Studying quantum beamstrahlung and nonperturbative QED with beam-beam collisions at FACET-II

FACET-II Science Workshop 2017 @ SLAC

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Investigating strong-field QED @ SLAC:

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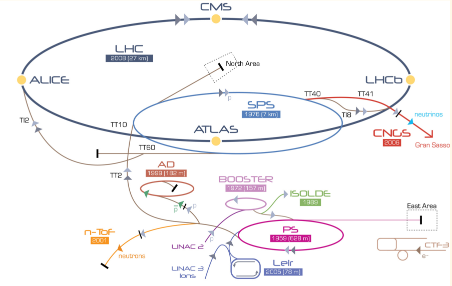
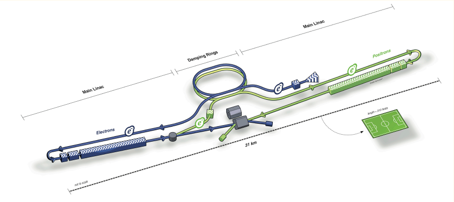
Motivation: beamstrahlung in future linear colliders

Large Hadron Collider (LHC)



International Linear Collider (ILC)



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- Why a linear electron-positron collider?
  - Clean interaction (unlike protons e^+/e^- are elementary particles)
  - Initial state known (protons: parton distribution functions)
  - TeV-scale e^+/e^- – ring not feasible due to synchrotron radiation energy loss
- Beamstrahlung at the interaction point:
  - High luminosity → high charge density → strong fields → beamstrahlung
  - **Beamstrahlung theory has never been tested in the quantum regime**

Motivation: studying nonperturbative QED

Important scales of QED

Energy	$\mathcal{E} = mc^2$	$10^6\,\mathrm{eV}$	relativistic effects
Length	$\lambda\!\!\!^{-}_C = \hbar c/(mc^2)$	$10^{-13}\,\mathrm{m}$	quantum fluctuations
Field strength	$E_{\mathrm{cr}} = (mc^2)^2/(\lvert e\rvert\,\hbar c)$	$10^{18}\,\mathrm{V/m}$	nonperturbative effects

Electron/positron mass (m) and charge ($e < 0$) determine fundamental scales

Relativity: Dirac equation

- Changed dispersion relation: $\epsilon = mv^2/2$ vs. $\epsilon = \gamma mc^2$
- Spin degree of freedom
- Antiparticles

Quantum fluctuations: QFT

- Virtual particles
- Lamb shift of atomic levels
- Anomalous magnetic moment
- Running coupling constant

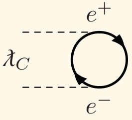
- At each fundamental scale the theory changes qualitatively
- Nature surprised us whenever we tested a fundamental scale
- **QED critical field E_{cr} has never been exceeded experimentally**

We use natural units from now on $\epsilon_0 = \hbar = c = 1$ (often restored for clarity)

The QED critical field & spontaneous pair production

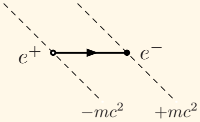
- According to quantum mechanics (Heisenberg uncertainty principle) the vacuum contains virtual electron-positron pairs (pictorial model)
- Spatial scale of these quantum fluctuations: $\lambda\!\!\!^{-}_C = \hbar/(mc)$
- If an electric field is able to transfers the rest energy $2mc^2$ to these pairs within their lifetime, they become real: $E_{\mathrm{cr}} = mc^2/(|e|\,\lambda\!\!\!^{-}_C)$

Vacuum fluctuations



Instead of being empty, the vacuum is filled with quantum fluctuations

Heuristic tunneling picture



"Tilted" energy levels $\longrightarrow$ tunneling
Probability: $\sim \exp\left(-\pi E_{\mathrm{cr}}/E\right)$

Critical field correspons to critical (laser) intensity $I_{\mathrm{cr}} = 4.6 \times 10^{29}\,\mathrm{W/cm^2}$:

	$\sim \hbar\omega$	Future facilities	I (intensity)	current
optical	$1\,\mathrm{eV}$	APOLLON, ELI,...	$10^{24-25}\,\mathrm{W/cm^2}$	$10^{22}\,\mathrm{W/cm^2}$
x-ray	$10\,\mathrm{keV}$	LCLS-II, XFEL,...	$10^{27}\,\mathrm{W/cm^2}$ (if focused)	$10^{21}\,\mathrm{W/cm^2}$

Reaching the QED critical field in a laboratory experiment

- The critical field cannot be reached directly (with existing technology)
- Fortunately, the electric/magnetic field is not Lorentz invariant:

$$\boldsymbol{E}' = \gamma(\boldsymbol{E} + \boldsymbol{\beta} \times \boldsymbol{B}) - \frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\boldsymbol{E}),$$

$$\boldsymbol{B}' = \gamma(\boldsymbol{B} - \boldsymbol{\beta} \times \boldsymbol{E}) - \frac{\gamma^2}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\boldsymbol{B})$$

- Decisive measure: electric field in the electron rest frame (E^*):

$$\chi = \frac{\sqrt{pF^2p}}{E_{\mathrm{cr}}mc^2} = \frac{E^*}{E_{\mathrm{cr}}}, \quad E_{\mathrm{cr}} = \frac{m^2c^3}{\hbar\,|e|} \approx 1.3 \times 10^{18}\,\mathrm{V/m}$$

Electron-laser collisions

$$\chi \approx 0.5741\,\frac{\epsilon}{10\,\mathrm{GeV}}\sqrt{\frac{I}{10^{20}\,\mathrm{W/cm^2}}}$$

I: laser intensity ϵ: electron energy (head-on collision)

Static magnetic field

$$\chi = \gamma\frac{B}{B_{\mathrm{cr}}}, \quad B_{\mathrm{cr}} = \frac{m^2c^2}{(\hbar\,|e|)}$$

If $\epsilon \gg mc^2$ and $E \ll E_{\mathrm{cr}}$, $B \ll B_{\mathrm{cr}}$: only χ is important

Ritus, J. Sov. Laser Res. **6**, 497–617 (1985); Di Piazza et al., Rev. Mod. Phys. **84**, 1177 (2012)

Probing the QED critical field in beam-beam collisions

Quantum parameter (beamstrahlung parameter)

$$\chi = \frac{|e|}{m^3}\sqrt{p^\mu F_{\mu\nu}^2 p^\nu}, \quad \chi_{\max} \approx \frac{2Nr_e^2\epsilon}{\alpha m \sigma_z(\sigma_x+\sigma_y)}$$

Compares the boosted electric field with the QED critical field, often the symbol $\Upsilon = \chi$ or $\Upsilon = 3\chi/2$ is used

(ϵ, p^μ: electron energy/four-momentum, $r_e \approx 2.8 \times 10^{-13}\,\mathrm{cm}$ N: particles per bunch, $\sigma_{x,y,z}$: r.m.s. bunch dimensions)

Facility		Energy [GeV]	#Particles [10^{10}]	$\sigma_x[\mu m]$	$\sigma_y[\mu m]$	$\sigma_z[\mu m]$	$\chi_{\max}$
ILC	base	250	2	0.474	0.0059	300	**0.15**
	upgrade	500					**0.30**
CLIC	base	190	0.37	0.045	0.0009	44	1.5
	upgrade	1500					12
FACET II	base	10	1.2	18	12	1.8	0.01
	upgrade		0.7	3	2	0.5	**0.13**
MACE	small-β^*	125	0.5	0.01	0.01	0.1	1300
	small-z		0.06			0.01	1700

K. Yokoya and P. Chen, Frontiers of Particle Beams, 415–445 (1992)

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FACET II +	**Plasma lense**	10	1	**0.04**	**0.04**	1	**5**
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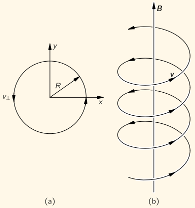
K. Yokoya and P. Chen, Frontiers of Particle Beams, 415–445 (1992)

Synchrotron radiation: classical description

Motion in a static B field



Transverse motion:

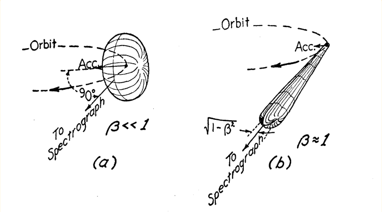
e^-/e^+ with energy ϵ: circular orbit, radius

$$\rho = \frac{\epsilon}{(|e|B)} = \frac{\gamma^2 \hbar}{\chi m c}$$

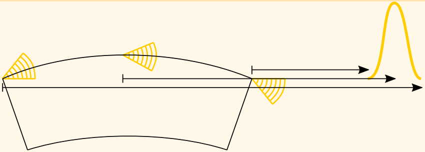
Longitudinal motion:

free propagation

Radiation pattern



Formation region and critical frequency



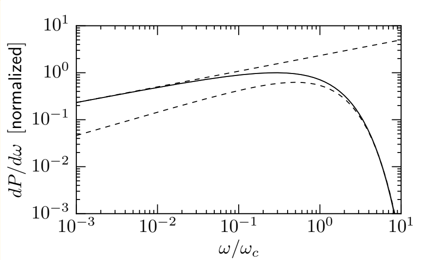
- **Formation length**: $l_f = \rho/\gamma$ (contributing circular segment)
- **Critical frequency:** $\omega_c \sim 1/T = c\gamma^3/\rho$ (typical frequency)

  [Fourier transform; burst duration: $T = \left(\frac{1}{v} - \frac{1}{c}\right) l_f \approx \rho/(c\gamma^3)$]

Jackson, *Classical Electrodynamics* (1999)

Synchrotron radiation: classical spectrum & total power

Synchrotron spectrum



Characteristic scalings

- Small frequencies:
  $dP/d\omega \sim (\omega/\omega_c)^{1/3}$
- Large frequencies:
  $dP/d\omega \sim \sqrt{\omega/\omega_c}\exp(-\omega/\omega_c)$

Critical frequency: $\hbar\omega_c = (2/3)\epsilon\chi$

Plot (left side): $\chi = 10^{-3}$, i.e. strong exponential suppression well before $\hbar\omega = \epsilon$

Total radiation power

Power P (energy per unit time) emitted per electron:

$$P \sim \alpha \cdot \frac{c}{l_f} \cdot \hbar\omega_c \sim \alpha \cdot \frac{mc^2}{\hbar}\frac{\chi}{\gamma} \cdot \epsilon\chi = \alpha\chi^2\frac{(mc^2)^2}{\hbar} \sim \frac{1}{m^4}$$

exact: $P = \alpha\chi^2(2/3)m^2$; $m^2 = 63.56 \times 10^6\,\mathrm{W}$; $\alpha \sim 1/\hbar$; $\chi \sim \hbar$

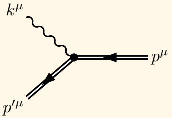
Intuitive derivation: photon emission probability per formation time c/l_f is α; typical energy of the radiated photon: $\hbar\omega_c$

Schwinger, *On the Classical Radiation of Accelerated Electrons*, Phys. Rev. **75**, 1912 (1949)

The QED critical field in classical electrodynamics (CED)

- Photon emission and pair production: related by a crossing symmetry
- However, pair production has a kinematic threshold ($\epsilon \geq mc^2$)
  $\longrightarrow$ Pair production is exponentially suppressed for $\chi_\gamma \ll 1$
- Photon emission with $\omega \gtrsim \omega_c$ shows similar suppression (tunneling exponent)
  $\longrightarrow$ Violation of the uncertainty principle is exponentially penalized

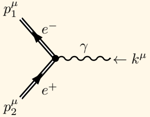
Photon emission



Photon emission probability if $\hbar\omega \gtrsim \chi\epsilon$:
$dP/d\omega \sim \exp\left[-(3\hbar\omega/(2\chi\epsilon)\right]$

$$\chi = \frac{\hbar c\,|e|}{(mc^2)^3}\sqrt{p^\mu F^2_{\mu\nu} p^\nu}$$

Electron-positron photoproduction



Probability to create a pair ($\chi_\gamma \ll 1$):
$\sim \exp\left[-8/(3\chi_\gamma)\right]$

$$\chi_\gamma = \frac{\hbar c\,|e|}{(mc^2)^3}\sqrt{k^\mu F^2_{\mu\nu} k^\nu}$$

- CED predicts a qualitative change at the critical field ($\chi \gtrsim 1$):
  $\longrightarrow$ emission of photons with $\hbar\omega > \epsilon$ feasible (wrong, CED breakdown)
- General conclusion: **If a theory predicts a qualitative change at a certain scale, one should test this scale experimentally!**

Why does radiation possess a classical limit?

Quantity	"Compensation" of quantum effects
$P = \frac{2}{3}\alpha\chi^2\frac{(mc^2)^2}{\hbar}$	light quantization ($\hbar\omega_c$) vs. emission probability (α)
$l_f = \frac{\gamma}{\chi}\frac{\hbar}{mc}$	Compton length [$\lambda\!\!\!^{-}_C = \hbar/(mc)$] vs. critical field ($\chi$)

- In the classical limit ($\chi \ll 1$) $\hbar$ must disappear from all quantities
- Therefore, the formation length has to be macroscopic ($l_f \gg \gamma\lambda\!\!\!^{-}_C$)

  $\longrightarrow$ Possible, as typical photon energy $\hbar\omega_c = (2/3)\epsilon\chi \ll \epsilon$ is very small, therefore, the uncertainty principle can be satisfied:

Momentum conservation

$$\begin{pmatrix}\epsilon\\ p_x\\ 0\\ 0\end{pmatrix} + \begin{pmatrix}0\\ q_x\\ q_y\\ 0\end{pmatrix} = \begin{pmatrix}\epsilon'\\ p'_x\\ p'_y\\ p'_z\end{pmatrix} + \begin{pmatrix}\omega\\ k_x\\ k_y\\ k_z\end{pmatrix}$$

ϵ ($\boldsymbol{p}$), ϵ' ($\boldsymbol{p}'$), ω ($\boldsymbol{k}$): energy (momentum) of the initial electron, final electron, emitted photon; $\boldsymbol{q}$: momentum transfered by the field

Uncertainty principle

- After some algebra: $|q_x| \sim \chi mc/\gamma$

  [$\epsilon' = \epsilon - \omega \longrightarrow p'_x \approx p_x - k_x - \omega/(2\gamma^2)$

  $\longrightarrow -q_x = \omega/(2\gamma^2) \sim \omega_c/\gamma^2 \sim \chi m/\gamma$]
- $k_y, k_z \sim k_x/\gamma$ are subleading ($1/\gamma$ cone)

$$|q_x|\, l_f \sim \frac{\chi mc}{\gamma}\frac{\hbar\gamma}{\chi mc} \sim \hbar$$

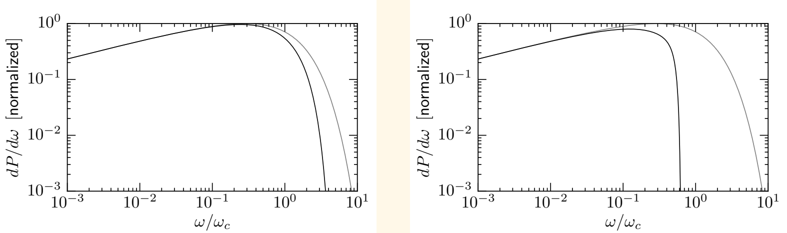
How does QED fix the problems of CED at the scale $\chi \gtrsim 1$?

- If $\chi \gtrsim 1$ classical electrodynamics predicts $\omega_c \sim \epsilon\chi \gtrsim \epsilon$ (not possible)
- The recoil ($\hbar\omega$) induced by the emitted photon becomes important
  $\longrightarrow$ quantization of the photon field must be taken into account
- Semiclassical approach: classical trajectory + photon recoil at the vertex

$$\frac{dP}{d\omega} = \frac{dP}{du}\frac{du}{d\omega}, \quad \frac{dP}{du} = -\alpha m^2 \frac{u}{(1+u)^3}\left\{ \int_z^\infty dt\ \mathrm{Ai}(t) + \frac{\mathrm{Ai}'(z)}{z}\left[2 + \frac{u^2}{(1+u)}\right]\right\},$$

$z = (u/\chi)^{2/3}$; $u \approx \omega/(\epsilon - \omega)$; $du/d\omega \approx (1+u)^2/\epsilon$; Classical limit: $u \approx \omega/\epsilon \sim \chi \ll 1$ ($z \sim 1$)

From classical to quantum electrodynamics



$\chi = 0.1$: onset of quantum corrections $\qquad$ $\chi = 1$: quantum corrections decisive

solid curve: quantum calculation; dotted curve: classical prediction

Baier, Katkov, Strakhovenko: *Electromagnetic Processes at High Energies in Oriented Single Crystals* (1998)

Suppression of radiation in the quantum regime

	classical scaling ($\chi \ll 1$)	quantum scaling ($\chi \gtrsim 1$)
formation length	$l_f = \frac{\gamma}{\chi}\frac{\hbar}{mc}$	$l_f = \frac{\gamma}{\chi}\frac{\hbar}{mc}(1+\chi/u)^{1/3}$
critical frequency	$\hbar\omega_c = \frac{2}{3}\epsilon\chi;\ \left[u \approx \frac{\omega}{\epsilon} \sim \chi\right]$	$\hbar\epsilon \gtrsim \omega_c;\ \left[u \approx \frac{\omega}{(\epsilon-\omega)} \gtrsim 1\right]$

- The emitted photon energy no longer increases with χ
- The formation length decreases slower due to the factor $(1+\chi/u)^{1/3}$

Total emitted power

$$P = -\alpha P_0 \chi^2 \int_0^\infty dz\, z \frac{4u^2+5u+4}{2(1+u)^4}\mathrm{Ai}'(z),$$

$$\frac{P}{\alpha P_0} \approx \frac{2^5\Gamma(2/3)}{3^5}(3\chi)^{2/3} \approx 0.37\chi^{2/3}\ (\chi \gg 1)$$

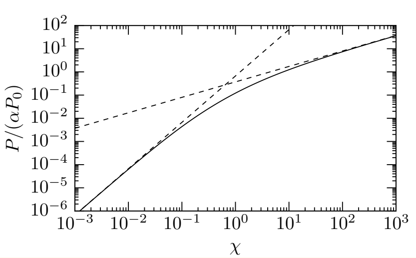
$$u = \chi z^{3/2},\ P_0 = (mc^2)^2/\hbar$$

Intuitive derivation

$$P \sim \alpha \cdot c/l_f \cdot \hbar\omega \sim \alpha\chi^{2/3}P_0$$

photon emission probability per formation time c/l_f is α; typical photon energy: $\hbar\omega \lesssim \epsilon$

General result vs. asymptotics



dashed lines: $\chi \ll 1$ and $\chi \gg 1$ asymptotics

Baier, Katkov, Strakhovenko: *Electromagnetic Processes at High Energies in Oriented Single Crystals* (1998)

Full quantum regime: intuitive derivation of the scaling

	$\hbar \sim \chi \ll 1$: independence of $\hbar$	$m^{-3} \sim \chi \gg 1$: independence of m
Formation length	$l_f = \frac{\epsilon}{\chi} \frac{\hbar c}{(mc^2)^2} \sim m$	$l_f = \frac{\epsilon}{\chi^{2/3}} \frac{\hbar c}{(mc^2)^2} \sim \hbar^{1/3}$
Power	$P \approx 0.66 \alpha \chi^2 \frac{(mc^2)^2}{\hbar} \sim \frac{1}{m^4}$	$P \approx 0.37 \alpha \chi^{2/3} \frac{(mc^2)^2}{\hbar} \sim \frac{1}{\hbar^{4/3}}$

- Even if ($\epsilon \gg mc^2$): physical observables depend on the rest energy mc^2
  $\longrightarrow$ Relativity (length contraction): $l_f \sim \epsilon$, independence of $\hbar$: $l_f \sim 1/\chi$
  $\longrightarrow$ Photon-electron coupling strength: $P \sim \alpha$, independence of $\hbar$: $P \sim \chi^2$
- **If $\chi \gtrsim 1$: $p^2 = m^2$ and $(e^2 pF^2 p)^{1/3}$ change "mass hierarchy"**
  $\longrightarrow$ The leading-order contribution should be independent of m
  [note: we cannot neglect $e^2 pF^2 p$ if $\chi \ll 1$, radiation requires field]

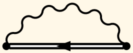
Approximations employed and their breakdown

- So far the only quantum correction is the recoil at the emission vertex
- Emission of virtual photons (radiative corrections) neglected
- Emission of multiple photons within one formation length neglected

These assumptions seem to break down in the regime $\alpha\chi^{2/3} \gtrsim 1$

Emission of virtual photons: radiative corrections

Field-induced mass shift



$$\frac{\delta m^2}{m^2} = \frac{\alpha}{\pi}\int_0^\infty \frac{du}{(1+u)^3}\,\frac{5+7u+5u^2}{3z}\,f'(z),$$

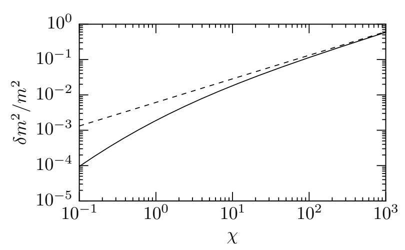
$$\Re(\delta m^2/m^2) \approx 0.84\alpha\chi^{2/3}\ (\chi \gg 1)$$

$$f(z) = \pi[\mathrm{Gi}(z) + i\,\mathrm{Ai}(z)],\ z = (u/\chi)^{2/3}$$

$\longrightarrow$ If $\alpha\chi^{2/3} \gtrsim 1$ $\delta m \approx m$!

$\longrightarrow$ higher-order diagrams important

General result vs. asymptotics



dashed line: $\chi \gg 1$ asymptotics

- Many conceptual questions related to $\alpha\chi^{2/3}\gtrsim 1$ remain unsolved
- So far the regime $\alpha\chi^{2/3}\gtrsim 1$ has been considered as very interesting but experimentally unaccessible, even in the far future

Now we have a realistic road map to this scale

$\longrightarrow$ Theory: strong reason to revisit & extend existing calculations

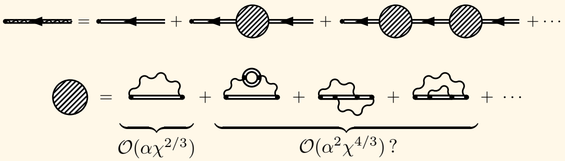
$\longrightarrow$ Experiment: very first access to strongly-coupled QED!

Ritus, Sov. Phys. JETP **30**, 1181 (1970); SM and Di Piazza, PRL **107**, 260401 (2011)

From CED to the fully nontperturbative quantum regime

- In general, also the emission of virtual photons must be taken into account
- If $\alpha\chi^{2/3} \gg 1$ the emission of virtual photons is no longer perturbative
  $\longrightarrow$ Conjecture by Ritus & Narozhny: $\alpha\chi^{2/3}$ is true expansion parameter

Radiative corrections inside a background field



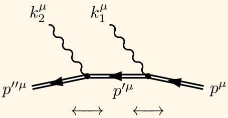
Exact electron wave function (top), Mass operator (bottom)

Different regimes of strong-field QED:

1. $\chi \ll 1$: **classical regime**
   Quantum effects are very small, pair production is exponentially suppressed
2. $\chi \gtrsim 1, \alpha\chi^{2/3} \ll 1$: **quantum regime (FACET II)**
   Recoil and pair production are important, but the radiation field is a perturbation
3. $\alpha\chi^{2/3} \gtrsim 1$: **fully nonperturbative regime (100 GeV collider)**
   Perturbative treatment of the radiation field breaks down

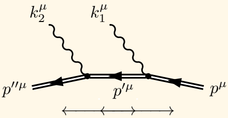
Emission of multiple photons: radiation reaction

Incoherent emissions



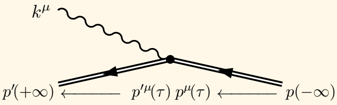
Emission vertices are well separated, standard approx. in numerical codes

Coherent emissions



Formation regions overlap, emission processes cannot be separated

Semiclassical description of photon emissions



- Classical motion between subsequent emissions
- Photon recoil changes the trajectory discontinuously

Classical radiation reaction $(\chi \ll 1)$

- Each emitted photon carries only a very small fraction of the electron energy
- The electron energy is changed *adiabatically* over many emissions

Quantum radiation reaction $(\chi \gtrsim 1)$

- The recoil of a *single* photon changes energy and trajectory significantly
- The changed electron trajectory strongly modifies the subsequent emissions

A. Di Piazza et al., Rev. Mod. Phys. **84**, 1177 (2012)

Summary: four options to study strong-field QED at SLAC

FACET-II beam and existing laser

FY 19-21

- 20 TW laser available (synchronized with FACET-II electron beam)
- Can start immediately together with FACET-II (required hardware exists)

FACET-II Facility for Advanced Accelerator Experimental Tests

$\chi \approx \mathbf{0.3}$

Upgraded laser & 10 GeV beam

FY 19-21

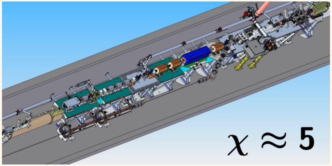
- 100 TW-class laser upgrade, e.g., 4 J, 35 fs, $10-100\,(\mu\mathrm{m})^2$ spot size
- Could also be combined with the LCLS electron beam (15 GeV)

$I \approx 5 \times 10^{20}\ \mathrm{W/cm^2}$, $\chi \approx \mathbf{1.5}$

10 GeV & 0.3 GeV "witness injector"

FY 22-25

- 175 kA 10 GeV + 300 MeV e^--beams
- Plasma-lense focus to $(40\,\mathrm{nm})^2$

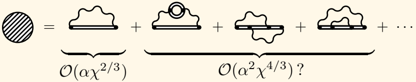


100 GeV e^-e^+ collider with $\chi \gtrsim 10^3$

Future facility ~ 20 years

- Full breakdown of perturbation theory
- No existing calculation applicable

$$\chi \gtrsim 10^3, \text{ i.e., } \alpha\chi^{2/3} \gtrsim 1$$



V. Yakimenko, ExHILP, September 5, 2017

Thank you for your attention
and your questions!