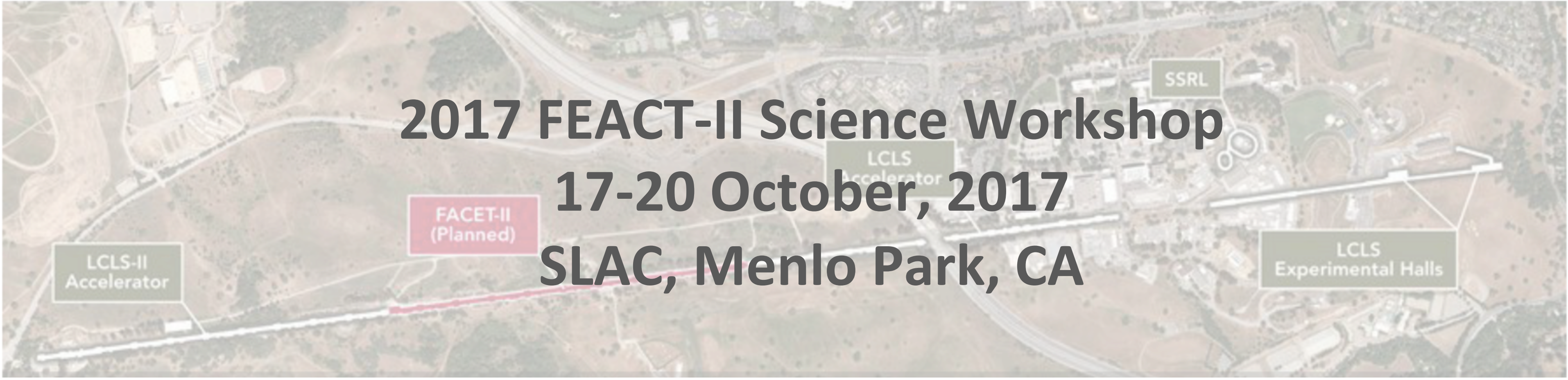

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SLAC, Menlo Park, CA

Blow-out wakes with high-order azimuthal modes

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UCLA/SLAC

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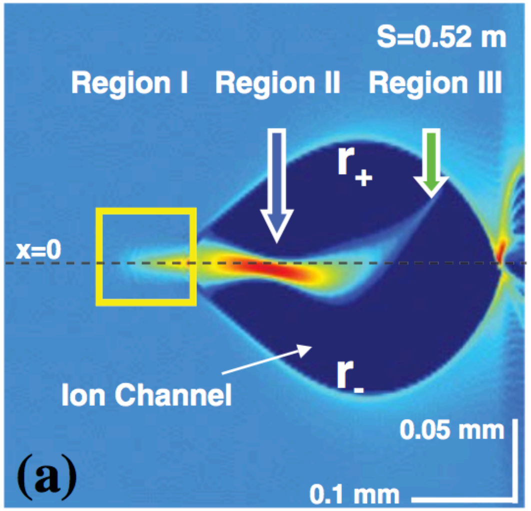
Motivation

- Blow-out wake excited by axial-symmetric beams[1]
  - ➢ linearly focusing field: $\vec{E}_\perp + \hat{z} \times \vec{B}_\perp = \frac{1}{2}\vec{r}$
  - ➢ transversely uniform acceleration field

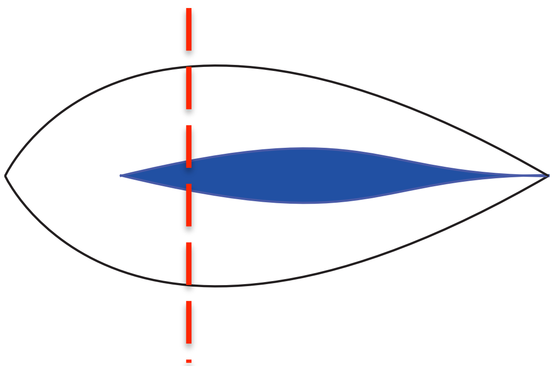
- Beams: alignment errors, asymmetric shape …
  - ➢ Modifications to the EM fields
  - ➢ Instabilities[2]

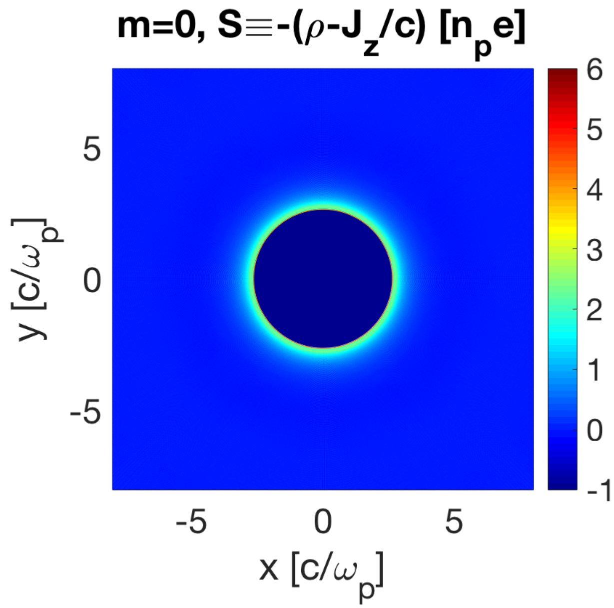


[1]W. Lu et al., PRL 96, 165002 (2006);
M. Tzoufras et al., PRL 101, 145002 (2008).
[2]David H. Whittum et al., PRL 67, 991 (1991);
C. Huang et al., PRL 99, 255001 (2007); …

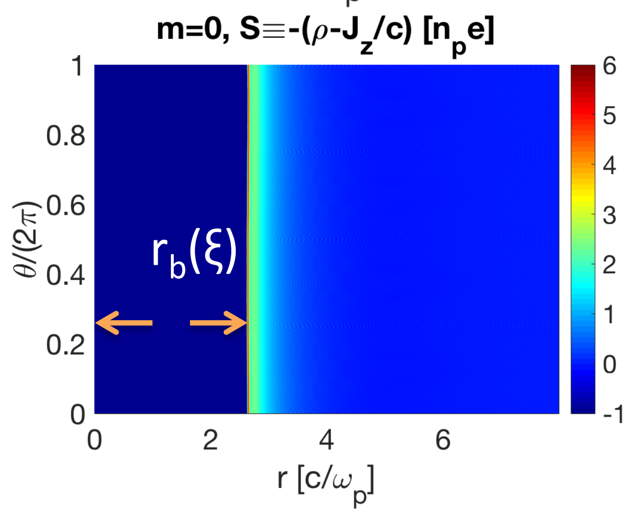
Method

axial-symmetric beams



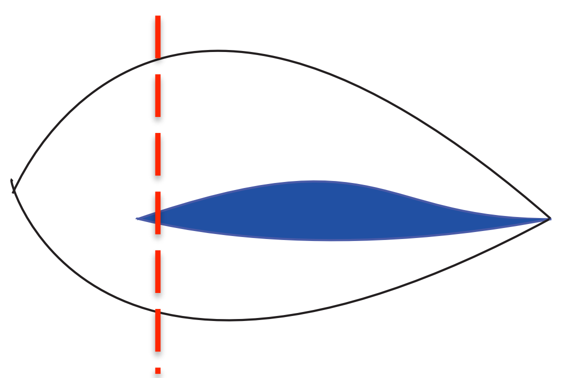


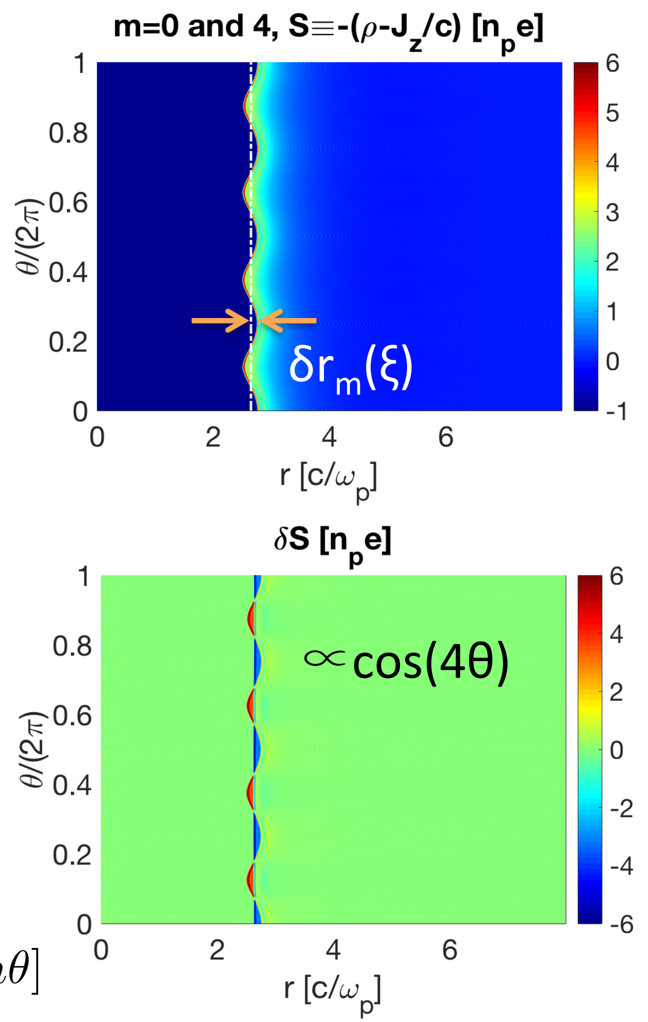
in Cartesian coordinates



in cylindrical coordinates

Method





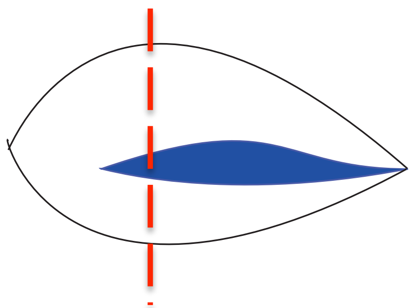
arbitrary beams:

- longi-trans correlations (m=1)
- different spot sizes (m=2)
- higher-order azimuthal components (m≥3)

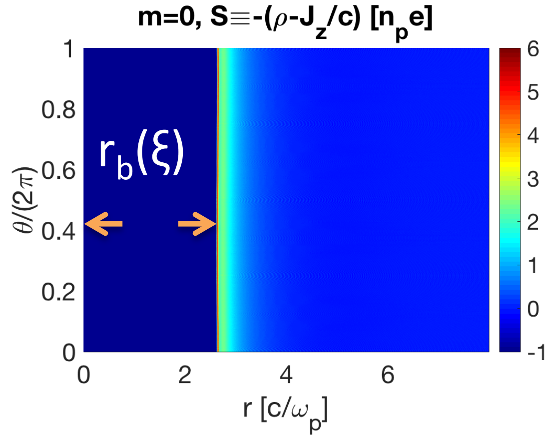
$$\rho(x,y) = \sum_{m=0}^{+\infty} [\rho_{m,c}(r)\cos m\theta + \rho_{m,s}(r)\sin m\theta]$$

Method

arbitrary beams

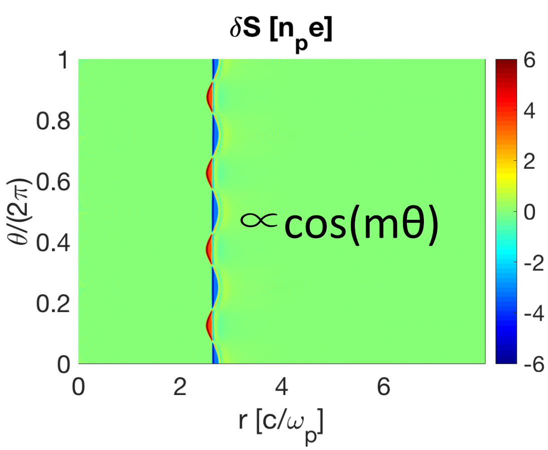


1. The m=0 wake from the m=0 beams



2. The m≥1 EM fields from the beams

3. The perturbation δr_m of the ion column boundary



4. Give the profile of high-order sheath structure S_m based on δr_m to calculate the ψ_m.

The EM fields of arbitrary relativistic beams

- 2D Poisson equation: $\nabla_\perp^2 \phi = -\rho$

  with the Green's function $G(x,y) = \int \frac{\mathrm{d}\vec{k}}{(2\pi)^2 k^2} \exp\left[i\vec{k}\cdot(\vec{r}-\vec{r}_0)\right]$

- Charge distribution: $\rho(x_0, y_0) = \sum_{m=-\infty}^{+\infty} \rho_m(r_0)\exp(im\theta_0)$

  Far from the source region:

$$E_r = \frac{1}{r}\int_0^{+\infty} r_0 \mathrm{d}r_0 \rho_0 + \sum_{m=1}^{+\infty} \frac{\cos m\theta}{r^{m+1}} \int_0^{+\infty} r_0^{m+1} \mathrm{d}r_0 \mathrm{Re}[\rho_m]$$

$$E_\theta = \sum_{m=1}^{+\infty} \frac{\sin m\theta}{r^{m+1}} \int_0^{+\infty} r_0^{m+1} \mathrm{d}r_0 \mathrm{Re}[\rho_m]$$

$$B_r = -E_\theta, B_\theta = E_r$$

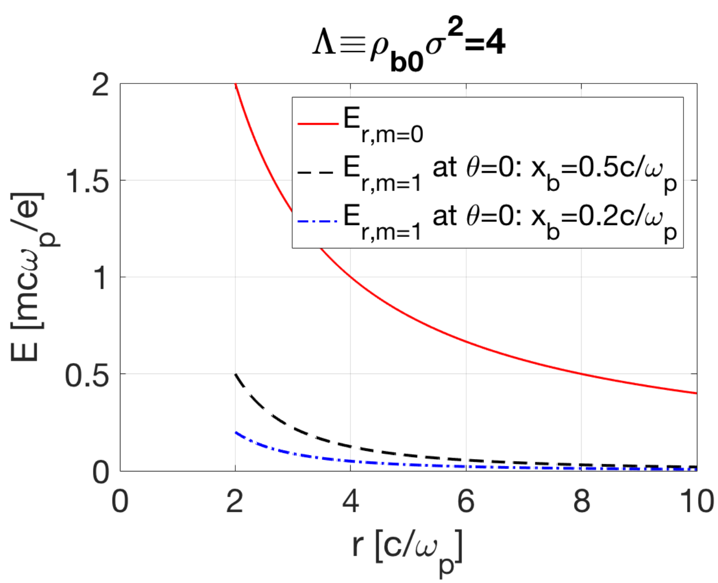
The EM fields of an arbitrary relativistic beam

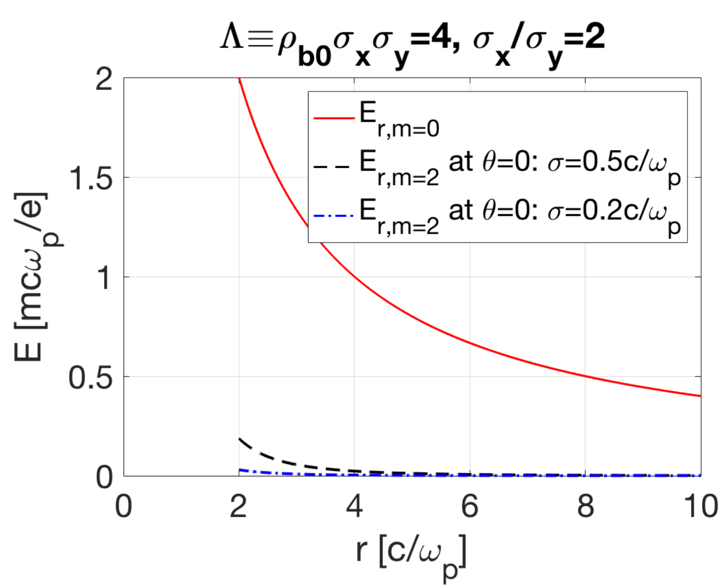
- with a transverse offset: $\rho_b(x,y) = \rho_{b0}\exp\left[-\frac{(x-x_b)^2}{2\sigma^2} - \frac{y^2}{2\sigma^2}\right]$

$$\vec{E}(r,\theta) \approx \frac{\rho_{b0}\sigma^2}{r}\left[\left(1+\frac{x_b}{r}\right)\cos\theta\hat{r} + \frac{x_b}{r}\sin\theta\hat{\theta}\right] \quad \text{when } r \gg \sigma$$

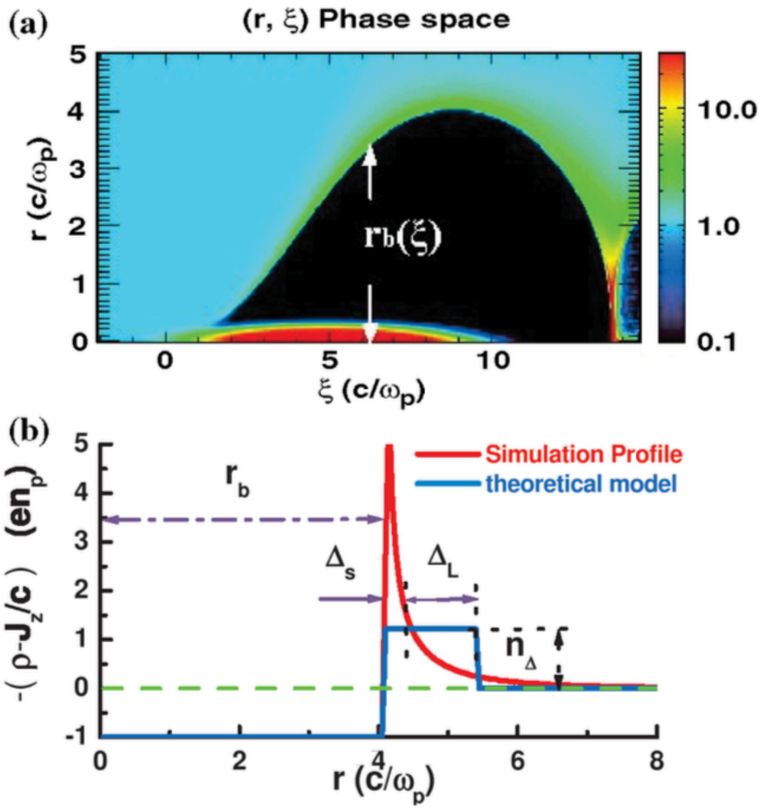
- with different spot sizes: $\rho_b(x,y) = \rho_{b0}\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)$

$$\vec{E}(r,\theta) \approx \frac{\rho_{b0}\sigma_x\sigma_y}{r}\left\{\left[1+\frac{\sigma_x^2-\sigma_y^2}{r^2}\cos 2\theta\right]\hat{r} + \left(\frac{\sigma_x^2-\sigma_y^2}{r^2}\sin 2\theta\right)\hat{\theta}\right\} \quad \text{when } r \gg \sigma$$





How to solve a m=0 blowout wake



$$\nabla_\perp^2 \psi = S \equiv -(\rho - J_z)$$

where $\psi \equiv \phi - A_z$

Describe the trajectory of a boundary plasma e⁻

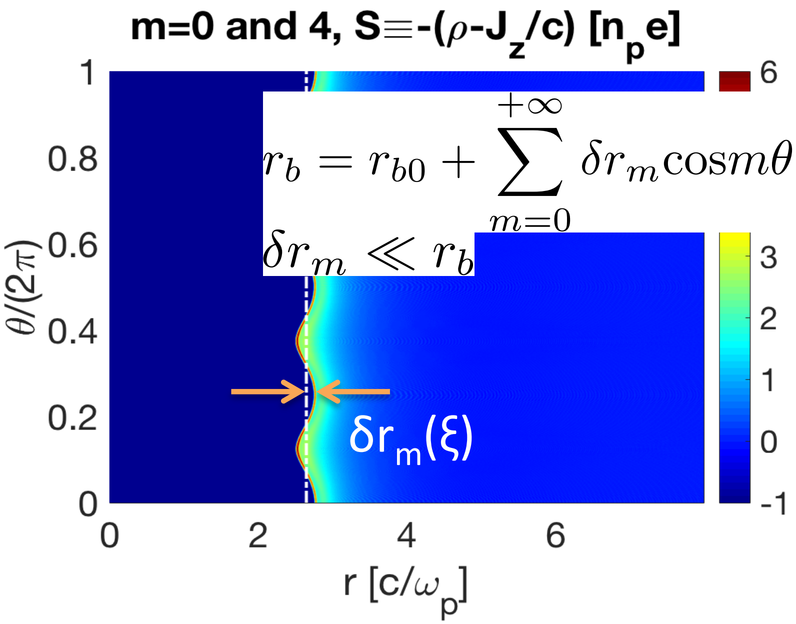
$$\begin{cases} \frac{\mathrm{d}P_\perp}{\mathrm{d}t} = -(E_r - \beta_z B_\theta) \\ \quad = -\frac{1}{2}r + (1-\beta_z)\frac{\lambda(\xi)}{r} - \frac{1}{2}(1-\beta_z)\frac{\mathrm{d}^2\psi_0}{\mathrm{d}\xi^2}r \\ \psi(r,\xi) = \psi_0(\xi) - \frac{r^2}{4} = \frac{r_b^2(\xi)}{4}(1+\beta) - \frac{r^2}{4} \end{cases}$$

$$\Rightarrow A(r_b)\frac{d^2 r_b}{d\xi^2} + B(r_b) r_b \left(\frac{dr_b}{d\xi}\right)^2 + C(r_b) r_b = \frac{\lambda(\xi)}{r_b}$$

$$\begin{cases} E_r - B_\theta = -\frac{\partial \psi(r,\xi)}{\partial r} = \frac{1}{2}r \\ E_z = \frac{\partial \psi(r,\xi)}{\partial \xi} \end{cases}$$

[1]W. Lu et al., PRL 96, 165002 (2006).

The perturbation to the ion column boundary δr_m



Describe the perturbation to the trajectory of a boundary e⁻

$$\frac{\mathrm{d}P_\perp}{\mathrm{d}t} = -(E_r - \beta_z B_\theta)$$

unperturbed + perturbed (small)

m^{th} EM fields

variation of 0^{th} EM fields due to $\delta r_m \cos m\theta$

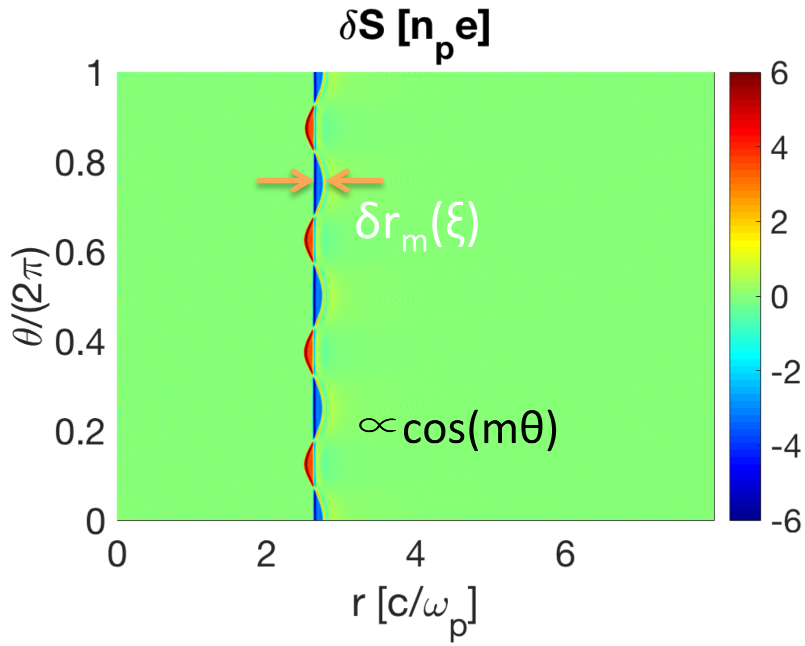
In the "short pulse limit": $\delta r''_m \approx \frac{\gamma_0}{(1+\psi_0)^2}(\vec{F}_{em} + \delta \vec{F}_0)\hat{r}$

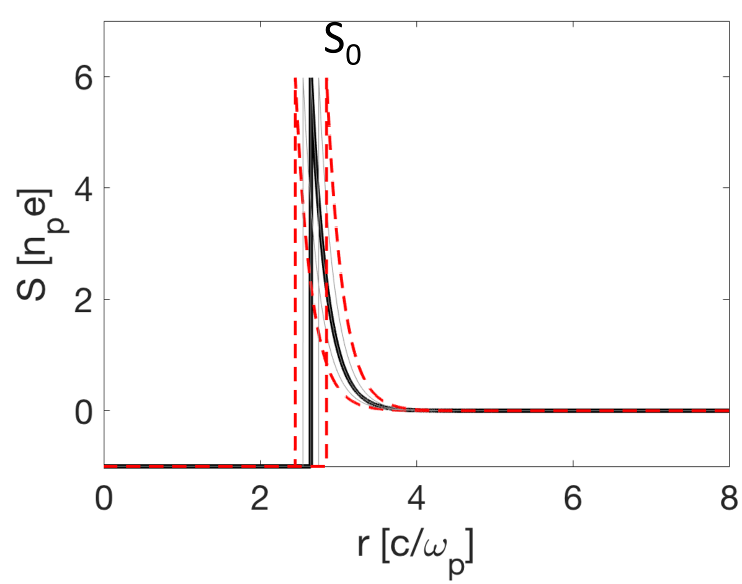
$$\delta r''_{m,c} + \frac{1}{1+\psi_0}\frac{\lambda}{r_{b0}^2}\delta r_{m,c} \approx \frac{1}{1+\psi_0}\frac{\int_0^{+\infty} r_0^{m+1}\mathrm{d}r_0 \mathrm{Re}[n_{b,m}]}{r_{b0}^{m+1}}$$

The pseudo-potential ψ_m from S_m

- The Poisson equation of ψ : $\nabla_\perp^2 \psi = S \equiv -(\rho - J_z)$

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\psi_m - \frac{m^2}{r^2}\psi_m = S_m \text{ , where } \psi = \sum_{m=0}^{+\infty} \psi_m \cos m\theta, S = \sum_{m=0}^{+\infty} S_m \cos m\theta$$





The pseudo-potential ψ_m from S_m: the constraint from the continuity eq.

- The Poisson equation of ψ : $\nabla_\perp^2 \psi = S \equiv -(\rho - J_z)$

➡ $\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\psi_m - \frac{m^2}{r^2}\psi_m = S_m$, where $\psi = \sum_{m=0}^{+\infty} \psi_m \cos m\theta, S = \sum_{m=0}^{+\infty} S_m \cos m\theta$

➢ The continuity equation of the charge

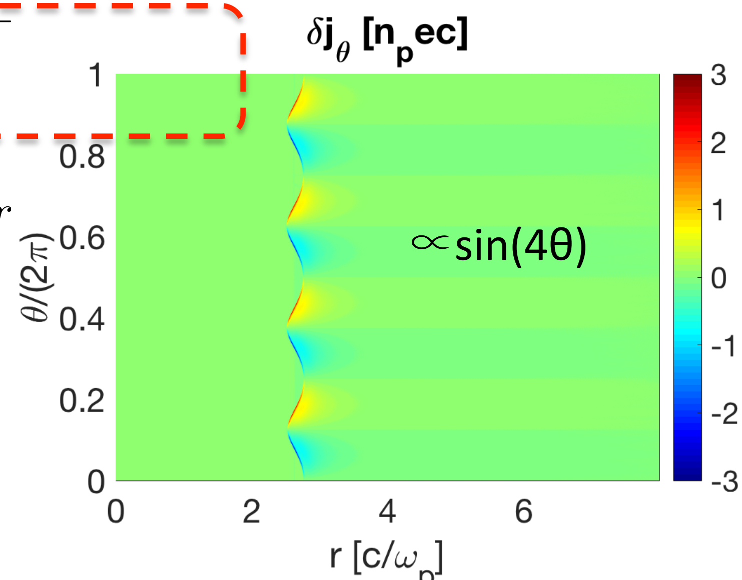
$$\frac{\partial}{\partial \xi}[-(\rho - J_z)] = \nabla_\perp \cdot \vec{J}_\perp$$

➢ After azimuthal expansion, $\frac{\partial S_m}{\partial \xi} = \frac{1}{r}\frac{\partial}{\partial r} r J_{r,m} + \frac{m}{r} J_{\theta,m}$

Integrate over r: $\frac{\mathrm{d}}{\mathrm{d}\xi}\int_0^{+\infty} r\mathrm{d}r S_m = m \int_0^{+}$

➢ when m=0, $\frac{\mathrm{d}}{\mathrm{d}\xi}\int_0^{+\infty} r\mathrm{d}r S_0 = 0 \rightarrow \int_0^{+\infty} r\mathrm{d}r$

➢ when m≥1, we need to make a connection between J_θ and n_Δ. Still working on this.



The pseudo-potential ψ_m

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}\psi_m - \frac{m^2}{r^2}\psi_m = S_m$$

$$\text{when } r \le r_{b0}(\xi), S_m = 0, \text{ then } \psi_m(r,\xi) = c_m(\delta r_m, r_{b0})r^m$$

And the mth EM fields inside the wake:

$$\vec{E}_{\perp,m} + \hat{z}\times\vec{B}_{\perp,m} = -\nabla_\perp\left[\psi_{m,c}(r,\xi)\mathrm{cos}m\theta\right]$$

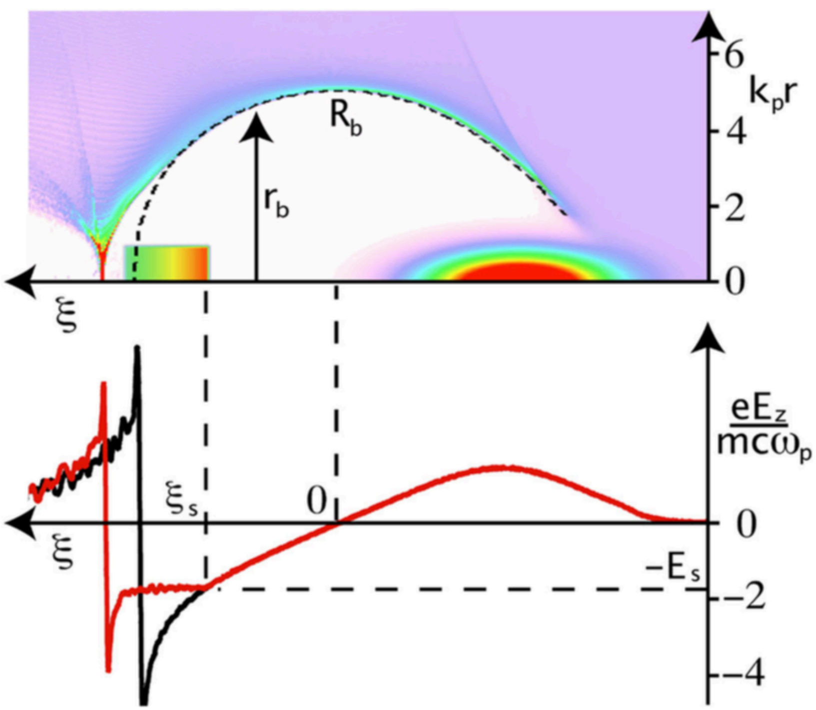
$$= -mc_m(\xi)r^{m-1}\left(\mathrm{cos}m\theta\hat{r} - \mathrm{sin}m\theta\hat{\theta}\right)$$

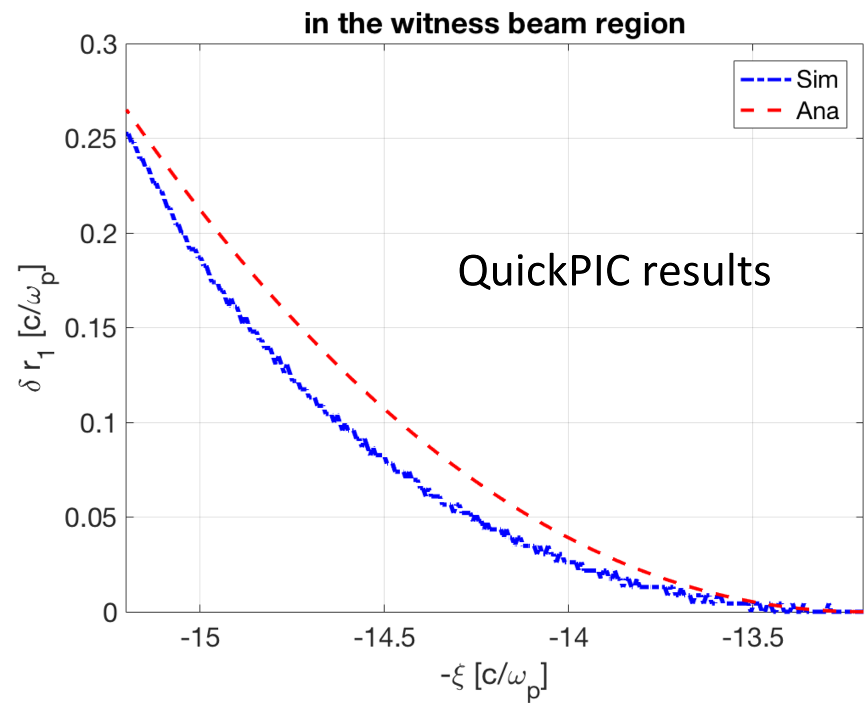
$$E_{z,m} = -\frac{\partial}{\partial\xi}\left[\psi_m(r,\xi)\mathrm{cos}m\theta\right]$$

$$= -\frac{\mathrm{d}c_m(\xi)}{\mathrm{d}\xi}r^m\mathrm{cos}m\theta$$

$$B_z = \frac{1}{r}\frac{\partial(rA_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial\theta} = (Ar^m + Br^{m-1})\mathrm{sin}m\theta$$

Comparison of δr_1 between simulations and formulas





$$\delta r''_{m,c} + \frac{1}{1+\psi_0}\frac{\lambda}{r_{b0}^2}\delta r_{m,c} \approx \frac{1}{1+\psi_0}\frac{\int_0^{+\infty} r_0^{m+1}\mathrm{d}r_0 \mathrm{Re}[n_{b,m}]}{r_{b0}^{m+1}}$$

M. Tzoufras, et al., PRL 101, 145002 (2008)

Similarities and differences between metal pipes/hollow channel and blowout wake

	Metal pipes[1] / plasma hollow channel[2]	Nonlinear blowout wake
structure	preformed	excited by m=0 components
perturbations (satisfy linear superposition)	all m modes	m≥1 modes
method	boundary conditions	study the profile of the sheath

[1]Chao Alexander, Physics of collective beam instabilities in high energy accelerators, 1993.
[2]C. B. Schroeder, et al., PRL 82.1177 (1999).

- Applications
  - Study the growth of “slice” emittance and energy spread in a static wake
  - Study instabilities
  - Study “general” beam loading with high-order modes
  - …

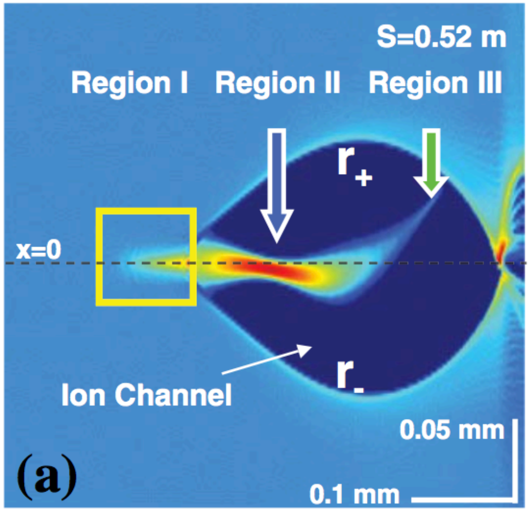
Instabilities induced by the high order modes

(1) m=1: hosing[1]

$$\delta r_1'' + \frac{\lambda}{r_{b0}^2}\frac{1}{1+\psi_0}\delta r_1 = \frac{\lambda}{r_{b0}^2}\frac{1}{1+\psi_0}x_b$$

$$\vec{E}_{\perp,1} + \hat{z}\times\vec{B}_{\perp,1} = -c_1(\delta r_1, r_{b0})\hat{x}$$

$$\rightarrow \frac{\partial^2 x_b}{\partial s^2} + \frac{1}{\gamma_b}\frac{\partial \gamma_b}{\partial s}\frac{\partial x_b}{\partial s} + \frac{1}{2\gamma_b}x_b = \frac{c_1(\delta r_1, r_{b0})}{\gamma_b}$$



(2) m=2: quadrupole instability

$$\delta r_2'' + \frac{\lambda}{r_{b0}^2}\frac{1}{1+\psi_0}\delta r_2 = \frac{\lambda}{r_{b0}^3}\frac{1}{1+\psi_0}\left(\sigma_x^2 - \sigma_y^2\right)$$

$$\vec{E}_{\perp,2} + \hat{z}\times\vec{B}_{\perp,2} = -2c_2(\delta r_1, r_{b0})x\hat{x} + 2c_2(\delta r_1, r_{b0})y\hat{y}$$

$$\rightarrow \frac{\partial^2 \sigma_x}{\partial s^2} + \frac{1-4c_2}{2\gamma_b}\sigma_x = \frac{\epsilon_{nx}^2}{\gamma_b^2\sigma_x^3}$$

$$\frac{\partial^2 \sigma_y}{\partial s^2} + \frac{1+4c_2}{2\gamma_b}\sigma_y = \frac{\epsilon_{ny}^2}{\gamma_b^2\sigma_y^3}$$

[1]David H. Whittum et al., PRL 67, 991 (1991);
C. Huang et al., PRL 99, 255001 (2007).

Is there a Green's function $W_{\parallel,\perp,m}(\xi_s, \xi - \xi_s)$?

- No for beams with m=0 component
  - m=0 component modify the wake shape: r_{b0}
  - The equation of the perturbation δr_m

$$\delta r''_{m,c} + \frac{1}{1+\psi_0}\frac{\lambda}{r_{b0}^2}\delta r_{m,c} \approx \frac{1}{1+\psi_0}\frac{\int_0^{+\infty} r_0^{m+1}\mathrm{d}r_0\mathrm{Re}[n_{b,m}]}{r_{b0}^{m+1}}$$

- Yes for a beam with pure high-order (m≥1) azimuthal components

$$\delta r''_{m,c} \qquad \approx \frac{1}{1+\psi_0}\frac{\int_0^{+\infty} r_0^{m+1}\mathrm{d}r_0\mathrm{Re}[n_{b,m}]}{r_{b0}^{m+1}}$$

$$\Rightarrow \delta r_m(\xi > \xi_s) = \frac{\int_0^{+\infty} r_0^{m+1}\mathrm{d}r_0\mathrm{Re}[n_{b,m}]}{r_{bs}^{m+1}\left[1+\psi_0(\xi_s)\right]}(\xi - \xi_s)$$

Summary

- We are working on developing a model for the nonlinear blowout wakes with high-order azimuthal modes.
- This model can help us to study the growth of the emittance, energy spread in such a wake, study the instabilities due to the coupling of high-order modes of the beams and the wake, and study “general” beam loading.

- Thanks!