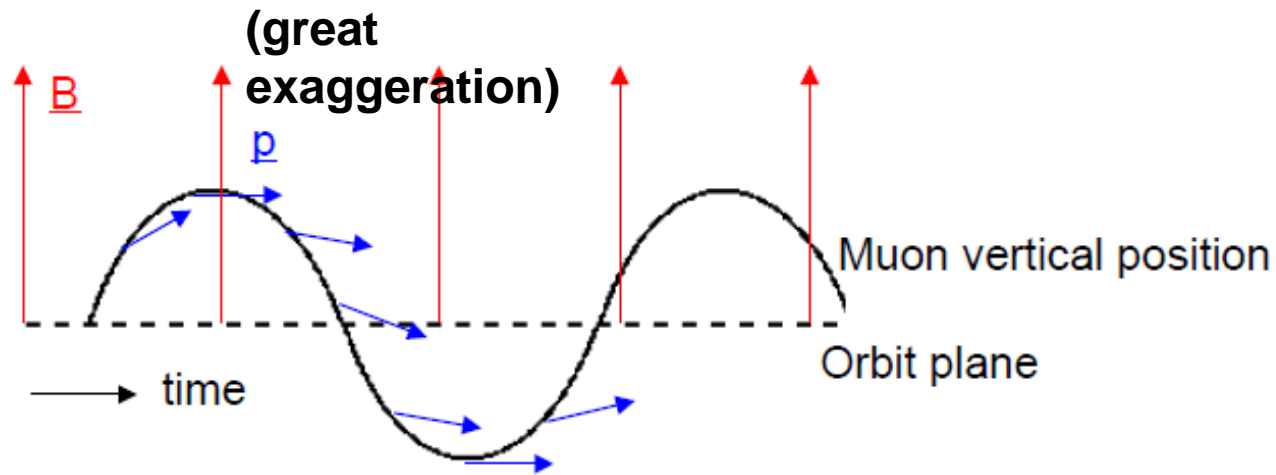
$\beta \times E$ and “ γ ” terms signs both flip depending on momentum

- \rightarrow No cancellation
- \rightarrow All off-momentum muons reduce effective a_μ

~ 0.5 ppm effect, net

The “pitch” correction owing to vertical betatron oscillations

$$\vec{\omega} = -\frac{q}{m} \left[a_{\mu} \vec{B} - a_{\mu} \frac{\gamma}{\gamma + 1} (\vec{\beta} \cdot \vec{B}) \vec{\beta} + \left(-a_{\mu} + \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$



Pitch effect reduces the magnitude of ω_a

$$\langle (\vec{\beta} \cdot \vec{B}) \vec{\beta} \rangle = \langle \beta_y B (\beta_x \hat{i} + \beta_y \hat{j} + \beta_z \hat{k}) \rangle = \langle \beta_y^2 \rangle B \hat{j}$$

$$C_p = \langle \psi^2 \rangle / 2 = n \langle y^2 \rangle / (2 R_0^2)$$

~0.25 ppm effect, net

Outlook

- The search for **New Physics** can profit from the “Indirect” approach ... as long as the measurement is very precise and the Standard Model expectations are clear
- We are presently in a bit of an **LHC lull** ... we anticipated more, but haven't seen it yet
 - The “TeV Scale” is so far not bearing unexpected fruit
- A few very sensitive experiments are pushing the envelope, but we don't yet know what will tip over the vase
 - EDMs ?
 - cLFV searches?
 - $0\nu\beta\beta$ programs ?
 - Beta decays? PVES ?
 - **MUON g-2 !!!**

STAY TUNED !



BACKUPS

E989 Scientific collaboration

7 countries
35 institutions
~192 authors



Domestic Universities

- Boston
- Cornell
- Illinois
- James Madison
- Kentucky
- Massachusetts
- Michigan
- Michigan State
- Mississippi
- Northern Illinois
- Regis
- UT Austin
- Virginia
- Washington

• National Labs

- Argonne
- Brookhaven
- Fermilab



Italy

- Frascati
- Molise
- Naples
- Pisa
- Roma 2
- Trieste
- Udine



China

- Shanghai



Germany

- Dresden



England

- Lancaster
- Liverpool
- University College London



Korea

- CAPP/IBS
- KAIST



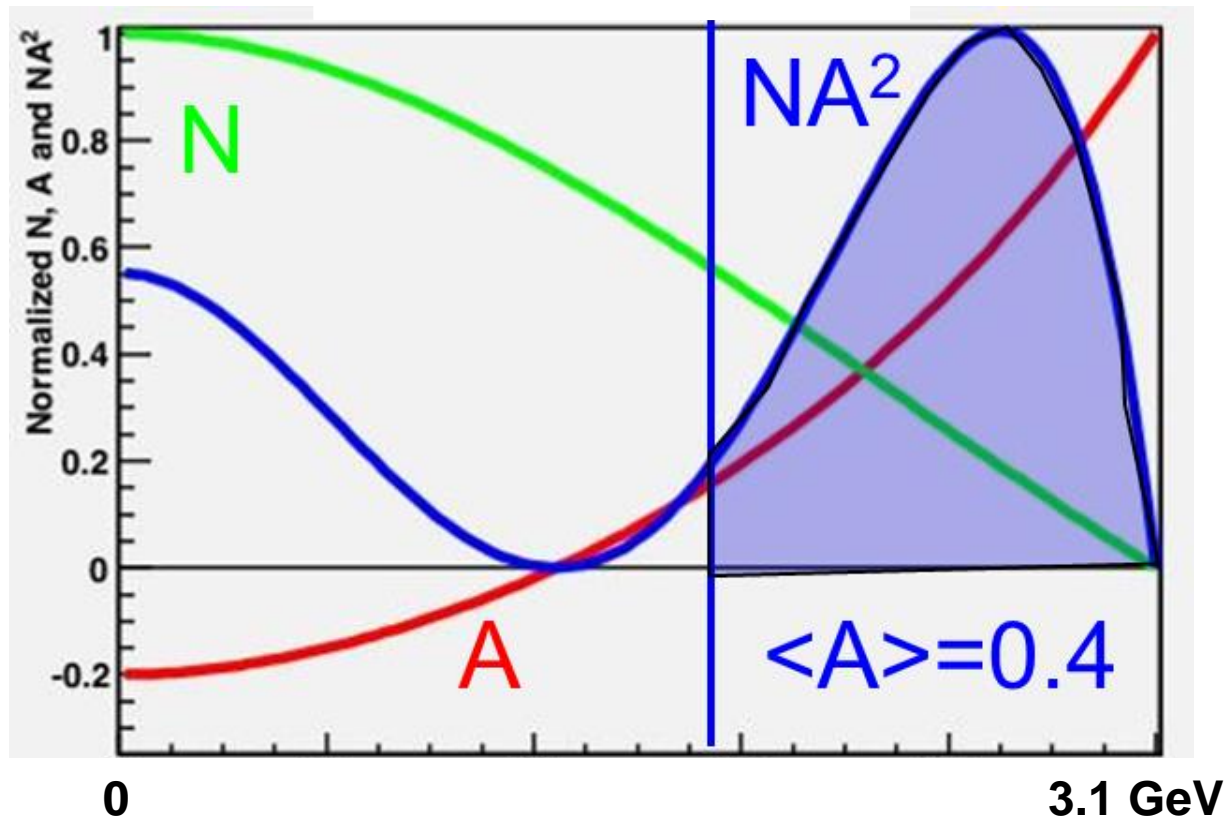
Russia

- JINR/Dubna
- Novosibirsk



Optimizing Statistical Error

$$\delta\omega_a = \frac{1}{\gamma\tau_\mu} \sqrt{\frac{2}{NA^2}}$$





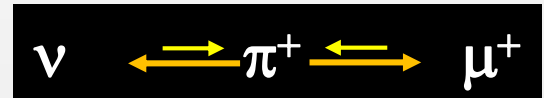
Muon Primer

- Mass $\sim 207 m_e$ (50 ppb)
 - ◆ $(m_\mu/m_e)^2 \approx 43,000$ times more sensitive to “new physics” through quantum loops compared to electrons (taus would be better!)

- Lifetime $\sim 2.2 \mu s$ (1 ppm)
 - ◆ High-intensity beams; can stop and study; can possibly collide

- Primary production: $\pi^+ \rightarrow \mu^+ \nu_\mu$ (99.98%)

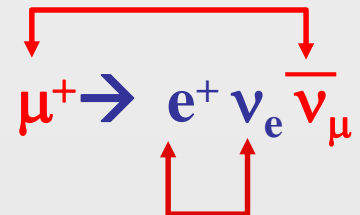
- ◆ Polarized naturally:



- Primary decay $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$ (~99%)

- ◆ Purely weak; distribution in θ and E reveals weak parameters

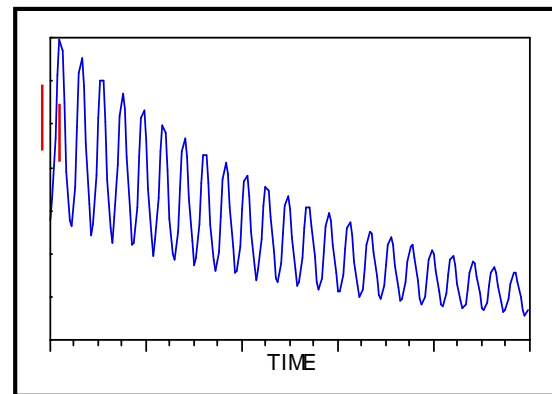
- Lepton number is conserved (BRs $< 10^{-12}$)



Two “blinded” frequency measurements are made. The ratio gives $a_\mu \equiv (g-2)/2$

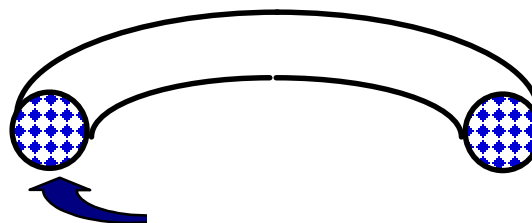
(1) Precession frequency

(1) Calorimeters



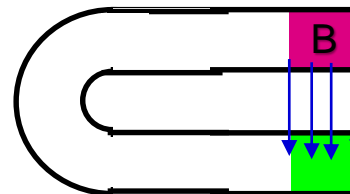
(2) Muon distribution

(2) Trackers & Models



(3) Magnetic field

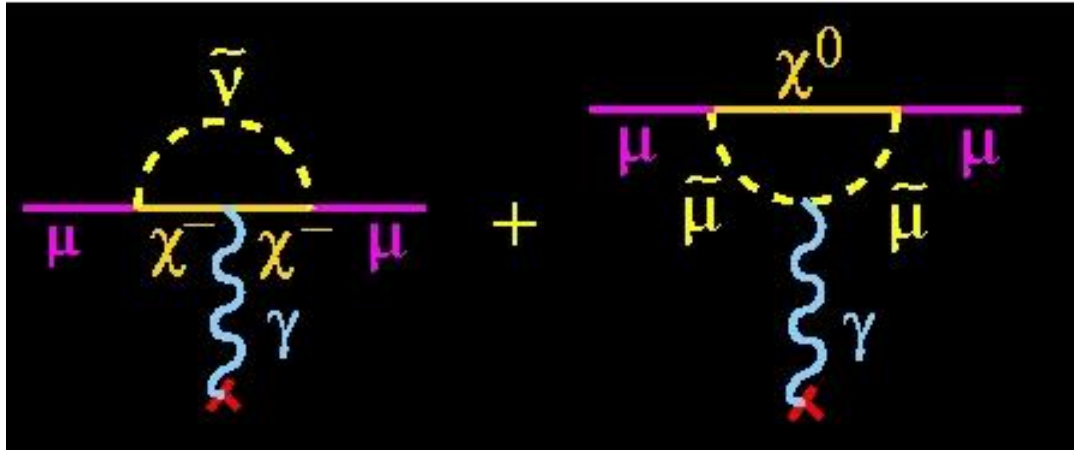
(3) proton pNMR



$$(g - 2) \propto \frac{(1)}{\langle \int (2)(3) \rangle}$$

How do we get each of these?

The attractive idea: SUSY



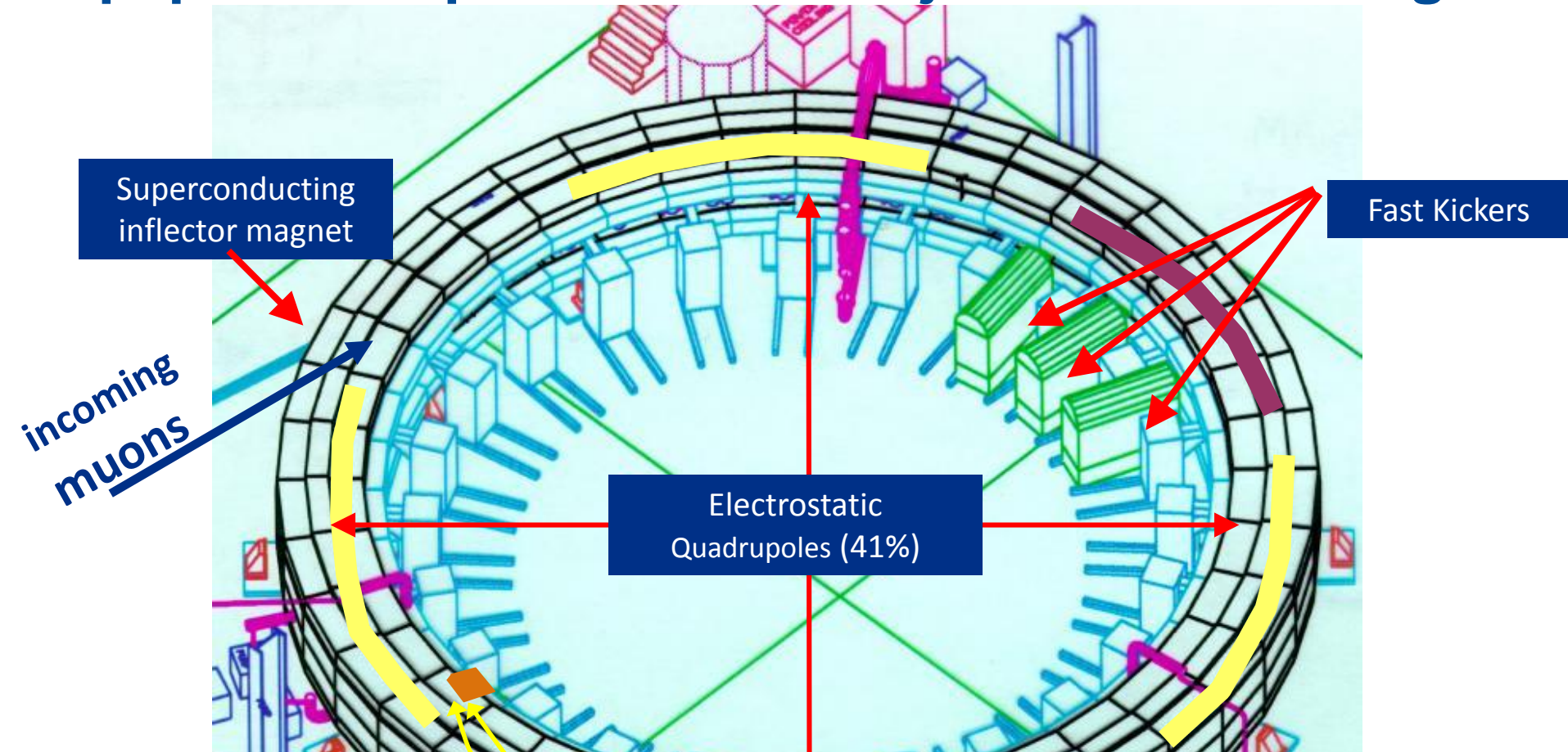
Difficulty to measure at the LHC

$$a_{\mu}^{\text{SUSY}} \approx 130 \times 10^{-11} \left(\frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \tan\beta \text{ sign}(\mu)$$

Recall, the deviation between Experiment and Theory is $\sim 280 \times 10^{-11}$, so the above calculation is interesting if you put in M_{SUSY} , and $\tan\beta$

$\tan\beta$? Ratio of the two vacuum values of the 2 neutral Higgses, typically estimated in range from **3 to 55**

Equipment to provide muon injection and storage



The steps:

1. Steer beam through SR magnet backleg corridor and through **Inflector** into ring
2. Provide well-timed angular **kick** to direct muons into storage region
3. Asymmetrically power **quads** to gently scrape edge muons on **collimators**
4. Relax quads to symmetric configuration for stable vertical containment

The injection hardware

Inflector field cancels the main g-2 magnet

