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Power efficiency vs instability (or, emittance vs beam loading)

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Fermilab/UChicago

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Our motivation for this study came from:

A Beam Driven Plasma-Wakefield Linear Collider: PWFA-LC
From Higgs Factory to Multi-TeV

J.P Delahaye / SLAC

On behalf of the E200 Collaboration

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Gradient and efficiency in Linear Colliders

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- High gradient acceleration requires high peak power and structures that can sustain high fields
  - Beams and lasers can be generated with high peak power
  - Dielectrics and plasmas can withstand high fields

→ **Beam-driven Plasma Wake-Field Accelerator (PWFA)**

Acc. structures		Accelerating field			Acceleration efficiency		
		Limit (MV/m)	By	Comment	Wall-Plug to RF or drive (%)	RF or drive to beam (%)	Total (%)
Super-Conducting	ILC	30-40	Magnetic field	Dyn. losses Cryogenics prop G^2	45	45 (pulsed + Cryo)	20
Normal Conducting	CLIC Two beam	100	RF break-downs	Peak RF Power ~ E^2	40	30	12
Dielectric	Laser driven	1000	RF break-downs		10	50	5
	Beam driven				?	50	?
Plasma	Laser driven	10000	Laser		10	50	5
	Beam driven		Drive beam		40	50	20

J.P.Delahaye @ MIT April 11,2013

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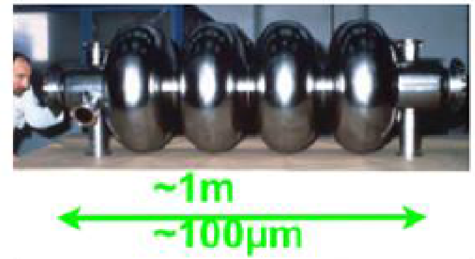
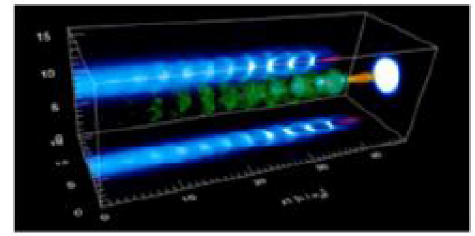
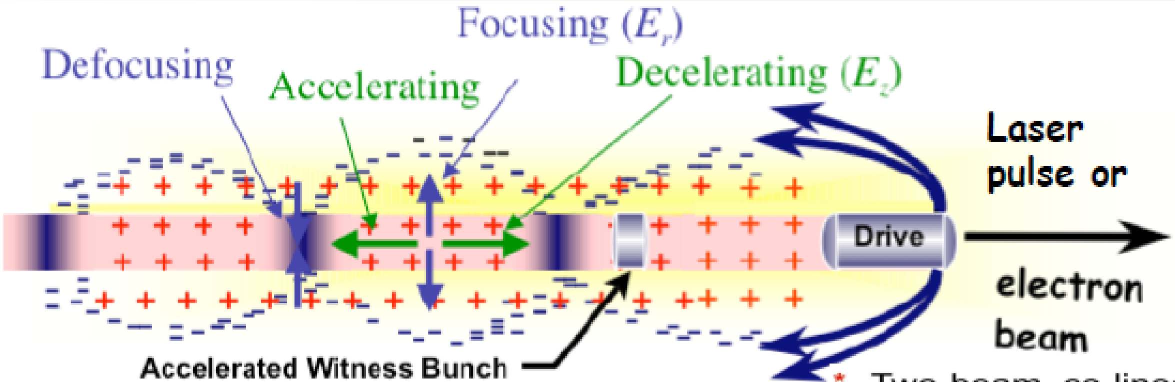
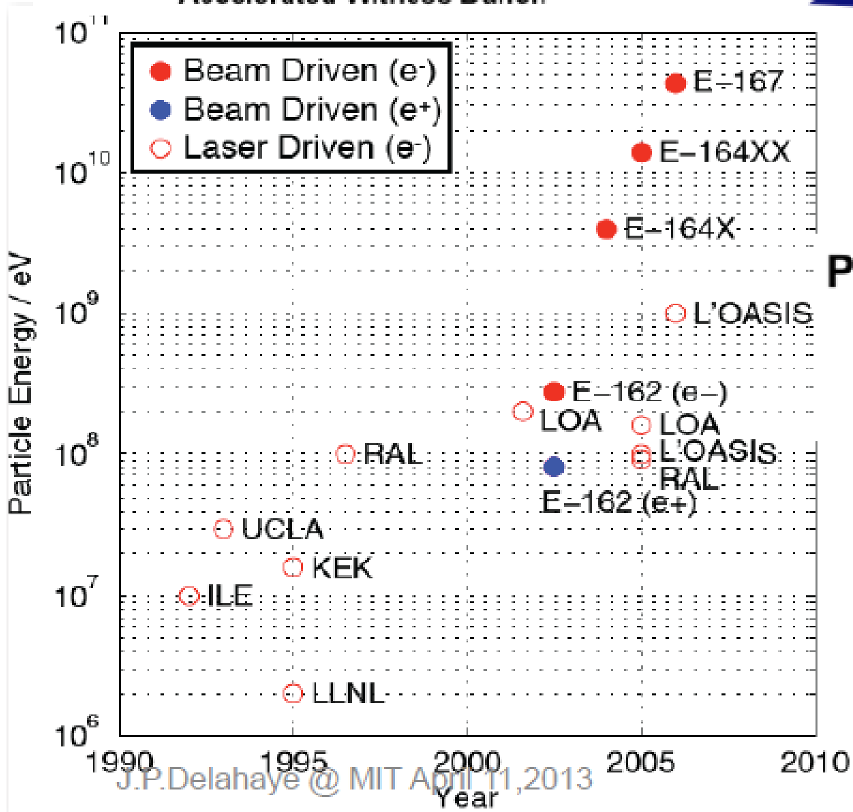
Plasma Acceleration (Beam-driven or Laser-driven)









- Two-beam, co-linear, plasma-based accelerator
- Plasma wave/wake excited by relativistic particle bunch
- Deceleration, acceleration, focusing by plasma
- Accelerating field/gradient scales as $n_e^{1/2}$
- Typical: $n_e \approx 10^{17}$ cm^{-3}, $\lambda_p \approx 100$ µm, G>MT/m, E>10 GV/m
- High-gradient, high-efficiency energy transformer

Peak Field For A Gaussian Bunch:

$$E = 6GV/m \frac{N}{2x10^{10}} \frac{20\mu}{\sigma_r} \frac{100\mu}{\sigma_z}$$ → >10GV/m

Extremely strong focusing:

$$B_\phi = 2\pi e \eta_p r$$ → > MT/m

Excellent power transfer efficiency:

$\eta_{drive\ to\ plasma} \sim 76\%$, $\eta_{plasma\ to\ main} \sim 66\%$ → $\eta_{drive\ to\ main}$ > 50%

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Is drive-to-beam 50% efficiency possible???

Conclusions

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PWFA a very promising technology:

Very high accelerating fields: effective 1 GV/m

Excellent power efficiency (Wall-plug to beam 20%)

Great flexibility of time interval

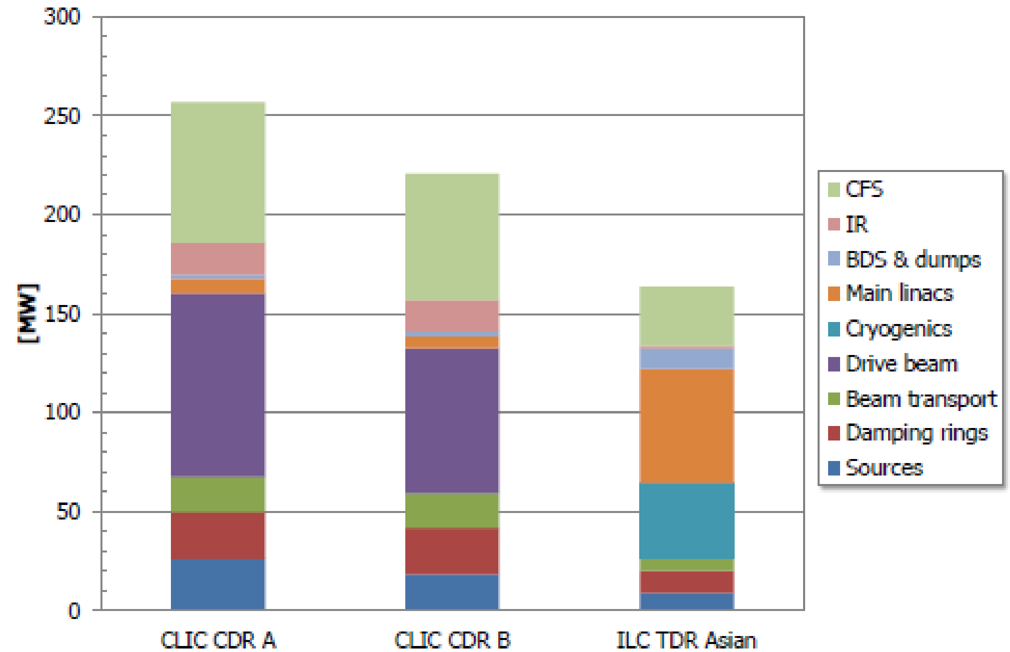
- CW or pulsed mode of operation
- An alternative for ILC energy upgrade?

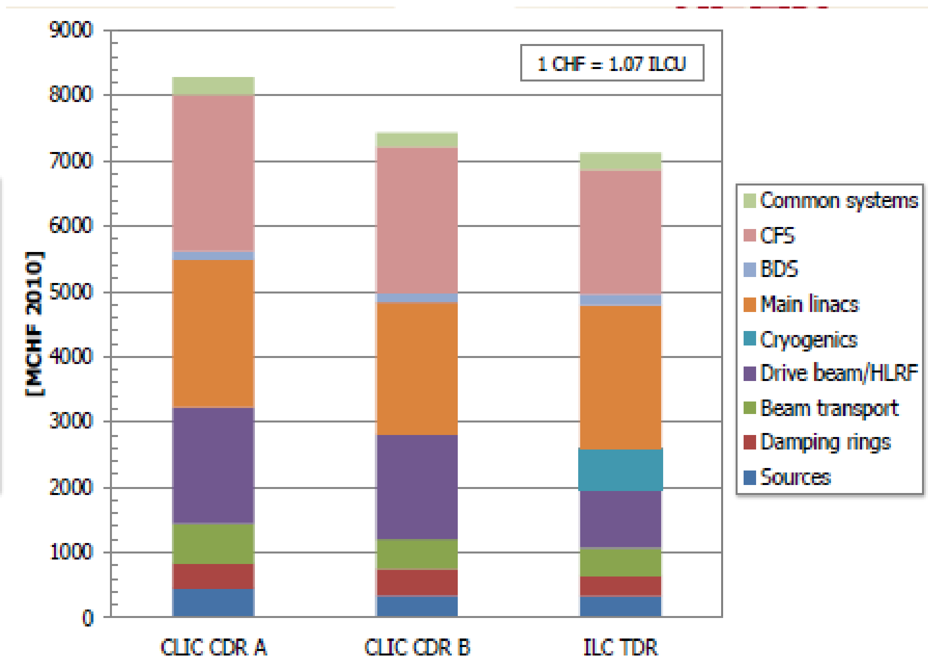
Many challenges still to be addressed;

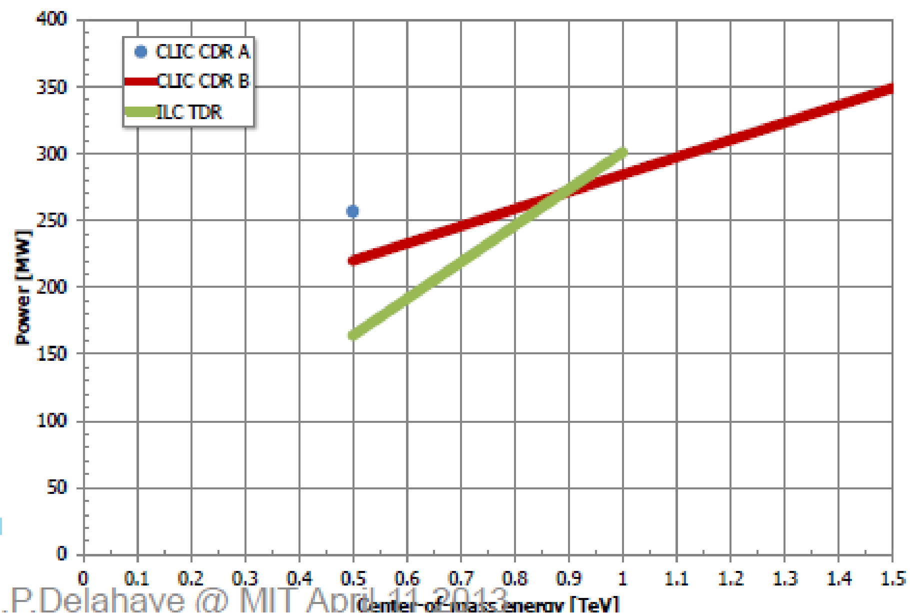
- Beam quality preservation, efficiency, positrons?
- Ambitious test facilities: FACET and FACET2
- Feasibility addressed early next decade?

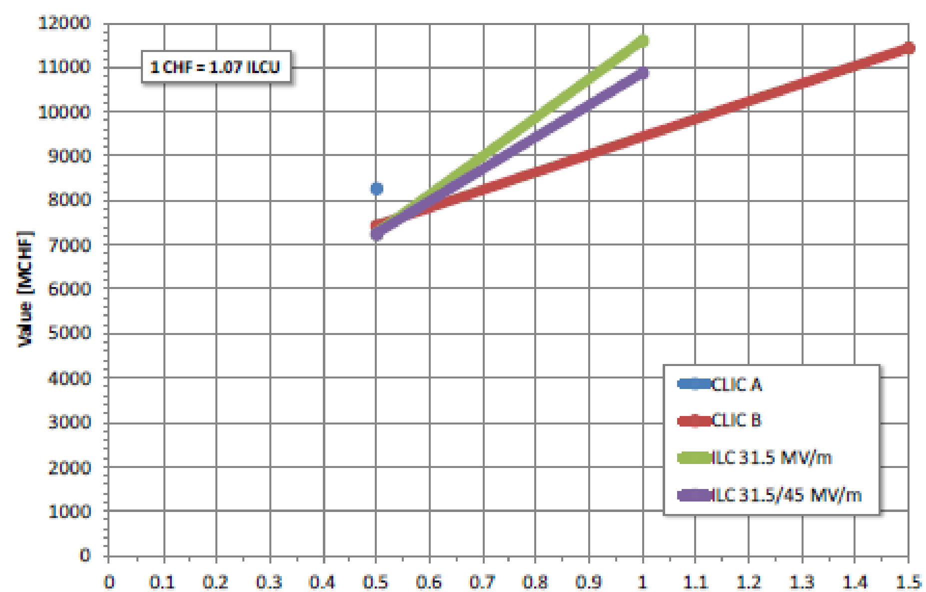
Thanks to excellent and expert collaboration: E200

Why is power efficiency important?
Because power = cost







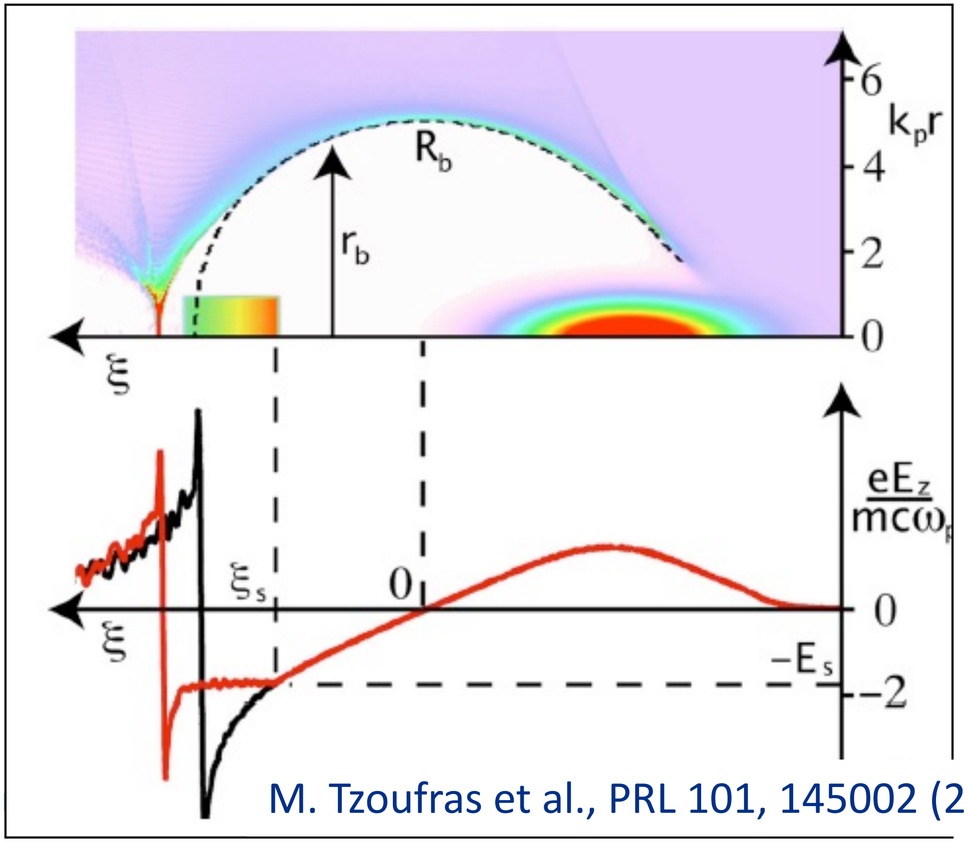


Acceleration in ILC cavities

- The ILC cavity: ~1 m long, 30 MeV energy gain; f_0 = 1.3 GHz, wave length ≈ 23 cm
- The ILC beam: 3.2 nC ($2x10^{10}$), 0.3 mm long (rms); bunches are spaced ~300 ns (90 m) apart
- Each bunch lowers the cavity gradient by ~15 kV/m (beam loading 0.05%); this voltage is restored by an external rf power source (Klystron) between bunches; (~0.5% CLIC)
- Such operation of a conventional cavity is only possible because the Q-factor is >> 1; the RF energy is mostly transferred to the beam NOT to cavity walls.

Acceleration in a blow-out regime

- The Q-factor is very low (~1) – must accelerate the trailing bunch within the same bubble as the driver!
- Cannot add energy between bunches, thus a single bunch must absorb as much energy as possible from the wake field.



To achieve L ~10^{34}, bunches should have ~10^{10} particles (similar to ILC and CLIC). In principle, we can envision a scheme with fewer particles/bunch and a higher rep rate, but the beam loading still needs to be high for efficiency reasons.

M. Tzoufras et al., PRL 101, 145002 (2008)

Transverse beam break-up (head-tail instability)

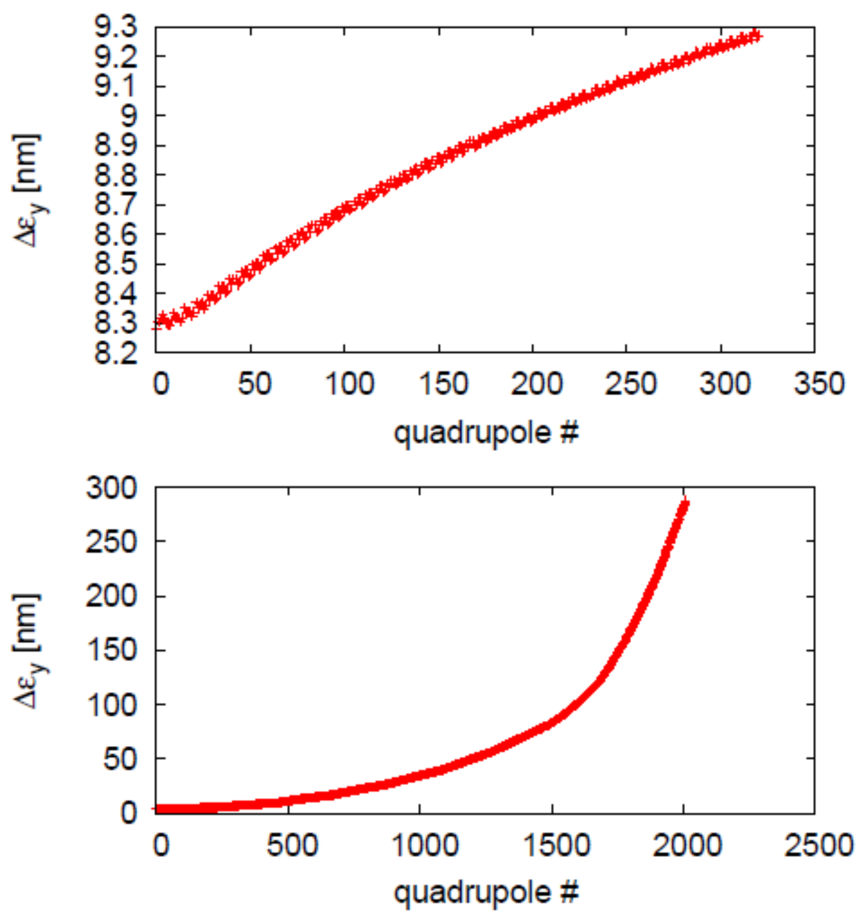
- Transverse wakes act as deflecting force on bunch tail
  - beam position jitter is exponentially amplified

Beam Stability

- Transverse stability of a beam with initial offset of σ_y
  - no energy spread assumed in the beam
  - emittance with respect to the beam axis is shown

$\Rightarrow$ acceptable for ILC (top)

$\Rightarrow$ would be intolerable for CLIC (bottom)



D. Schulte, 6th Linear Collider School 2011, Main Linac Basics 69

Short-range transverse wake

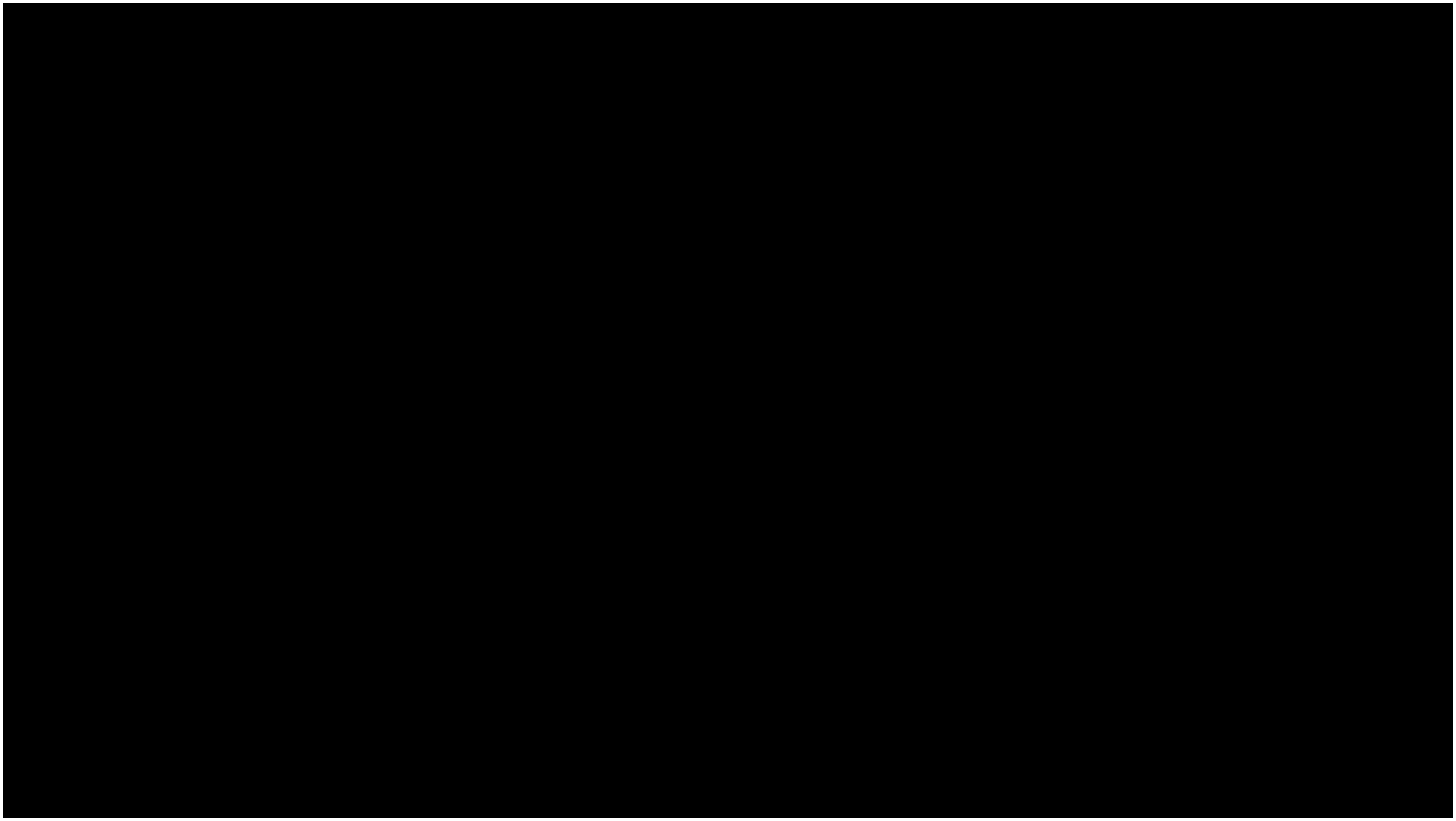
$$W_{\perp}(z) = \frac{8z}{a^4}$$

$a \approx 35$ mm (ILC)

$a \approx 3.5$ mm (CLIC)

$a \sim 0.1$ mm (PWFA)

Case I: ~50% power efficiency



Courtesy of UCLA

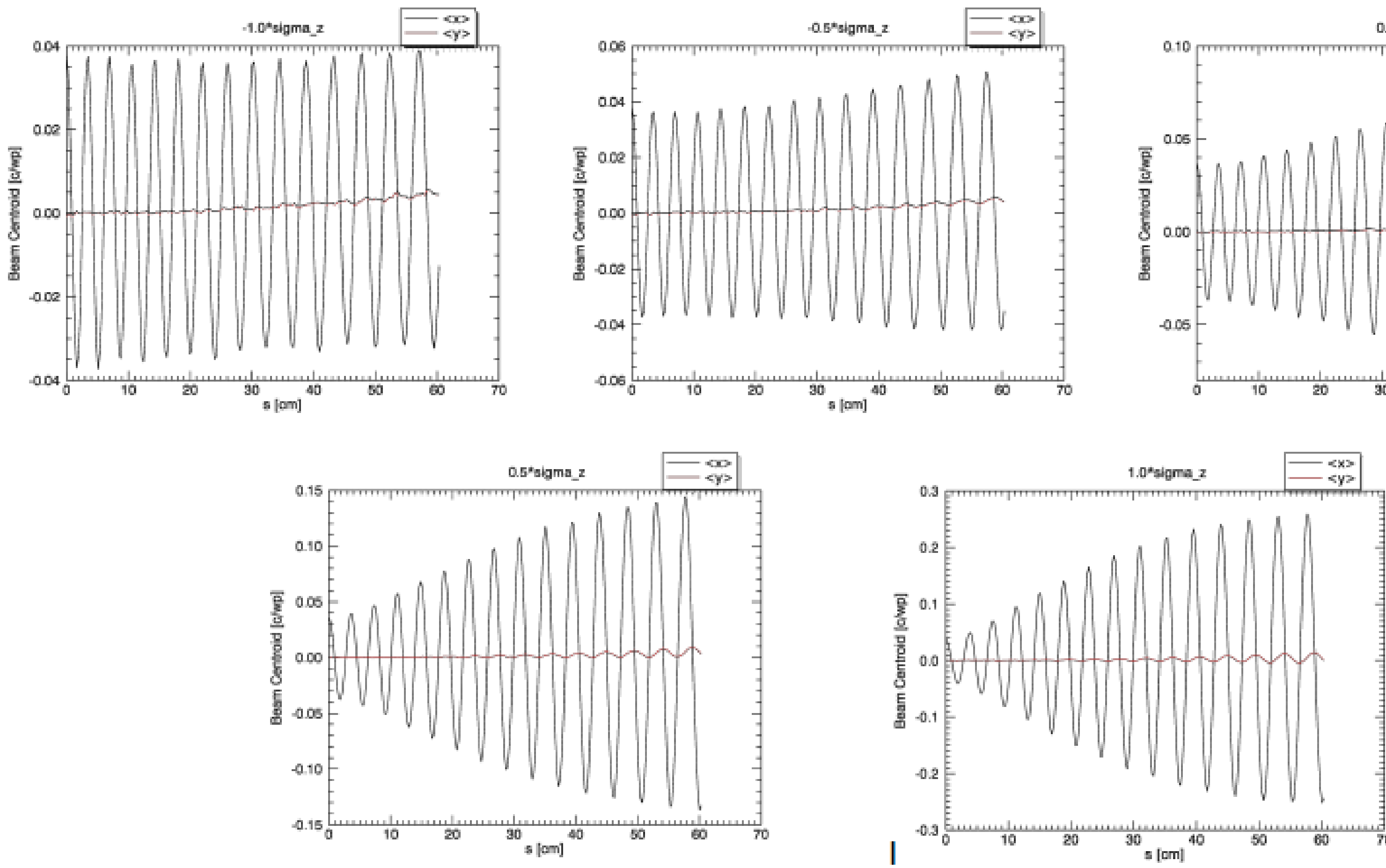
Hosing Study for FACET II: Case I

Drive Beam: E = 10 GeV, I_{peak}=15 kA
σ_r = 3.65 μm, σ_z = 12.77 μm ,
N =1.0 x 10^{10} (1.6 nC), ε_N = 10 μm

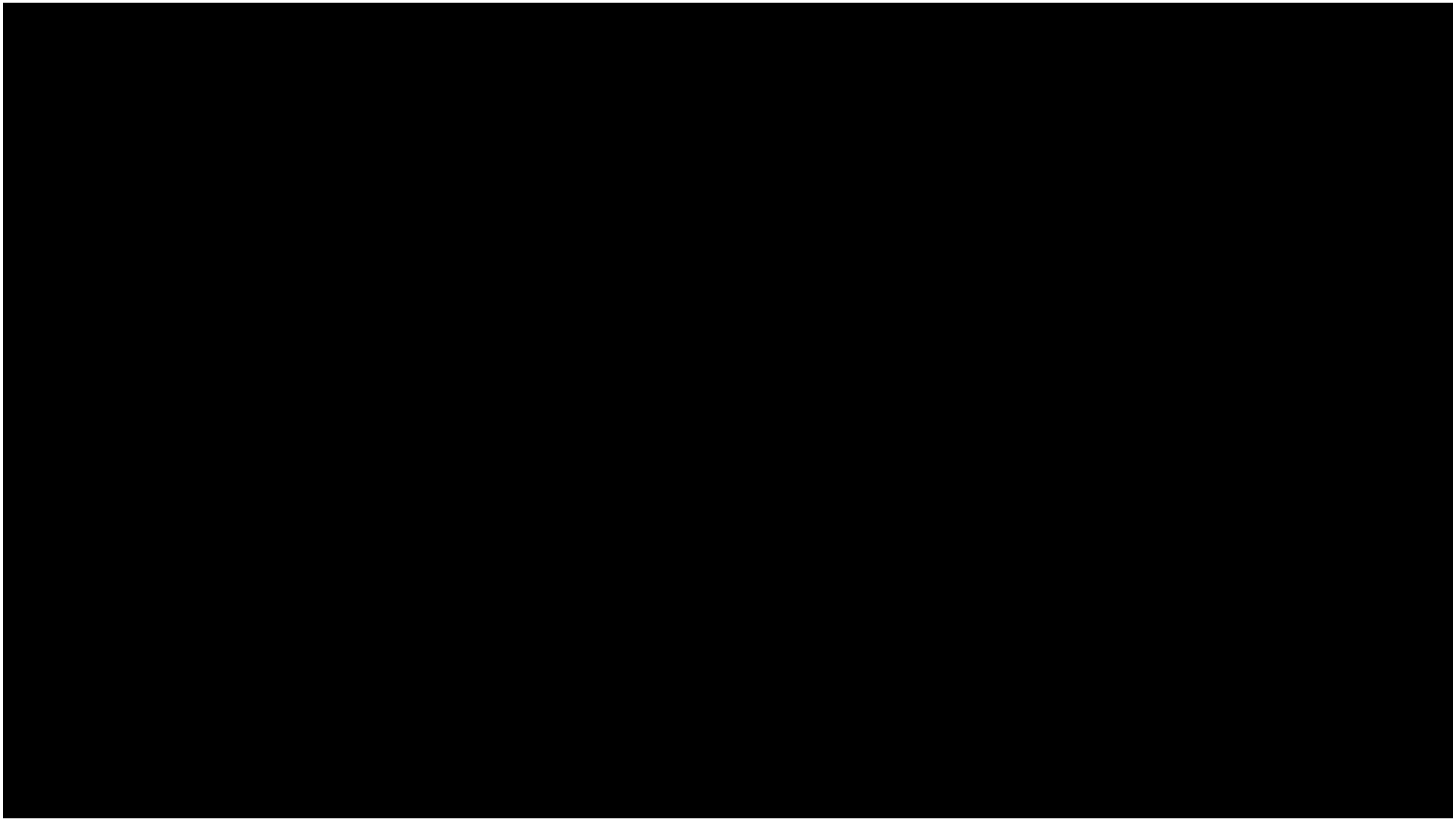
Trailing Beam: E = 10 GeV, I_{peak}=9 kA
σ_r = 3.65 μm, σ_z = 6.38 μm ,
N =4.33 x 10^9 (0.69 nC), ε_N = 10 μm
(transversely offset by 1 μm)

Distance between two bunches: 150 μm
Plasma Density: 4.0 x 10^{16} cm^{-3}

Trailing beam centroid vs s in different slices



Case II: ~25% power efficiency



Courtesy of UCLA

UCLA

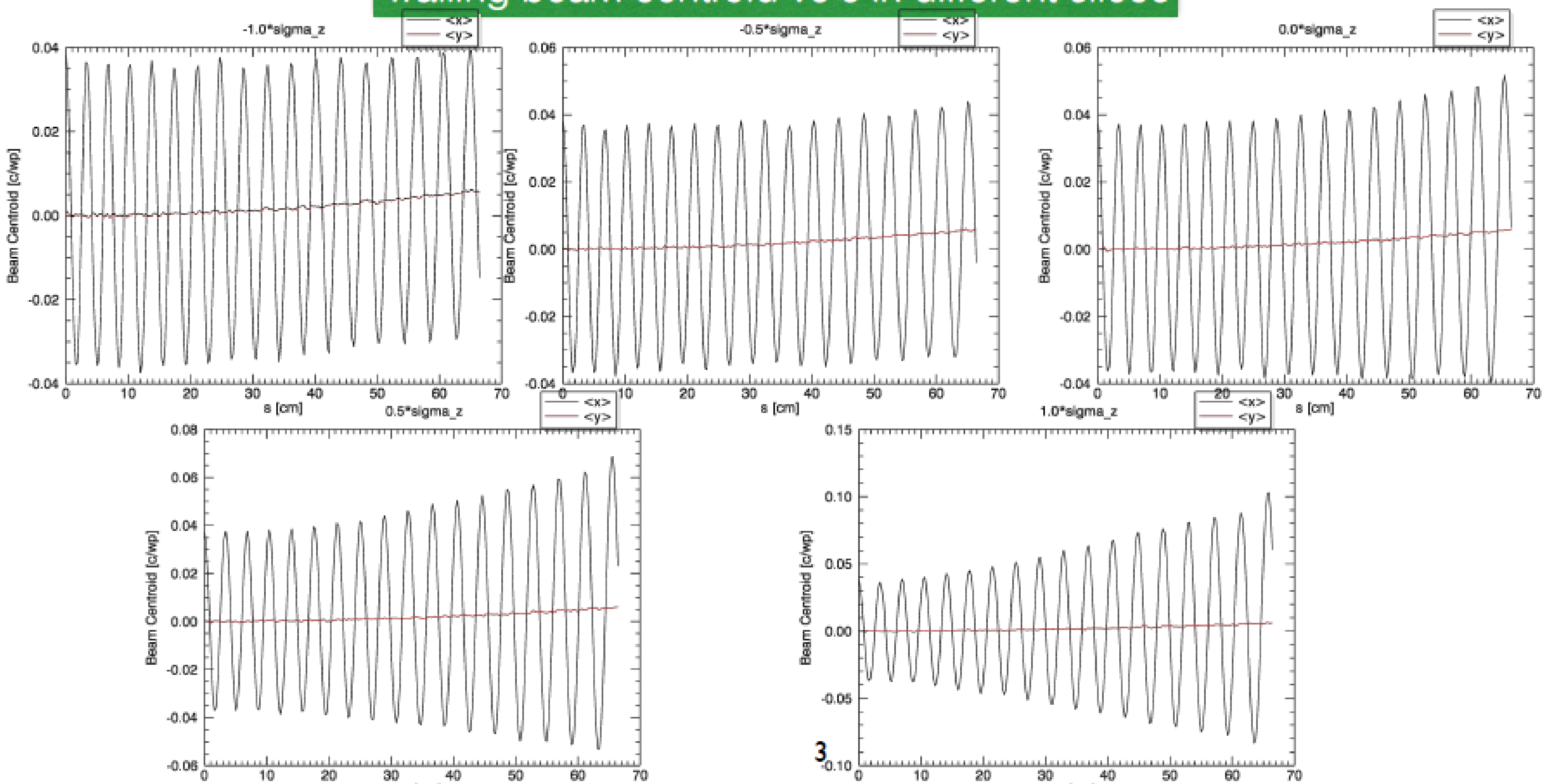
Hosing Study for FACET II: Case II

Drive Beam: E = 10 GeV, I_{peak}=15 kA
σ_r = 3.65 μm, σ_z = 12.77 μm ,
N =1.0 x 10^{10} (1.6 nC), ε_N = 10 μm

Trailing Beam: E = 10 GeV, I_{peak}=9 kA
σ_r = 3.65 μm, σ_z = 6.38 μm ,
N =4.33 x 10^{9} (0.69 nC), ε_N = 10 μm
(transversely offset by 1 μm)

Distance between two bunches: 108 μm
Plasma Density: 4.0 x 10^{16} cm^{-3}

Trailing beam centroid vs s in different slices



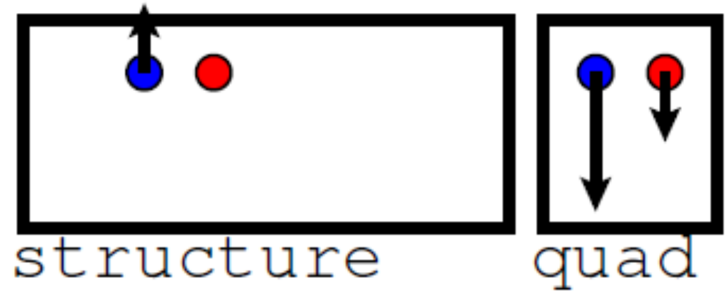
Beam breakup in various collider concepts

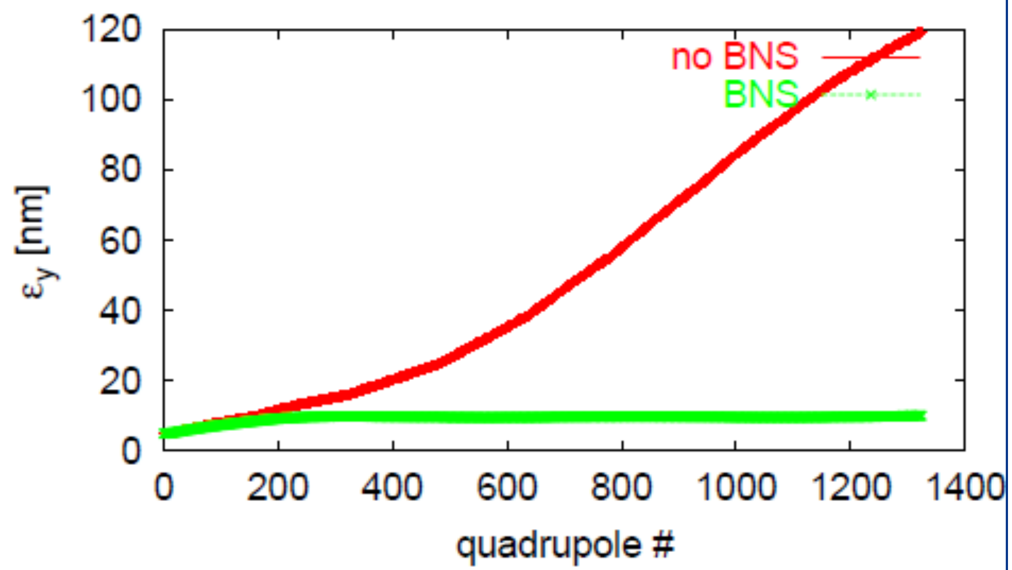
- ILC
  - Not important; bunch rf phase is selected to compensate for long wake and to minimize the momentum spread
- CLIC
  - Important; bunch rf phase is selected to introduce an energy chirp along the bunch for BNS damping (~0.5% rms). May need to be de-chirped after acceleration to meet final-focus energy acceptance requirements
- PWFA – the subject of our study
  - Critical; BNS damping requires a large energy chirp (see below). De-chirping and beam transport is very challenging because of plasma stages (small beta-function in plasma ~1 cm). In essence, requires a “final-focus” optics between every stage.

CLIC strategy: BNS damping + < μm alignment of cavities

Achieving Beam Stability

- Transverse wakes act as defocusing force on tail
  - ⇒ beam jitter is exponentially amplified
- BNS (Balakin, Novokhatsky, and Smirnov) damping prevents this growth
  - manipulate RF phases to have energy spread
  - take spread out at end





D. Schulte, 6th Linear Collider School 2011, Main Linac Basics 70

Strategy was also used at the SLC…

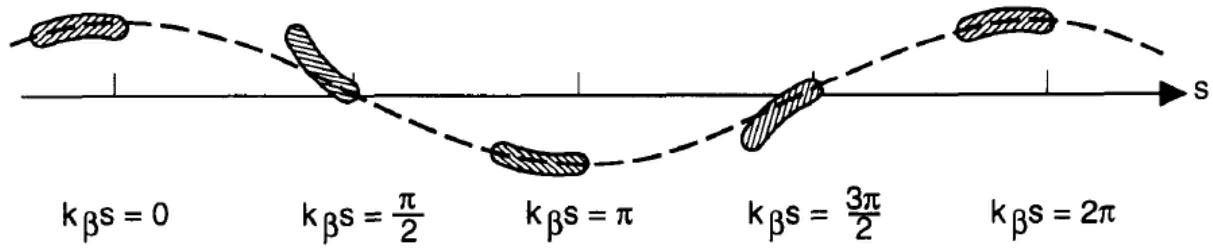


Figure 3.3. Sequence of snapshots of a beam undergoing dipole beam breakup instability in a linac. Values of $k_\beta s$ indicated are modulo 2π. The dashed curves indicate the trajectory of the bunch head.

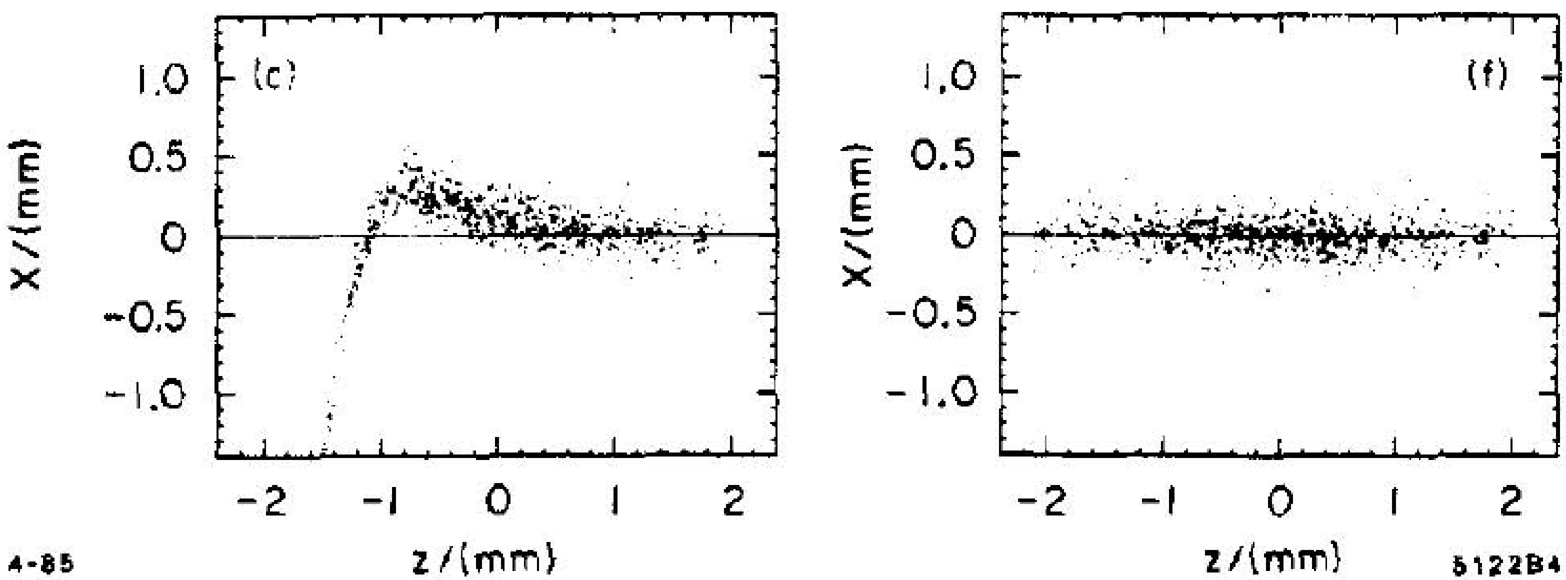


Figure 34: Multiparticle simulation of a particle bunch passing through the SLAC linac without (left) and with BNS damping (right) [36].

We start with the Lu plasma bubble equation

- We assume the driving bunch intense enough to produce an electron-free plasma bubble with radius $R_b >> k_p^{-1}$. According to Lu et al. :

$$r_b \frac{d^2 r_b}{d\xi^2} + 2\left(\frac{dr_b}{d\xi}\right)^2 + 1 = \frac{2}{\pi n_0 r_b^2}\frac{dN_d}{d\xi} \qquad E_{\parallel} = -2\pi n_0 e r_b \frac{dr_b}{d\xi}$$

$$R_b = \frac{L_d}{\sqrt[4]{2}} \sqrt[4]{\frac{8N_d}{\pi n_0 L_d^{\,3}}\left(\sqrt{\frac{8N_d}{\pi n_0 L_d^{\,3}} + 1} - 1\right)}$$

$$R_b \approx \left(\frac{2^7 N_d^3}{\pi^3 L_d n_0^3}\right)^{1/8}, \quad \frac{N_d}{n_0 L_d^3} >> 1$$

$$\text{Example: } N_d = 10^{10};\ n_0 = 4\times 10^{16}\ \text{cm}^{-3};\ L = 25\ \mu\text{m}$$

$$R_b k_p \approx 3.2$$

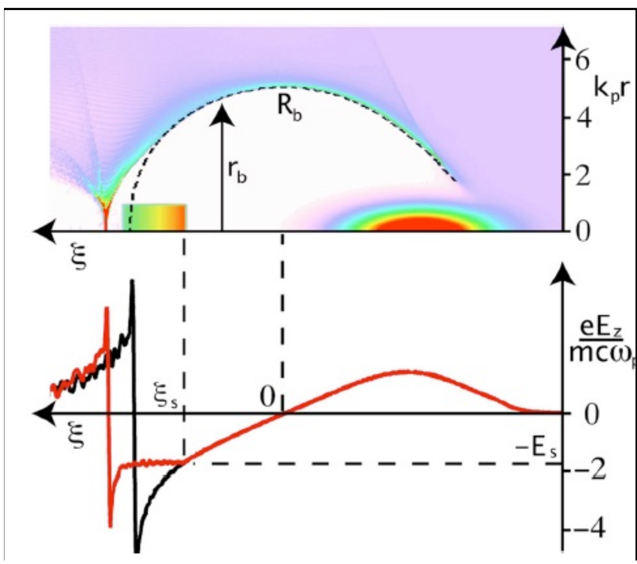
Power transfer from drive to trailing bunches

- Following M. Tzoufras et al., PRL 101, 145002 (2008)

Trapezoidal line density distribution → constant electric field

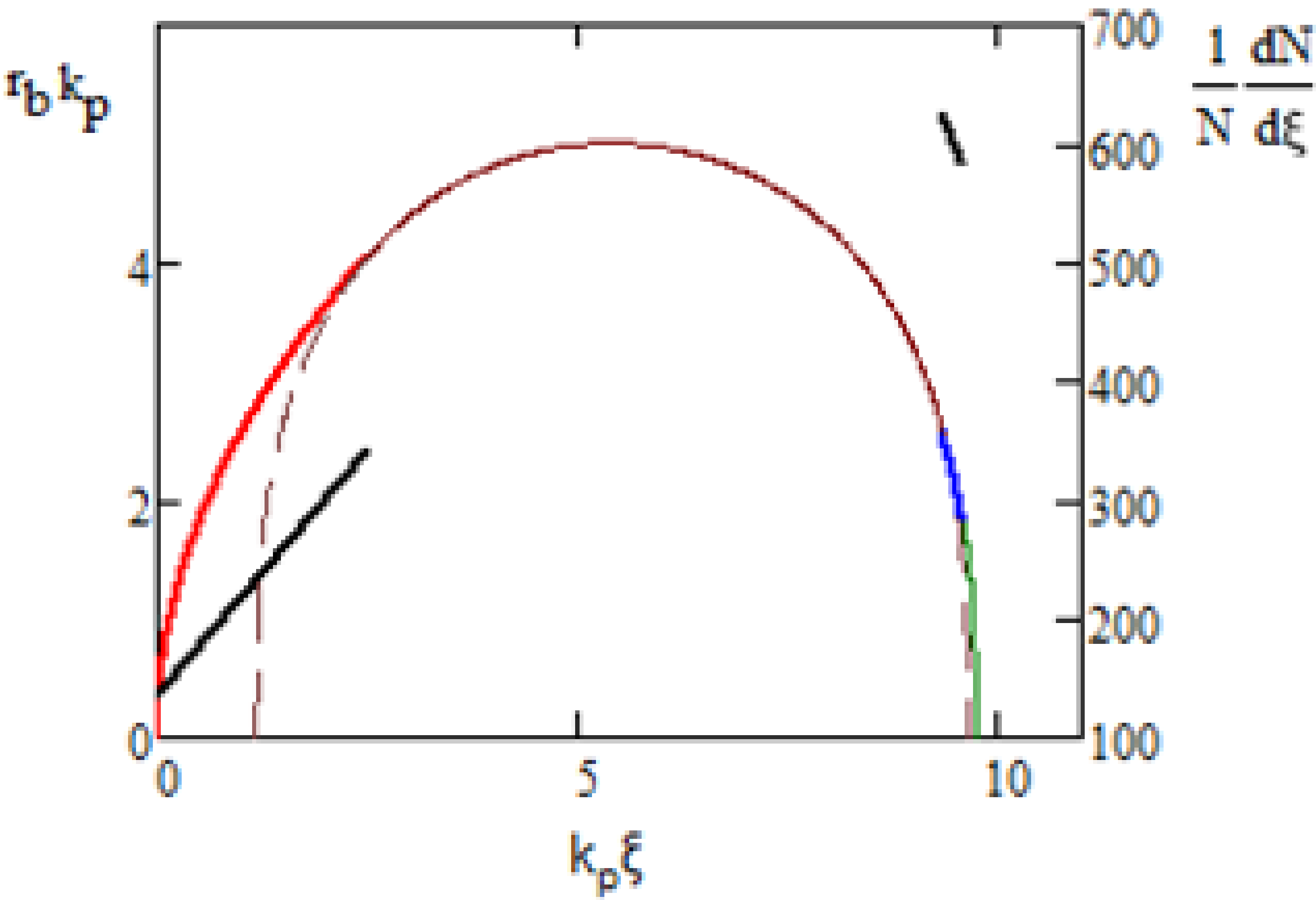
$$P = eN_d E_d c = \frac{\pi^2}{4} e^2 n_0^2 c R_b^4$$

$$P_t = ecN_t E_t = \frac{\pi^2 e^2 n_0{}^2 c}{4} \left(r_{t2}{}^2 - r_{t1}{}^2\right)\left(\frac{R_b{}^4}{r_{t2}{}^2} + r_{t1}{}^2\right)$$



$$\eta_P = \frac{P_t}{P} = \frac{r_{t2}{}^2 - r_{t1}{}^2}{R_b{}^2}\left(\frac{R_b{}^2}{r_{t2}{}^2} + \frac{r_{t1}{}^2}{R_b{}^2}\right)$$

Example



The power transfer efficiency of 50% and the transformer ratio of 2. For $n_0=10^{17}$ cm^{-3} the drive bunch parameters are chosen to be $R_b k_p=5$, $L_d k_p=2.5$ yielding the decelerating field of E_d = 50 GV/m and $N_d=3.55\cdot10^{10}$. The trailing bunch parameters are: $r_{t2}=0.518R_b$, $r_{t1}=0.373R_b$, E_t = 100 GV/m, $N_t=8.86\cdot10^9$.

Instability of the trailing bunch

- The *Beam Break-up* (BBU) instability is characterized by the ratio of the wake deflection force to the focusing force.

$$F_r = -2\pi n_0 e^2 r$$ Focusing force

$$F_t \equiv F(\xi_1) = e^2 r \int_{\xi_1 - L_t}^{\xi_1} \frac{dN_t}{d\xi} W_\perp(\xi_1, \xi) d\xi$$ Defocusing force (varies along bunch)

- Need to find $W_\perp\left(\tilde{\xi}\right)$ for the bubble regime.
- First, in a quasilinear regime,

$$W_\perp = 2\frac{k_p}{\sigma_\perp^{\ 2}}\left(\frac{\Delta n}{n}\right)_e \sin\left(k_p\left(s - s'\right)\right)\ln\left(\frac{\rho_{\max}}{\rho_{\min}}\right), \quad k_p = \frac{\omega_p}{c}$$

  - where $\sigma_\perp$ is the rms size of plasma channel
  - For a hollow channel $\frac{\Delta n}{n} \sim 1$

$$W_\perp \approx 2k_p^{\ 3}\sin\left(k_p\left(s - s'\right)\right)\ln\left(2\right), \quad \sigma_\perp \approx k_p^{-1}$$

Wakes in the bubble regime

Longitudinal (from the Lu equation): $$W_{\parallel} = \frac{4}{r_b^2}; \quad (\Delta z << r_b, k_p^{-1})$$

(*similar to a dielectric channel and periodic array of cavities*)

For reference, see: A. V. Fedotov, R. L. Gluckstern, and M. Venturini (PRST-AB 064401 (1999))

Transverse :

$$W_{\perp} \approx \frac{2}{r_b^2}\int W_{\parallel} dz = \frac{8\Delta z}{r_b^4}; \quad (\Delta z << r_b, k_p^{-1})$$

$r_b(z) >> k_p^{-1}$ -- local bubble radius at bunch location, z

(*This is true for a dielectric channel, array of cavities and resistive wall*)

For reference, see also: Karl Bane, SLAC-PUB-9663 and S. S. Baturin and A. D. Kanareykin, PRL 113, 214801 (2014) .

Recent findings: $\tilde{r}_b(z) \to r_b(z) + k_p^{-1}$ to account for bubble wall thickness

Our estimate for the transverse wake

$$W_{\perp}\left(\xi,\xi_2\right) \approx \frac{8\tilde{\xi}}{r_b\left(\xi\right) r_b^{\;3}\left(\xi_2\right)} \theta(\tilde{\xi}) \,, \quad \tilde{\xi} = \xi - \xi_2$$

$$r_b(\xi) >> k_p^{-1}$$

– $\theta(x)$ is the Heaviside step function.

- We believe this estimate is on the "low" side. The actual wake is likely to be greater.
- Now, let's find the ratio of the defocusing (wake) force to the focusing force:

$$\eta_t = -\frac{F_t}{F_r} = \frac{r_{t2}}{r_{t1}} \int_0^{L_t} d\xi \frac{L_t - \xi}{r_b^{\;3}(\xi)} \times \left[r_{t2}\left(\frac{R_b^{\;4}}{r_{t2}^{\;4}} - 1\right) - 2\left(\xi\sqrt{2\left(\frac{R_b^{\;4}}{r_{t2}^{\;4}} - 1\right)} - r_{t2}\right)\right]$$

- Recall that $$\eta_P = \frac{P_t}{P} = \frac{r_{t2}^{\;2} - r_{t1}^{\;2}}{R_b^{\;2}}\left(\frac{R_b^{\;2}}{r_{t2}^{\;2}} + \frac{r_{t1}^{\;2}}{R_b^{\;2}}\right)$$

The efficiency-instability relation

$$\eta_t \approx \frac{{\eta_P}^2}{4\left(1-\eta_P\right)}, \quad \frac{r_{t2}}{R_b} \leq 0.7$$

- This formula does not include any details of beams and plasma, being amazingly universal!
- Note: this formula is an estimate from a "low side". On a "high side", we estimate it as: $\eta_t \approx {\eta_P}^2 / \left(4\left(1-\eta_P\right)^2\right)$
- Example: $\eta_P = 50\% \rightarrow \quad 0.125 < \eta_t < 0.25$

$$\eta_P = 25\% \rightarrow \quad 0.021 < \eta_t < 0.028$$

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Instability development

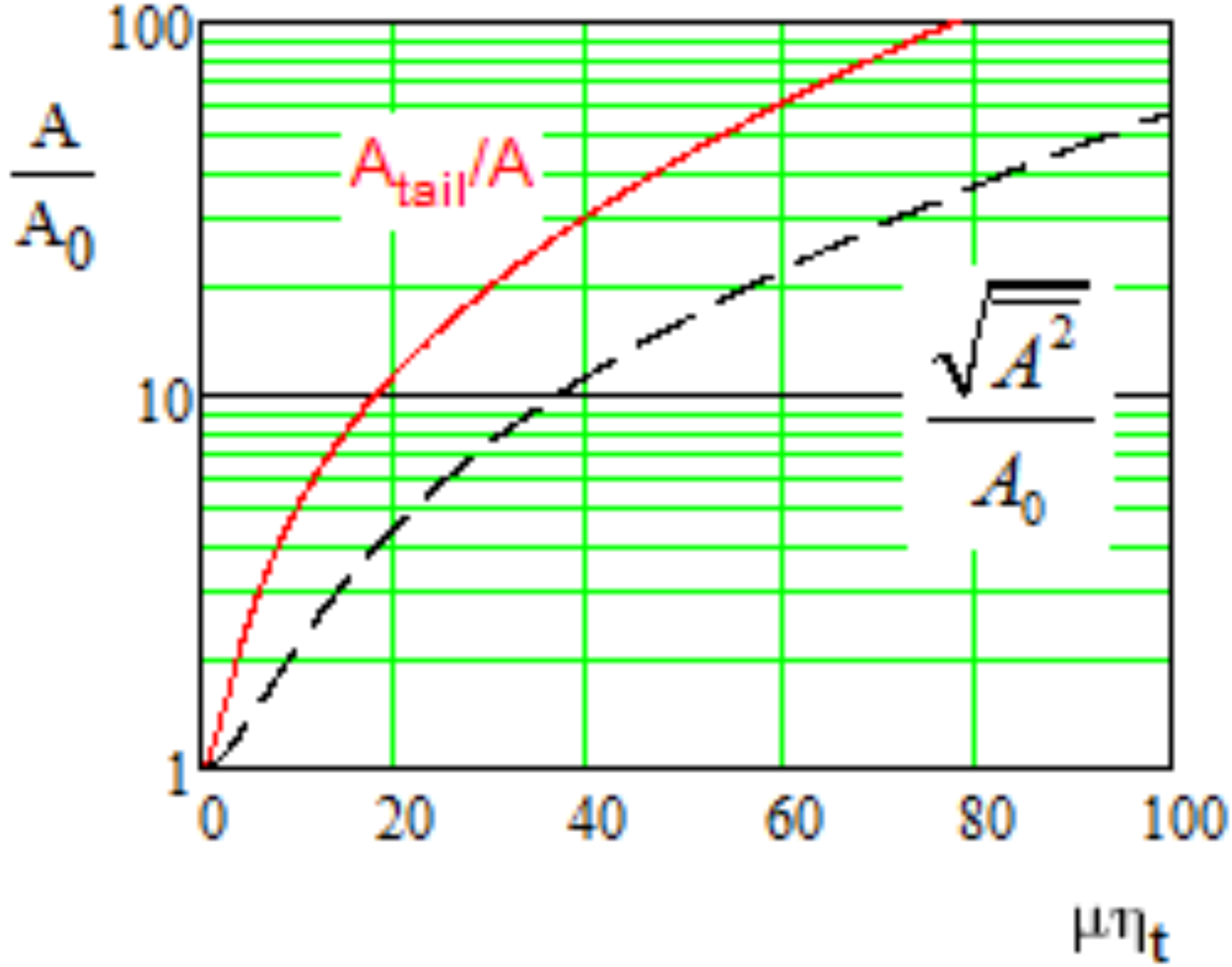
$$\frac{d^2X}{d\mu^2}+\frac{X}{1+\Delta p/p}=\frac{2\eta_t}{(1+\Delta p/p)L_t^2}\int_0^{\xi}X(\xi')(\xi-\xi')d\xi'.$$

$$X=\frac{x}{\sqrt{\beta}}\sqrt{\frac{p}{p_0}};\quad \beta=k_p^{-1}\sqrt{2\gamma}\qquad d\mu=dz/\beta$$

- For $\eta_t \ll 1$ and $\Delta p/p=0$ it was solved in:
  - C. B. Schroeder, D. H. Whittum, and J. S. Wurtele, "Multimode Analysis of the Hollow Plasma Channel Wakefield Accelerator", Phys. Rev. Lett. **82**, n.6, 1999, pp. 1177-1180.
- Approximate solutions (it's a very good fit, <10% deviation):

$$\frac{A}{A_0}=\exp\left(\frac{(\mu\eta_t)^2}{10+1.4(\mu\eta_t)^{1.57}}\right);\quad \begin{matrix}\mu\eta_t\le 100\\ \eta_t\le 0.1\end{matrix}$$

$$\frac{\sqrt{\overline{A^2}}}{A_0}=\exp\left(\frac{(\mu\eta_t)^2}{60+2.2(\mu\eta_t)^{1.57}}\right);\quad \begin{matrix}\mu\eta_t\le 100\\ \eta_t\le 0.1\end{matrix}$$



- Note that A is a normalized particle amplitude. For a constant plasma density and without instability A would stay constant, while the initial physical amplitude x should decrease as $1/\gamma^{\frac{1}{4}}$

Examples (FACET-II)

$$\text{Plasma: } n_0 = 4\times10^{16}\ \text{cm}^{-3},\ 60\ \text{cm long channel}$$

- p_i=10 GeV/c for both the drive and the trailing bunches, and the final momentum of trailing bunch p_f=21 GeV/c, N_d=1x10^{10} and N_t=4.3x10^9

$$\eta_P = 50\%,\ \eta_t \approx 0.12,\ \mu\eta_t \approx 11.5 \ \rightarrow\ \frac{A}{A_0} \approx 5.8$$

- If one reduces the power efficiency:

$$\eta_P = 25\%,\ \eta_t \approx 0.021,\ \mu\eta_t \approx 2 \ \rightarrow\ \frac{A}{A_0} \approx 1.3$$

- Of course, the final momentum is now p_f=15.5 GeV/c (for the same number of particles)

$$\delta\varepsilon_n = \frac{\delta x^2}{2\beta_i}\gamma_i\left(\frac{\overline{A^2}}{A_0{}^2}\right),\quad \beta_i = \frac{\sqrt{2\gamma_i}}{k_p}$$

BNS damping

- Assume a constant long. density trailing bunch. Chromatic detuning of tail particles allows to keep amplitudes constant

$$\frac{1}{1+\frac{\Delta p}{p}} - \frac{2\eta_t}{\left(1+\frac{\Delta p}{p}\right)L_t{}^2}\int_0^{\xi}(\xi-\xi')d\xi' = 1$$

$$\frac{\mathrm{D}p(\xi)}{p} = -\eta_t \frac{\xi^2}{L_t^2}$$

- We believe that the collider final focus optics and transitions between stages can not tolerate $\frac{\Delta p}{p} > 1\%$, so $\eta_t \le 0.01$

- This limits the power transfer efficiency to < 18%

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Conclusions

- We have found a universal efficiency-instability relation for plasma acceleration. Should allow for tolerance and instability analysis without detailed computer simulations.
- We considered only the ideal "trapezoidal" distributions. Real-life distributions are worse (from the efficiency perspective).
- In a blowout regime, plasma focusing is just strong enough to keep the instability in check for low power efficiencies (<25%)
  - Even for such efficiencies, external focusing and hollow channels are not viable concepts because of transverse instability.
  - Presents obvious difficulties for positrons
- BNS damping is possible but external optical systems limit the momentum spread to ~1% max. Thus, the power efficiency (drive to trailing) can not exceed ~18%.

Summary

- We wish FACET-II success and would like to be part of its science program.
- Our conclusions require confirmation by computer simulations and by experiments, especially in regimes not covered by the Lu equations (small bubble size).