

# High fields computational challenges

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## **Implementation of QED effect**

PIC loop + QED + Merging algorithm

## **Strong field QED with lasers**

Laser-beam + cascades + vacuum birefringence

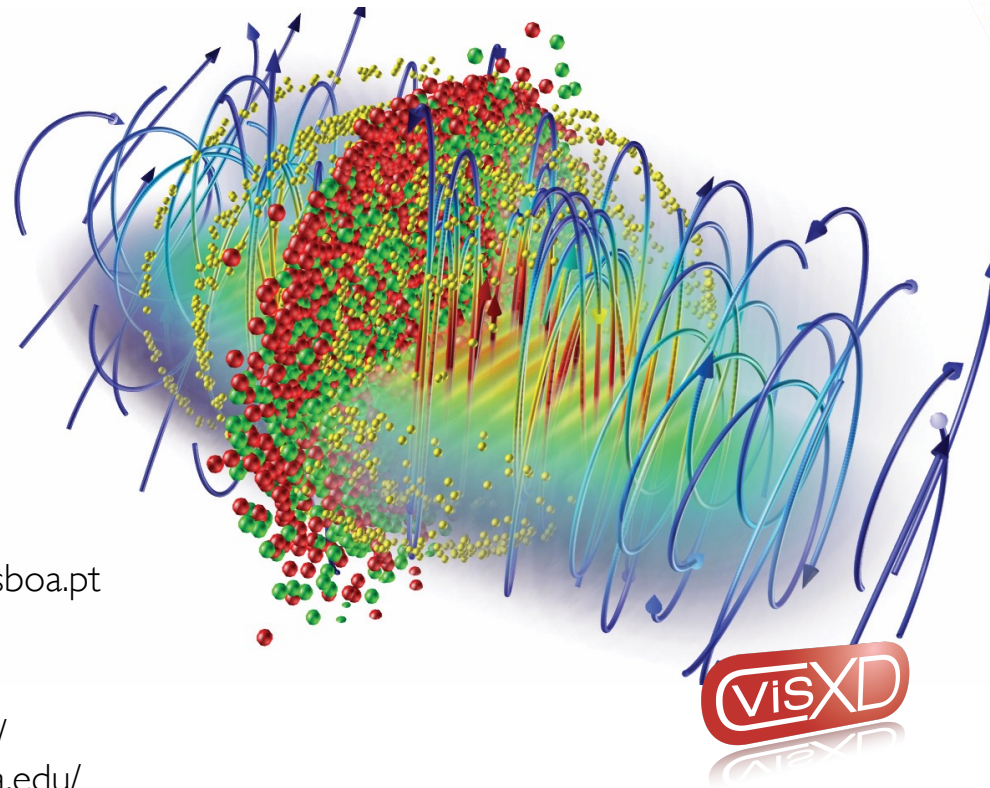
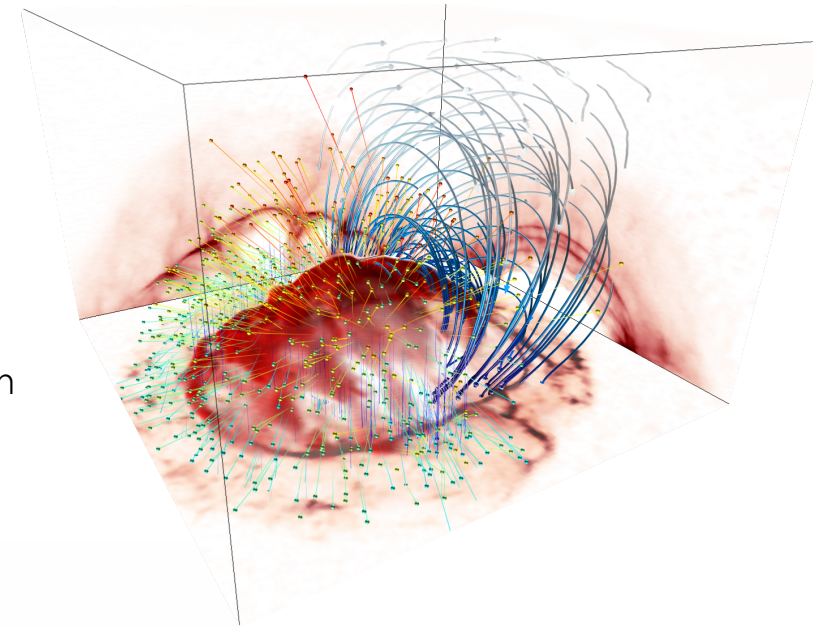
## **Beam-beam physics**

Disruption + strong field QED with LWFA and SLAC



## osiris framework

- Massively Parallel, Fully Relativistic Particle-in-Cell (PIC) Code
- Visualization and Data Analysis Infrastructure
- Developed by the osiris.consortium  
⇒ UCLA + IST



## code features

- Scalability to ~ 1.6 M cores
- Dynamic Load Balancing
- GPGPU and Xeon Phi support
- Particle merging
- **QED module**
- Quasi-3D
- Current deposit for NCI
- Collisions
- **Radiation reaction**
- Ponderomotive guiding center

Ricardo Fonseca

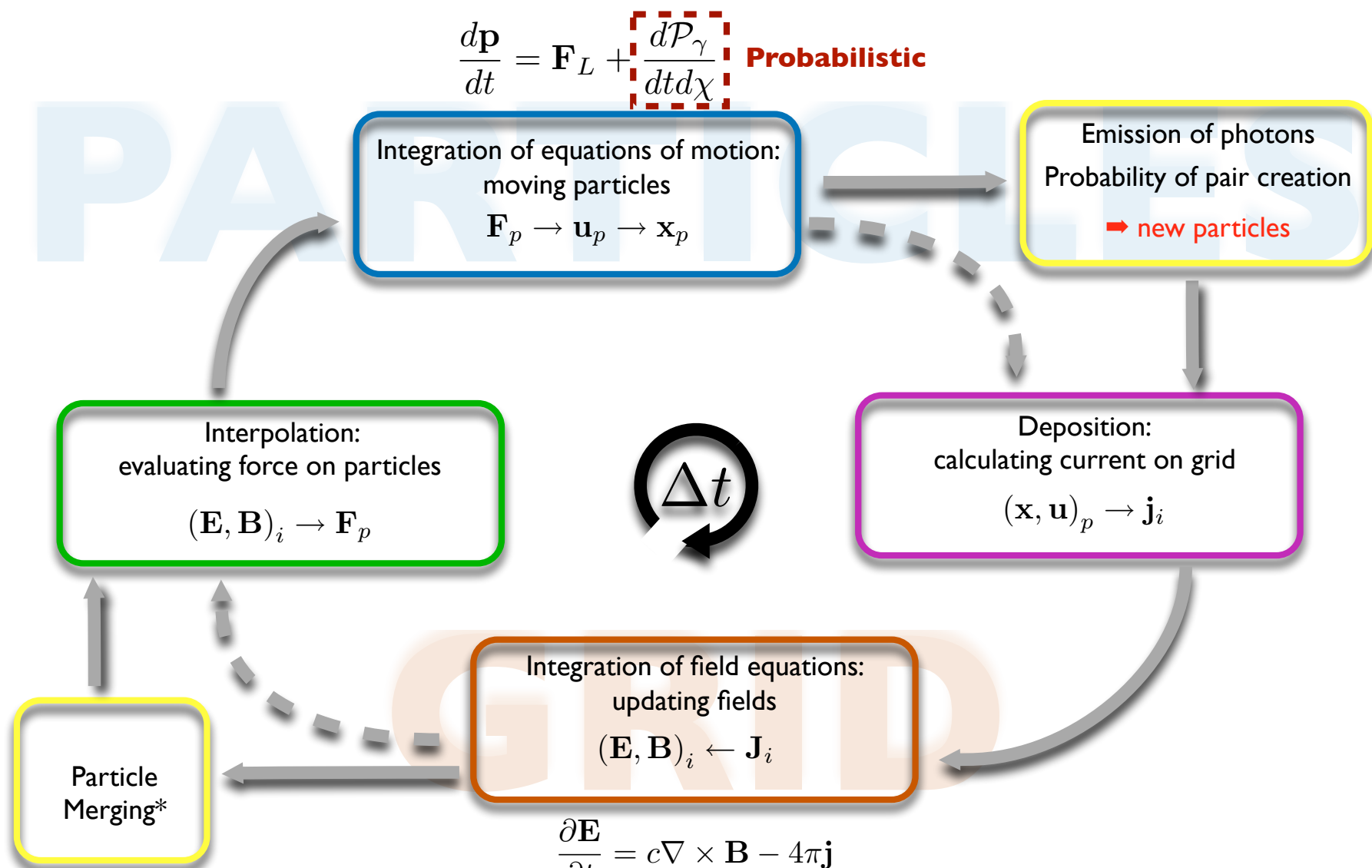
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<http://epp.tecnico.ulisboa.pt/>

<http://plasm asim.physics.ucla.edu/>



E.N Nerush et al., *PRL* 106, 035001 (2011)  
 C. P. Ridgers et al., *PRL*, 108, 165006 (2012)  
 M. Lobet et al., *PRL* 115, 215003 (2015)  
 A. Gonoskov et al., *PRE* 92, 023305 (2015) \*M.Vranic et al., *CPC*191, 65-73 (2015)

 **Schwinger field**

$$E_s = \frac{m^2 c^3}{e \hbar}$$

→ **Pair creation probability :**

$$W \propto \exp(-\pi E_s / E)$$

 **Let us introduce the parameter**

$$\chi = \frac{E}{E_s}$$

 **And generalized in any frame**

$$\chi = \frac{1}{E_s} \sqrt{(\gamma \mathbf{E} + \frac{\mathbf{p}}{mc} \times \mathbf{B})^2 - (\frac{\mathbf{p}}{mc} \cdot \mathbf{E})^2}$$

**Other configuration with lower  $E$   
should allow pair creation !**

$$\chi \simeq \frac{\gamma E_{\perp}}{E_s}$$

## Radiation Reaction

### Different types of Radiation reaction models

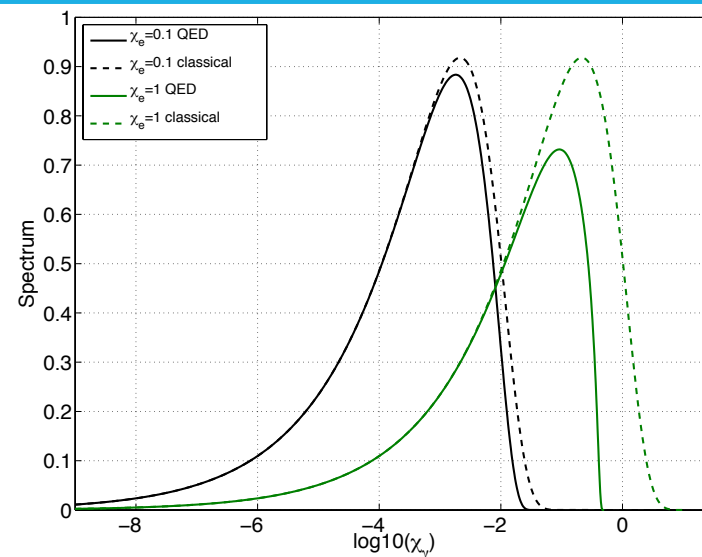
$$\frac{d\mathbf{p}}{dt} = \mathbf{F}_L + \begin{cases} \mathbf{F}_{rad} & \text{Continuous damping rate*} \\ \frac{d^2 P}{dt d\chi_\gamma} & \text{QED probabilistic approach**} \end{cases}$$

### Implementation in PIC codes

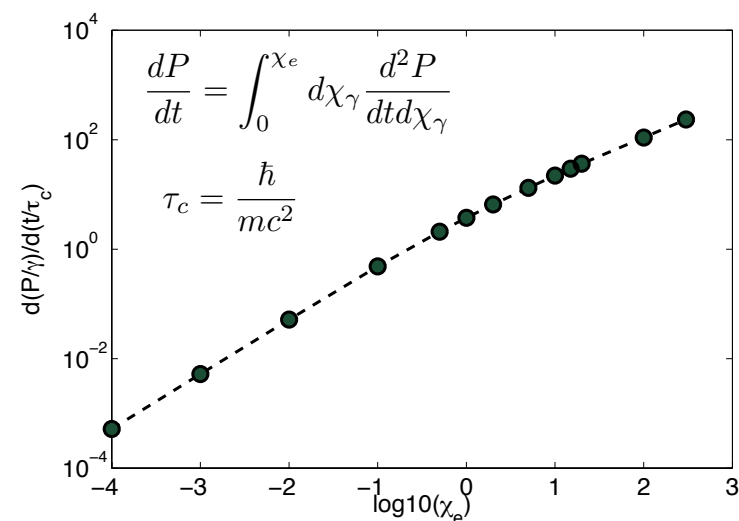
- Continuous damping rate: particle pusher with  $\mathbf{F}_{rad}$   $\gamma < 10$
- QED probabilistic approach: particle pusher + Monte Carlo module
  - every  $\Delta t$  : probability of photon emission
  - Select a photon in QED synchrotron spectrum
  - Update particle momentum due to quantum recoil
- The QED approach can be generalized to any external EM fields under the conditions:  $t_{carac}(\vec{E}, \vec{B}) \gg t_{coh} \implies a_0 \gg 1$ 
  - quasi-static fields
  - weak fields  $\chi_e^2 \gg \text{Max}(f, g)$   $(f, g) \ll 1$

$$f = F_{\mu\nu}^2 / E_{crit}^2 \quad g = F_{\mu\nu}^* F_{\mu\nu} / E_{crit}^2 \quad E_{crit} = m^2 c^3 / e\hbar \quad \chi_{e,\gamma} = \frac{|F_{\mu\nu} p_{e,\gamma}^\nu|}{E_{crit} m c}$$

## Synchrotron Spectrum



## Emission rate



\* Landau & Lifshitz (Theory of Fields)

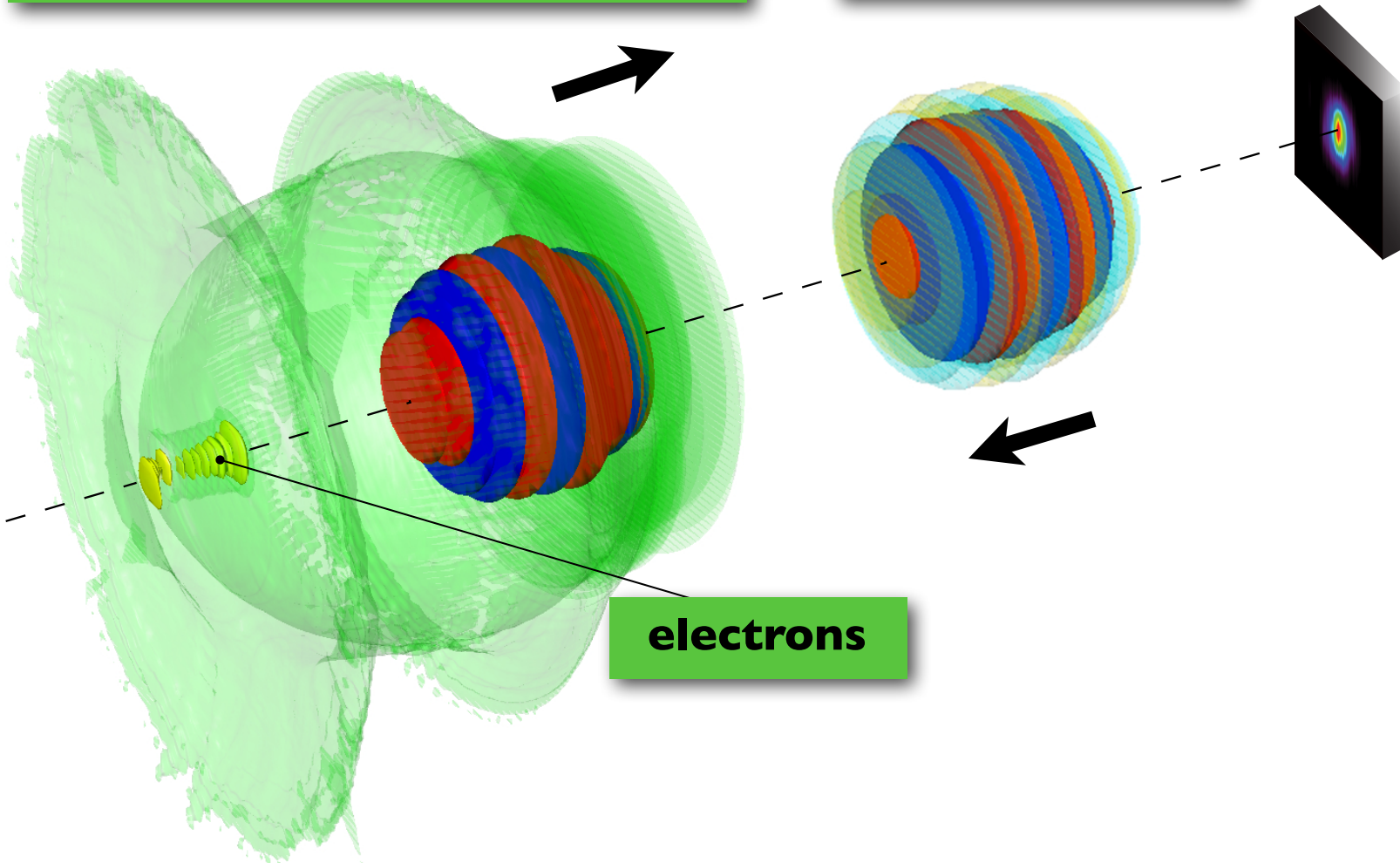
\*\* A.I. Nikishov & V.I. Ritus (1967), N.P. Kelpikov (1954), V.N. Baier & V.M. Katkov (1967)

# All-optical radiation reaction

~ 40% energy loss for a 1 GeV beam at  $10^{21}$  W/cm<sup>2</sup>

laser wakefield accelerator in  
bubble regime

second laser  
 $I \sim 10^{21}$  W/cm<sup>2</sup>



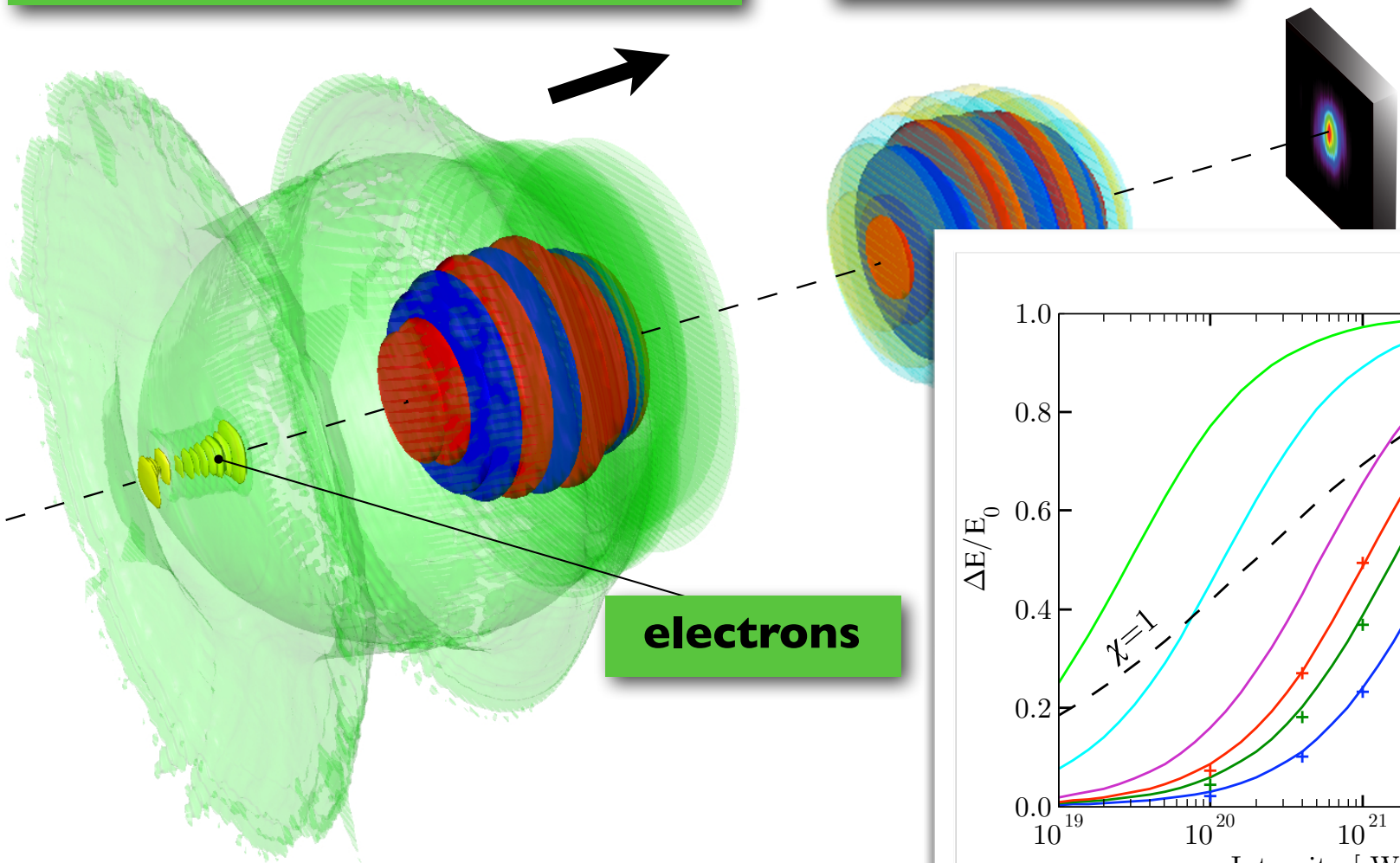
electrons

# All-optical radiation reaction

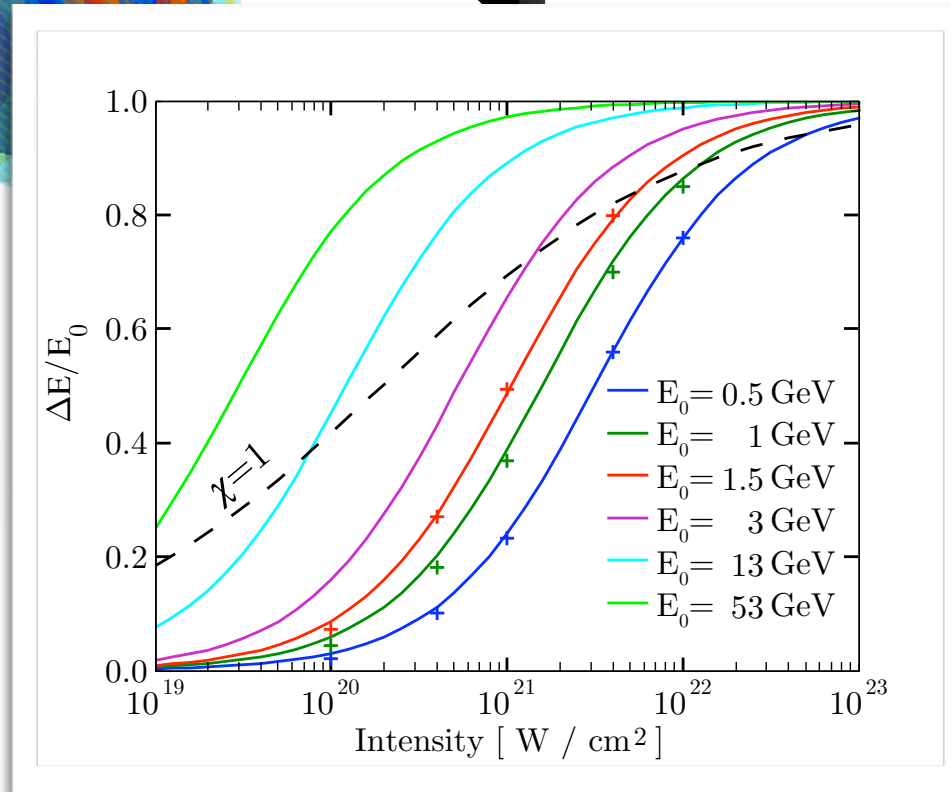
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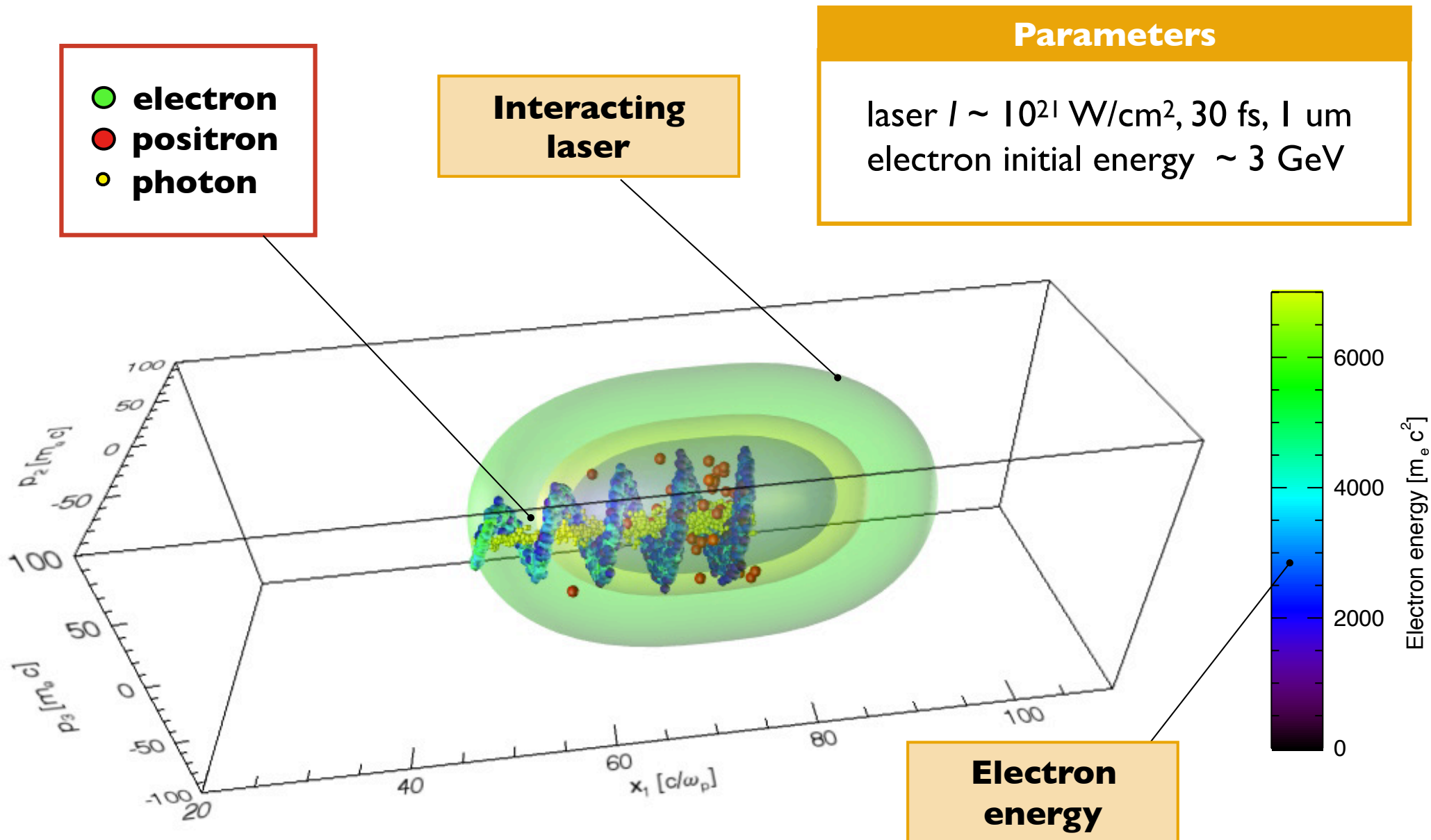
electrons





# Pairs can be produced already at $\chi \simeq 1$

~ 200 pairs obtained per 1 000 000 interacting electrons



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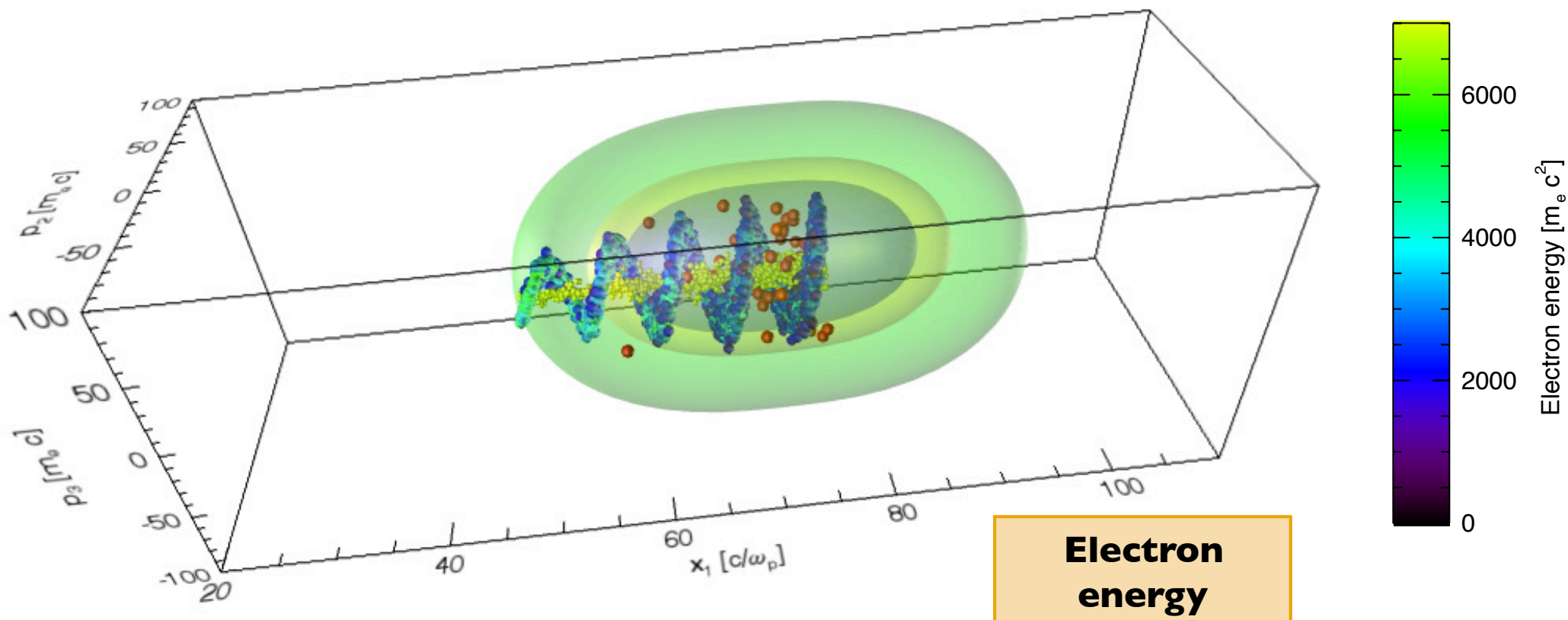
~ 200 pairs obtained per 1 000 000 interacting electrons

## Parameters

laser  $I \sim 10^{21}$  W/cm<sup>2</sup>, 30 fs, 1  $\mu$ m  
electron initial energy  $\sim 3$  GeV

- electron
- positron
- photon

Interacting laser



Electron energy

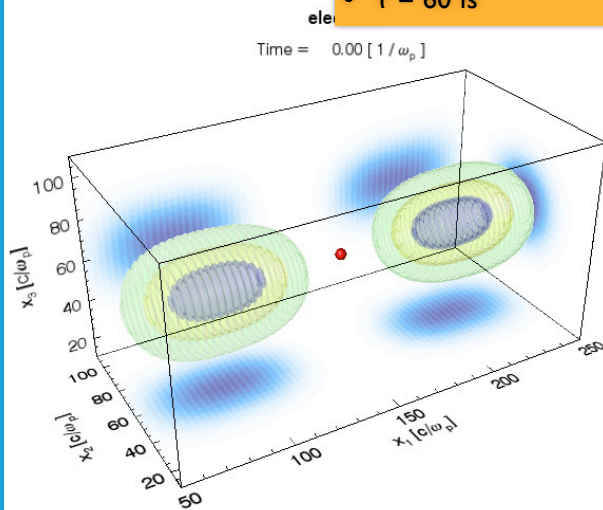
# Standing waves configurations

## Linear

- electron
- positron

### Parameters

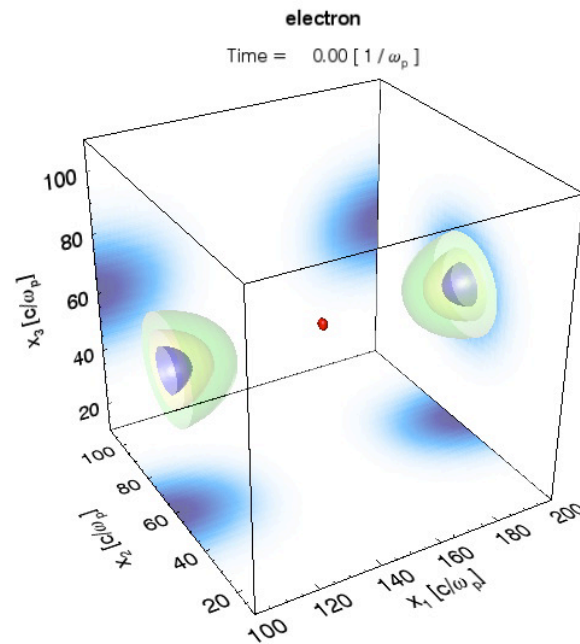
- absorbing boundaries
- $a_0 = 1000$
- $\lambda_0 = 1 \mu\text{m}$
- $W_0 = 3.2 \mu\text{m}$
- $\tau = 60 \text{ fs}$



Particles remain in the  $x_1$ - $x_2$  plane

## Double clockwise

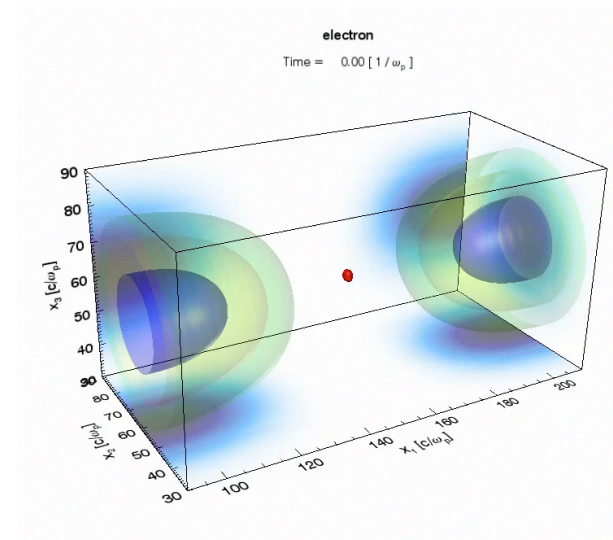
- electron
- positron
- photon



Particles explore the whole space

## Clockwise-anti

- electron
- positron
- photon



Particles rotate mainly in the  $x_2$ - $x_3$  plane

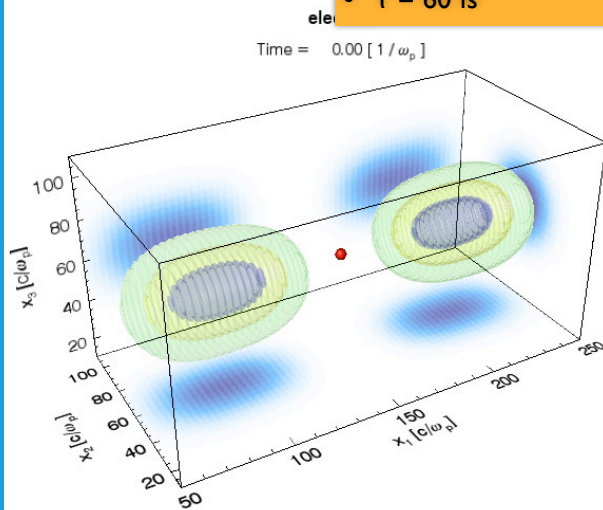
# Standing waves configurations

## Linear

- electron
- positron

### Parameters

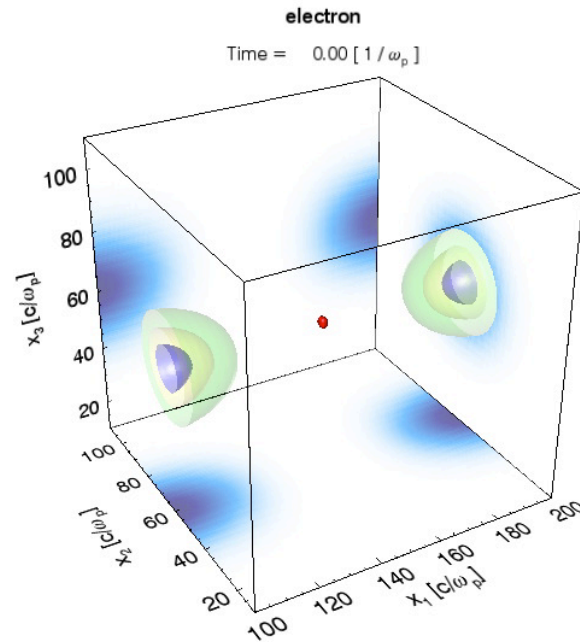
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**Particles remain in the  $x_1$ - $x_2$  plane**

## Double clockwise

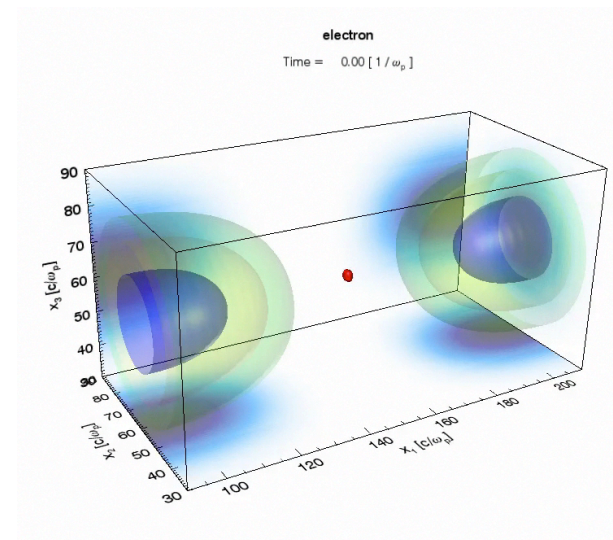
- electron
- positron
- photon



**Particles explore the whole space**

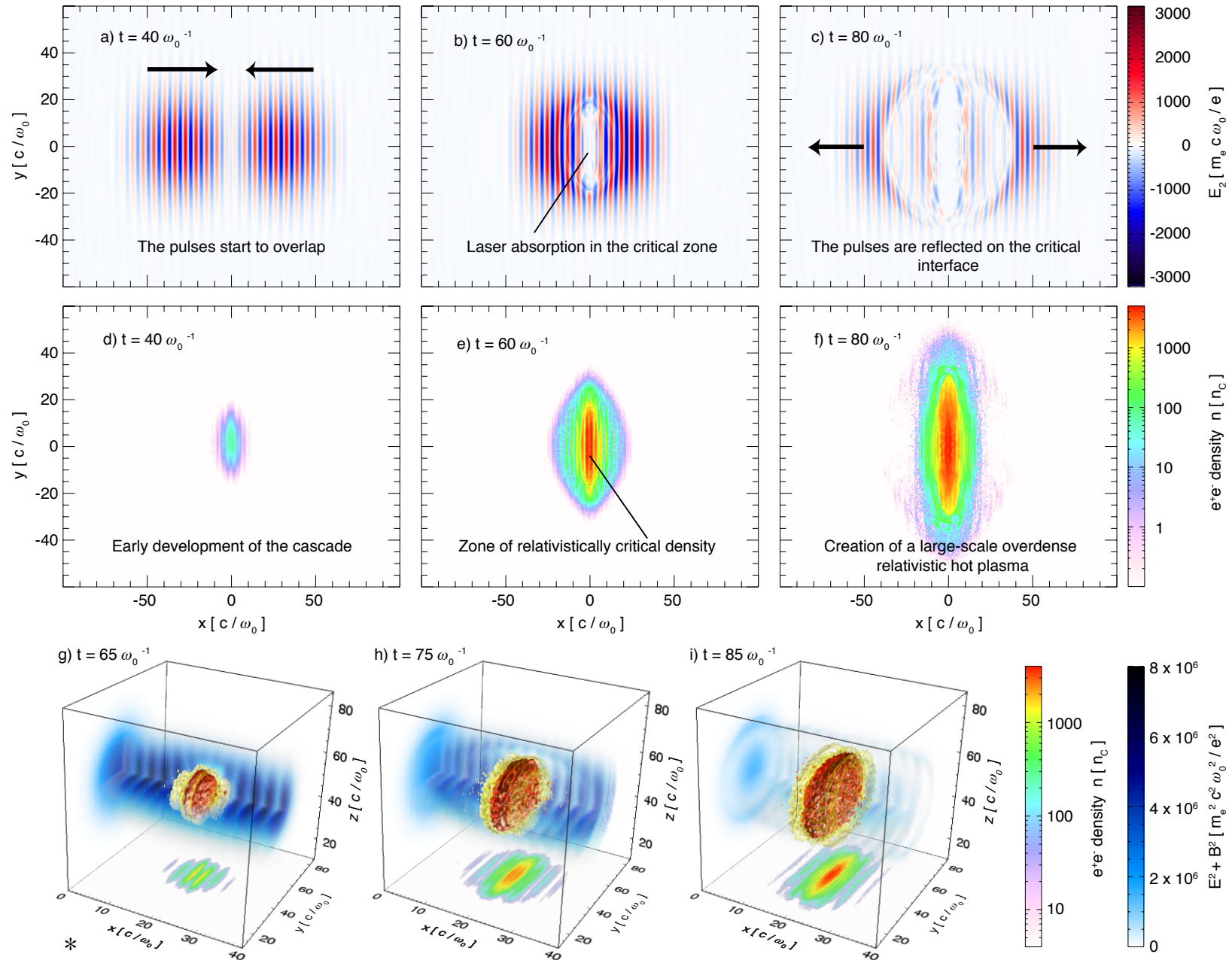
## Clockwise-anti

- electron
- positron
- photon



**Particles rotate mainly in the  $x_2$ - $x_3$  plane**

# Interaction between self-consistent created pair plasma and the lasers



## Heisenberg-Euler corrections to Maxwell's Equations

- Electron-positron fluctuations give rise to an effective polarization and magnetization of the vacuum which can be treated in an effective form as corrections to Maxwell's equations\*.

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} = 0 \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = 0 \quad \vec{\nabla} \cdot \vec{D} = 0$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

- With the effective vacuum polarization and magnetization

$$\vec{P} = 2\xi \left[ 2(E^2 - c^2 B^2) \vec{E} + 7(\vec{E} \cdot \vec{B}) \vec{B} \right]$$

$$\vec{M} = -2\xi \left[ (2(E^2 - c^2 B^2) \vec{B} - 7(\vec{E} \cdot \vec{B}) \vec{E}) \right]$$

- **Relevance for extreme astrophysical scenarios?**

J. Pétri, Mon. Not. Roy. Astron. Soc (2015)



- **Unprecedented intensities will allow to probe the quantum vacuum!**

**What laser properties will be affected?**

A. Di Piazza et.al, Rev. Mod. Phys. 84, 1177–1228 (2012).



STFC  
Central Laser Facility

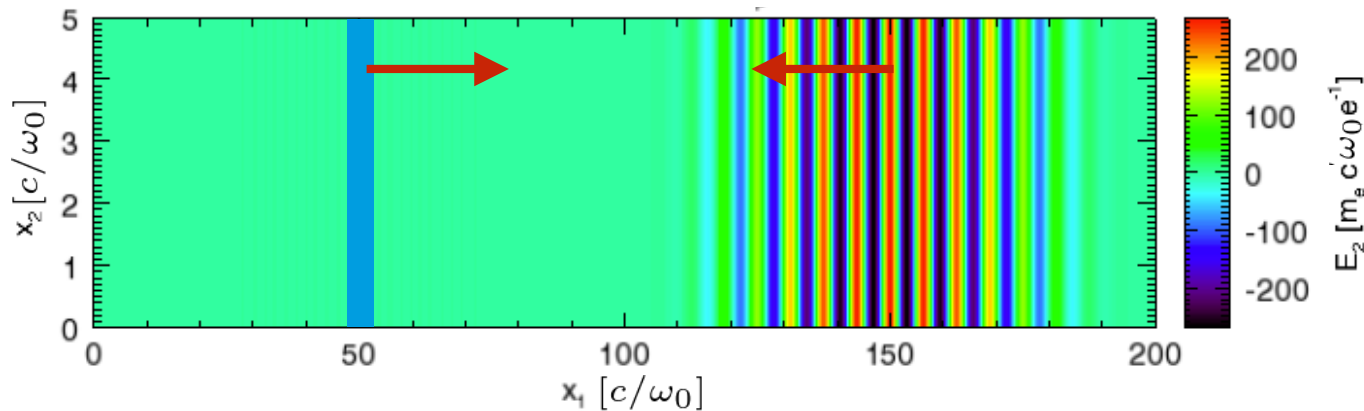
- **Extract observable consequences of fundamental QED predictions.**



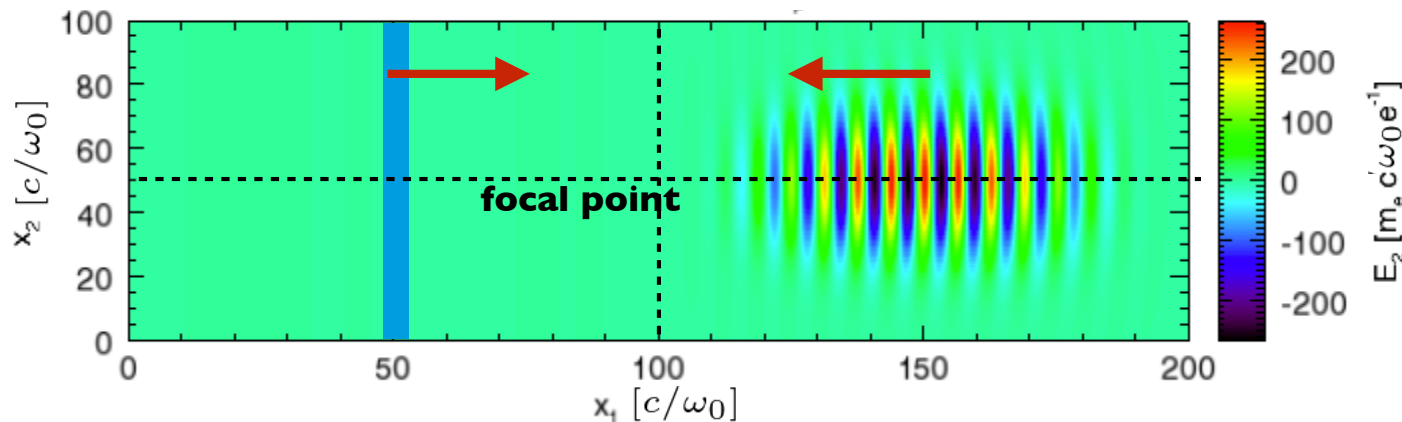
\* W. Heisenberg and H. Euler, Z. Physik 98, 714 (1936).

## Plane Wave transverse pump profile vs Gaussian pump beam profile

### Plane Wave Transverse Pump Profile



### Gaussian Pump Beam Profile



#### Pump pulse

$$I_0 = 10^{23} \text{ [W/cm}^2\text{]}$$

$$\lambda_0 = 1 \text{ [\mu m]}$$

Gaussian transverse profile

$$W_0 = 20 \text{ [c/\omega}_0\text{]}$$

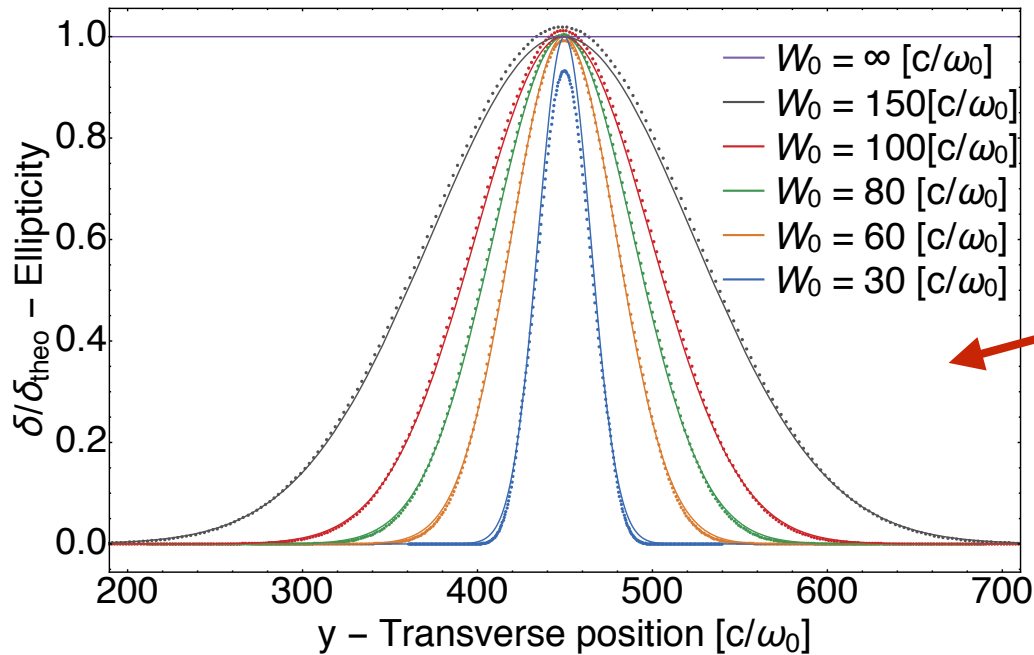
#### Probe pulse

$$I_p = 10^{18} \text{ [W/cm}^2\text{]}$$

$$\lambda_p = 10 \text{ [nm]}$$

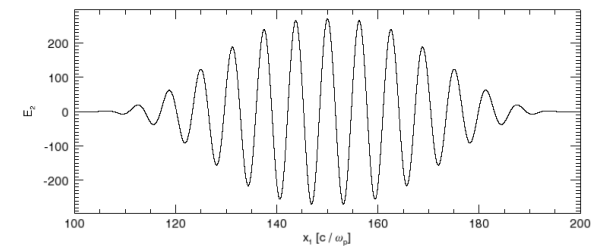
$$pol_p = \frac{\pi}{4} \text{ [rad]}$$

## Plane Wave transverse pump profile vs Gaussian pump beam profile



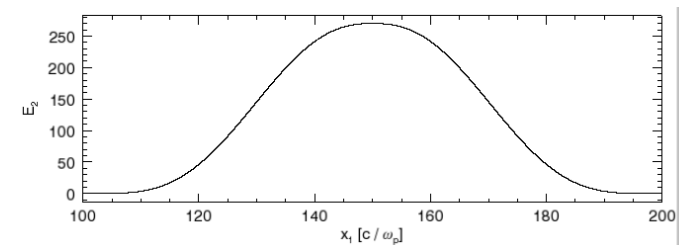
### Oscillating gaussian profile

$$\delta = \frac{3}{2} \sqrt{\pi} k_p \sigma \xi E_0^2$$



### Non-oscillating gaussian profile

$$\delta = 3 \sqrt{\pi} k_p \sigma \xi E_0^2$$





## OSIRIS

### Pusher

Usual Boris-Pusher with classical and/or quantum radiation reaction with recoil

### Beam field solver

Solves solves self-consistently Maxwell's equations for all particles in the box (beam + new created particles)

### QED processes (first principles)

- Non-linear Compton scattering
- Breit-Wheeler pair production (spin averaged)
- Linear Compton scattering

## CAIN

### Pusher

Crank-Nicolson or subsampling trajectory with 4<sup>th</sup> order Runge-Kutta

### Beam field solver

- Solves Poisson equation for transverse instantaneous electric field applied to each slice considering only the charges in the beam.
- The transverse magnetic field obeys

$$\mathbf{E} = -\boldsymbol{\beta} \times \mathbf{B}$$

### QED processes with Spin effects

- Non-linear Compton scattering
- Breit-Wheeler
- Bethe-Heitler
- Landau-Lifschitz pair production with spin effects
- Linear Compton scattering

\* P.Chen et al., SLAC-PUB-6583 (1994) and <https://ilc.kek.jp/~yokoya/CAIN/cain235> (2003)

## Disruption parameter

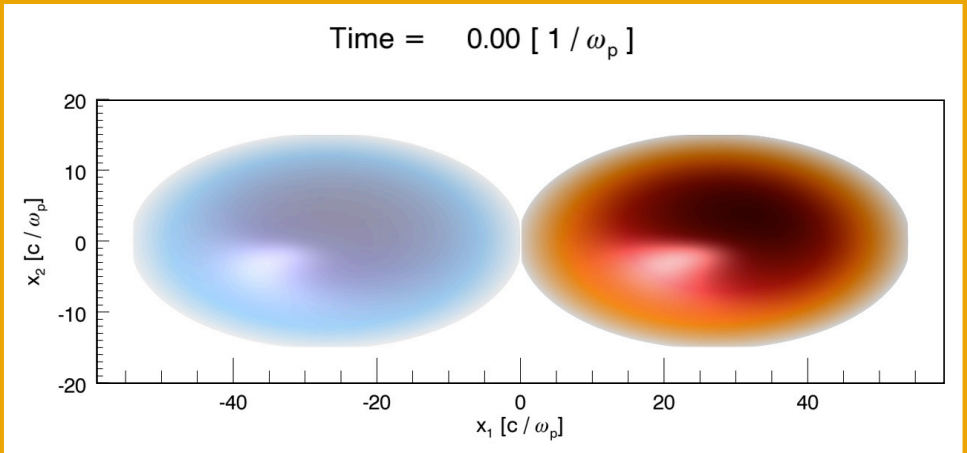
The disruption parameter relates to the number of pinching points of the beams during their interaction time

$$E_{\perp} \simeq B_{\perp} \sim en_0\sigma_0$$

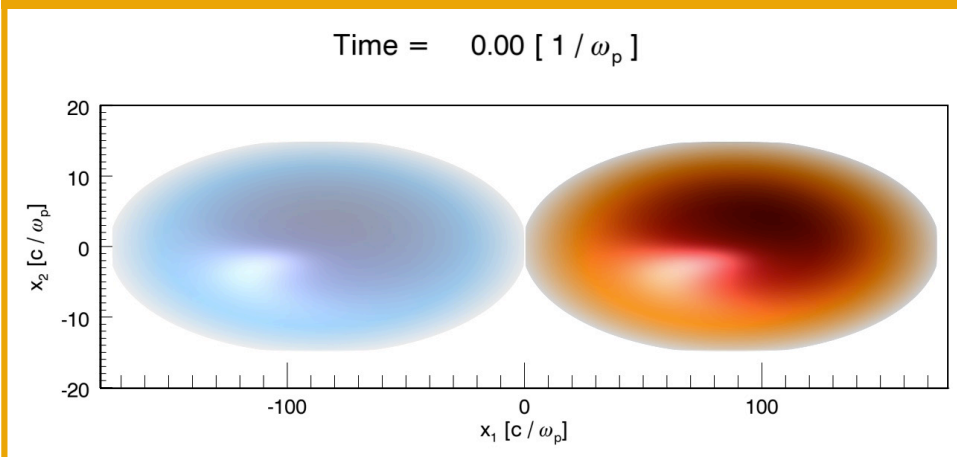
$$E_{\parallel} \sim E_{\perp}/\gamma$$

$$D = \frac{r_e N \sigma_z}{\gamma \sigma_0^2}$$

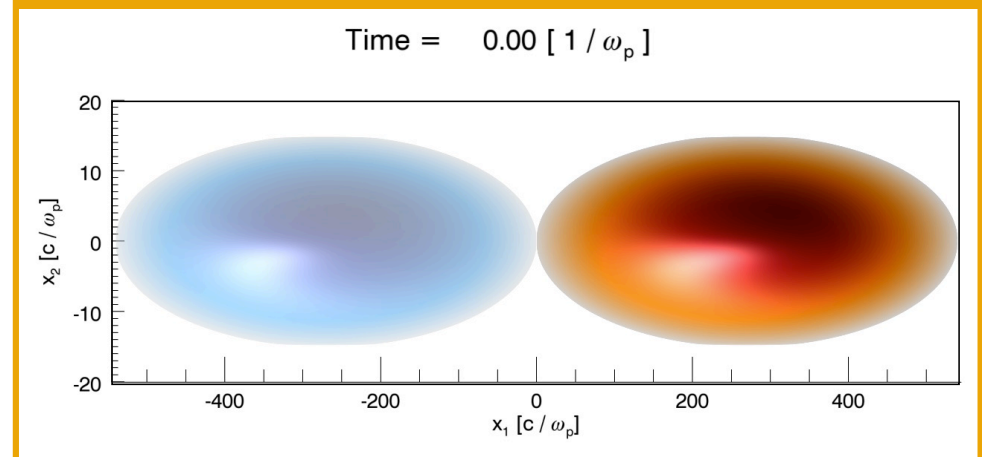
## Low disruption regime $D < 1$

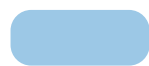


## Transition regime $1 < D < 10$



## Confinement regime $D > 10$



-  Electron beam density
-  Positron beam density

P. Chen, S. Rajagopalan and J. Rosenzweig, *PRE* 40 (1989)

P. Chen and K. Yokoya et al., *PRD* 38 (1988)

T. Katsouleas et al., *PoP* 1384 (1990)

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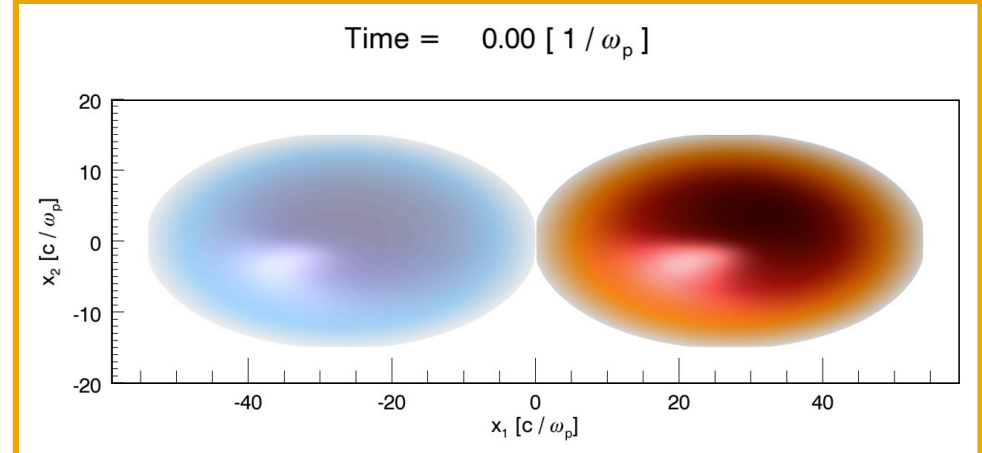
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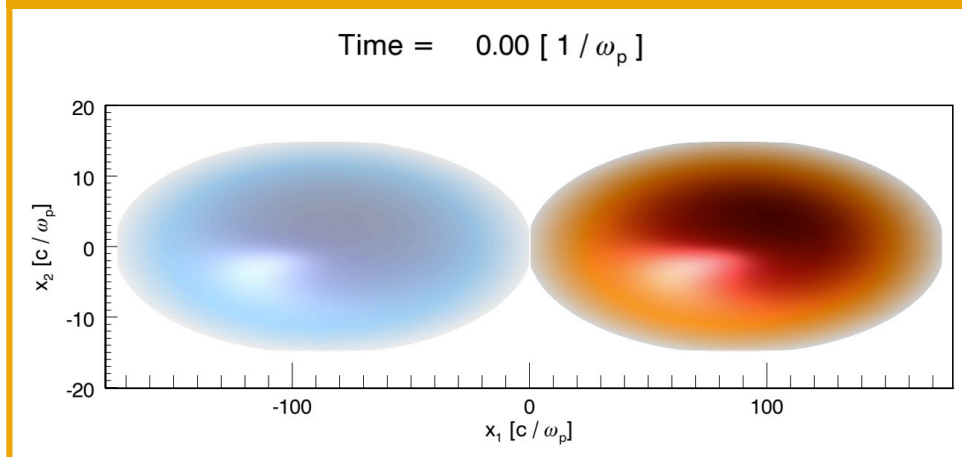
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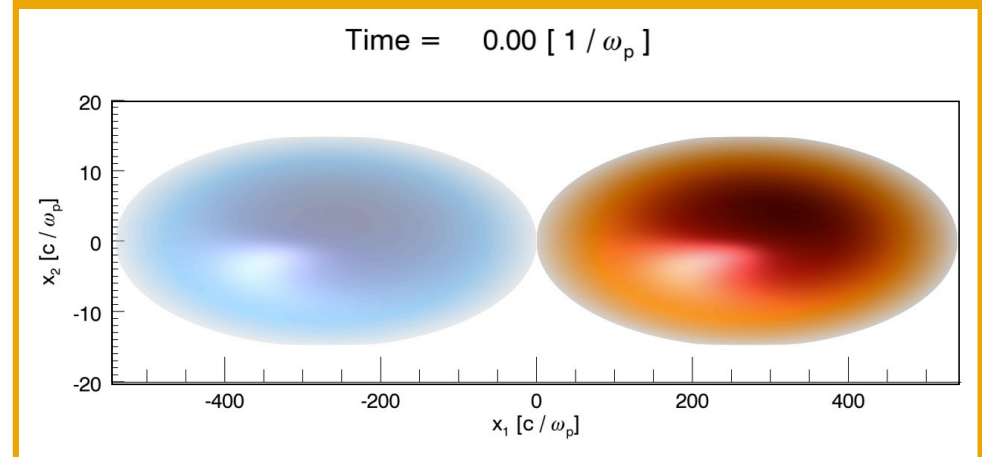
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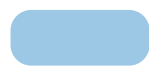



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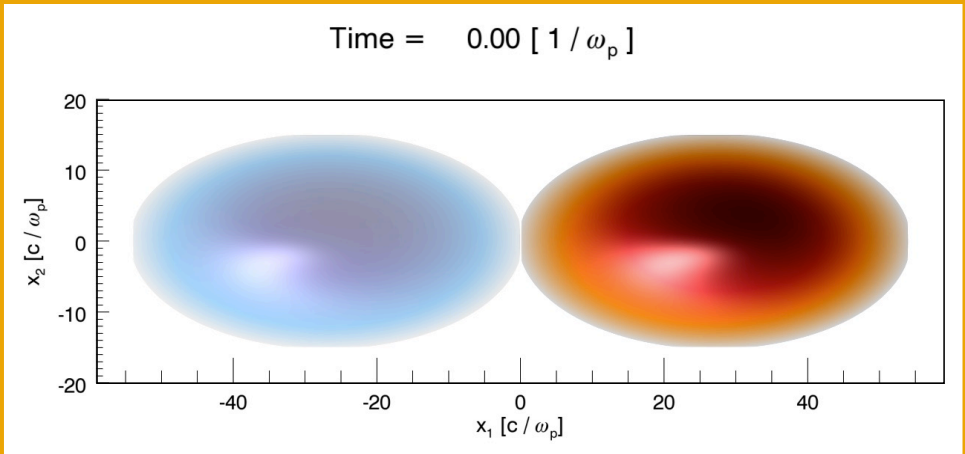
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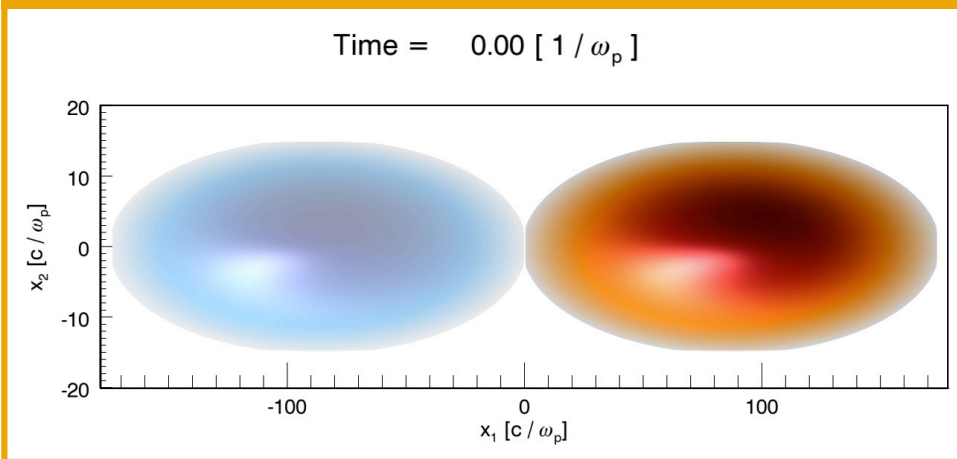
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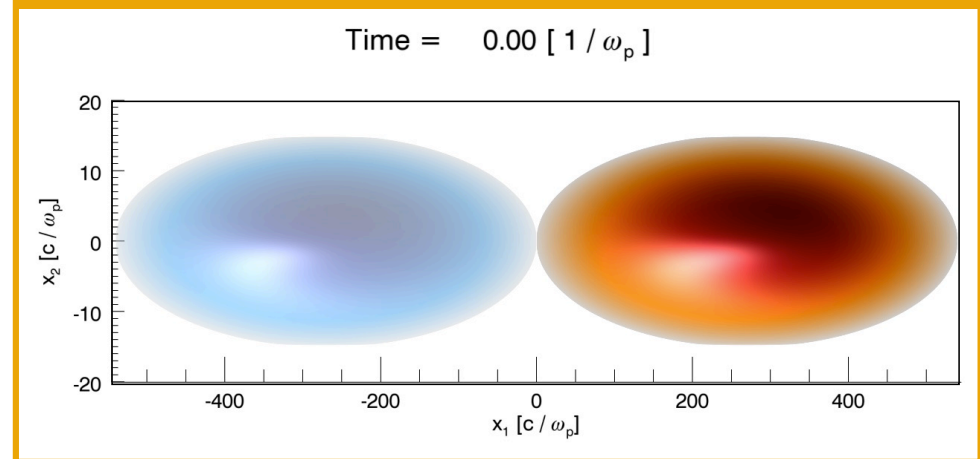
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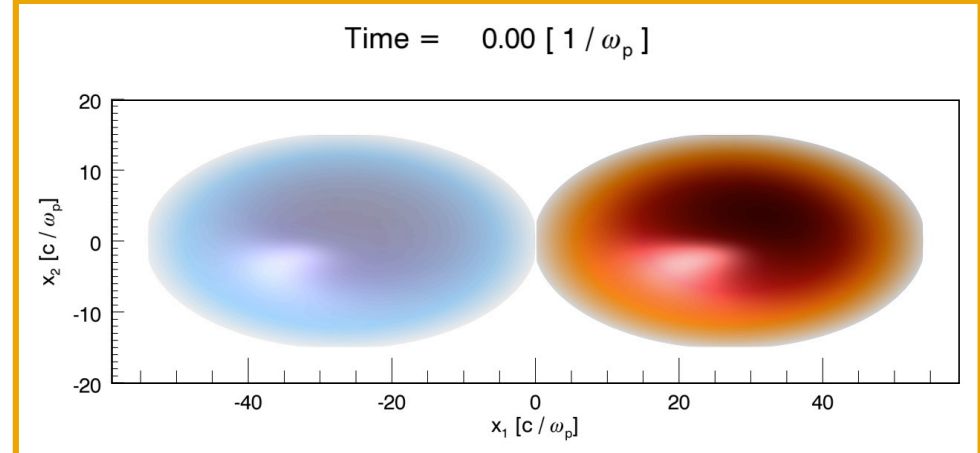
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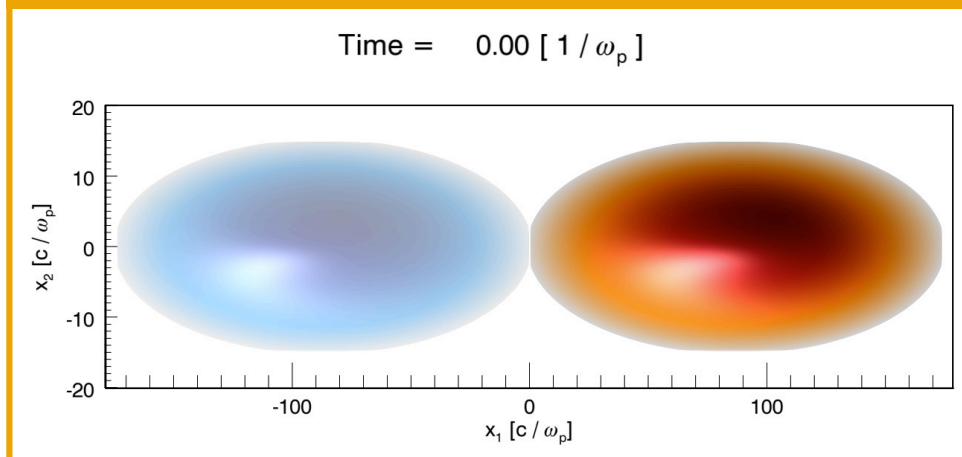
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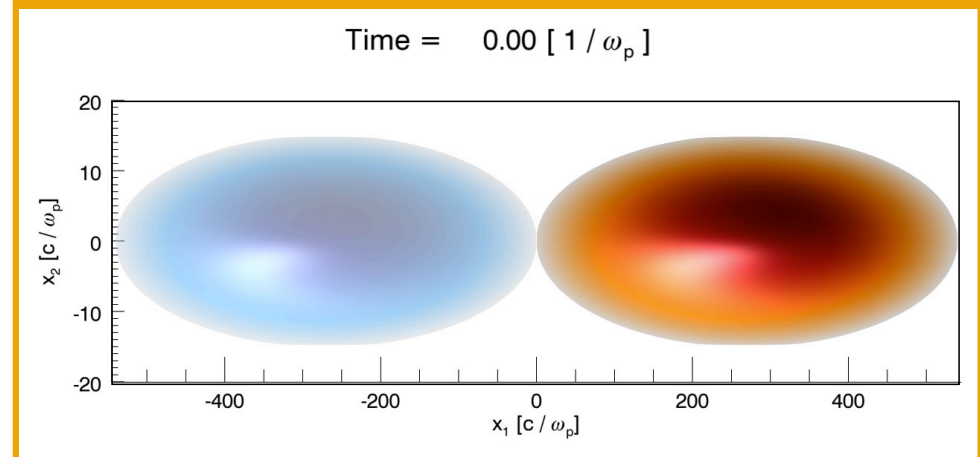
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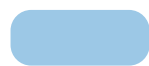



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## Quantum regime

Parameter  $\chi$  measures the closeness to the quantum regime, with  $E_s$  the Schwinger limit

$$\chi = \frac{\gamma E}{E_s}$$

## Classical vs quantum

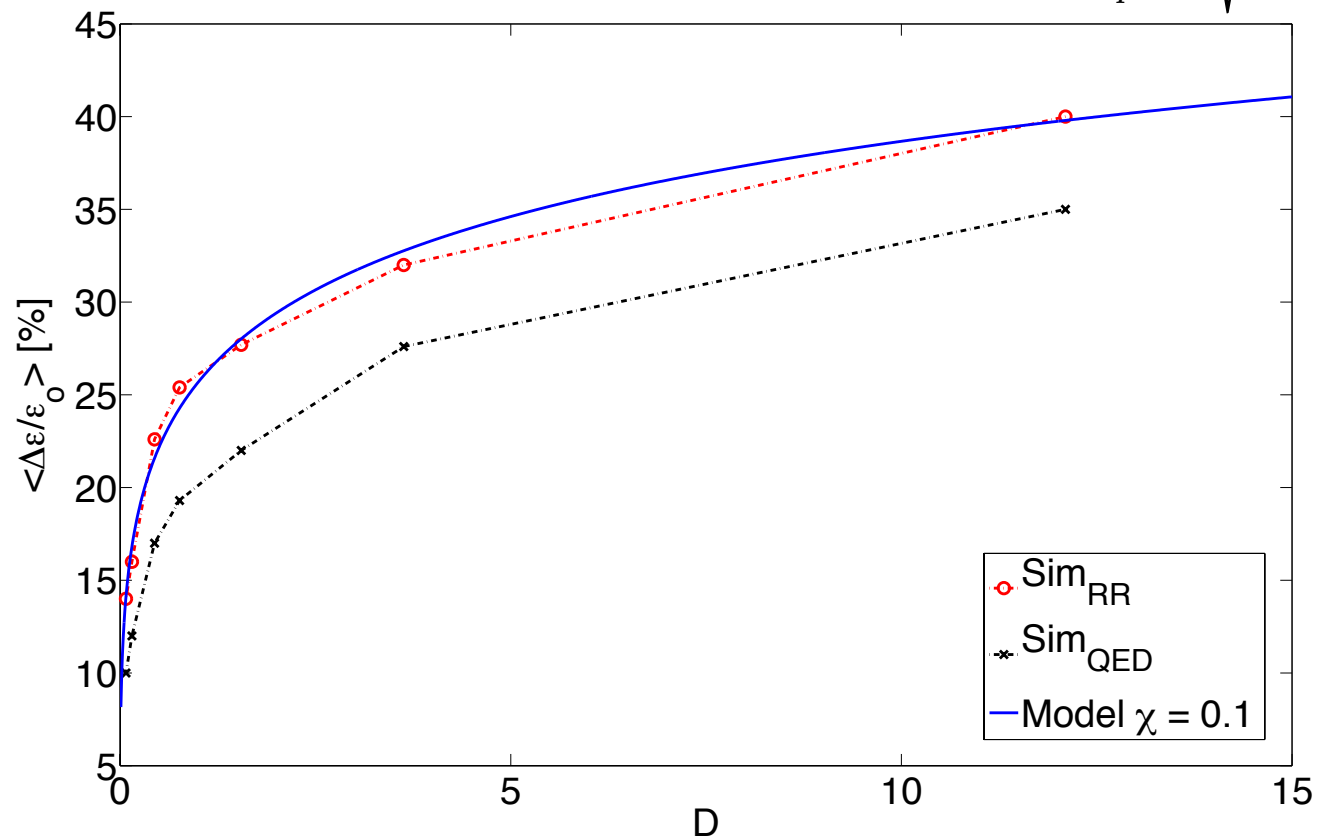
Close to the quantum regime ( $\chi \rightarrow 1$ ), the classical model overestimates the radiative losses, thus a quantum approach is required

## Energy loss of the beam close to the quantum regime

Average energy loss derived from the classical Landau-Lifshitz radiation reaction force

$$\left\langle \frac{\Delta\epsilon}{\epsilon} \right\rangle = 1 - \frac{\text{atan}(\sqrt{b})}{\sqrt{b}}$$

$$b = \frac{\pi}{3} (2\pi)^{1/4} \alpha \frac{mc^2}{\hbar\omega_p} \chi_0^2 \sqrt{\frac{D}{\gamma_0}}$$



## Fast development of plasma wakefield acceleration technology

Single-stage up to 5 GeV  
Multi-stage configuration

	LWFA parameters
<b>N</b> [ $10^{10}$ ]	2-6
<b>Energy</b> [GeV]	5-30
<b>Length</b> [ $\mu\text{m}$ ]	3
<b>Spot size</b> [ $\mu\text{m}$ ]	1x1
<b>D</b>	$10^{-5} - 10^{-2}$

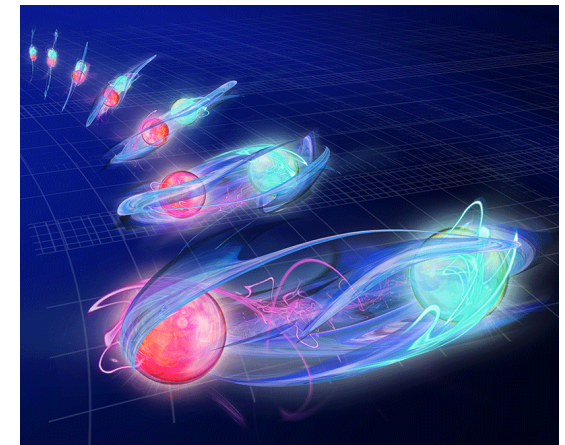
## New LWFA beams colliders may approach the quantum regime $\chi \sim 1$



courtesy of <http://www.klassikmagazine.com/seb-janiak-artist/>

Beam particles can emit  $\gamma$  ray  
interacting with  
the oncoming beam EM field

Hard photons  
can decay in pairs

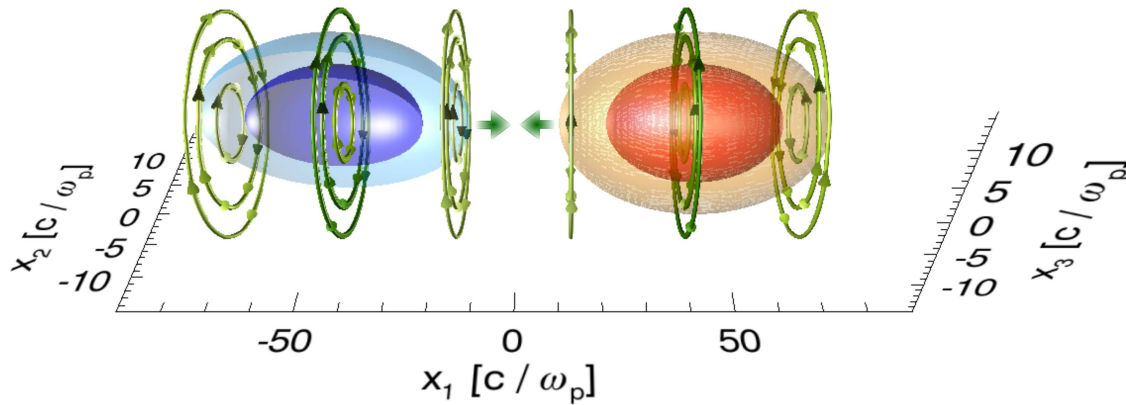


courtesy of <http://physics.aps.org/articles/v9/67>

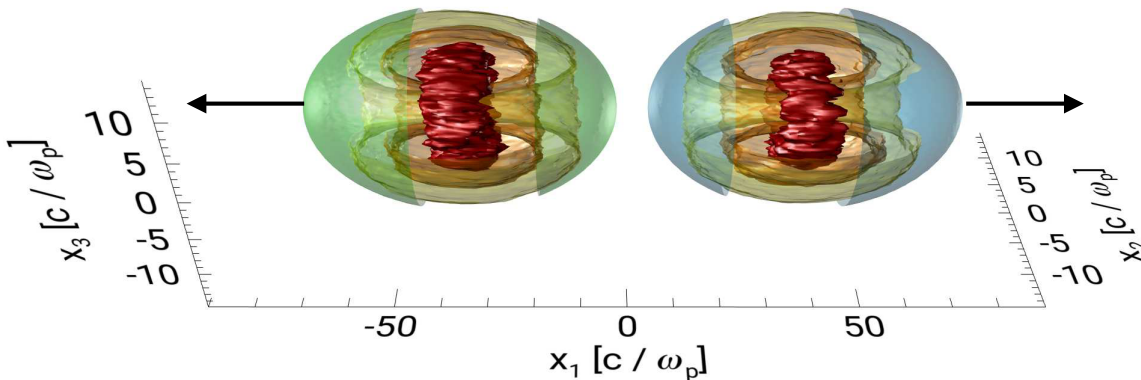
# Beamstrahlung approaching the QED regime

## Before interaction

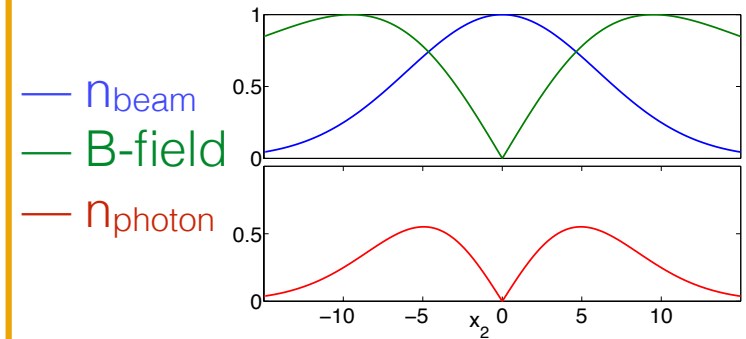
electron beam → B field line  
positron beam



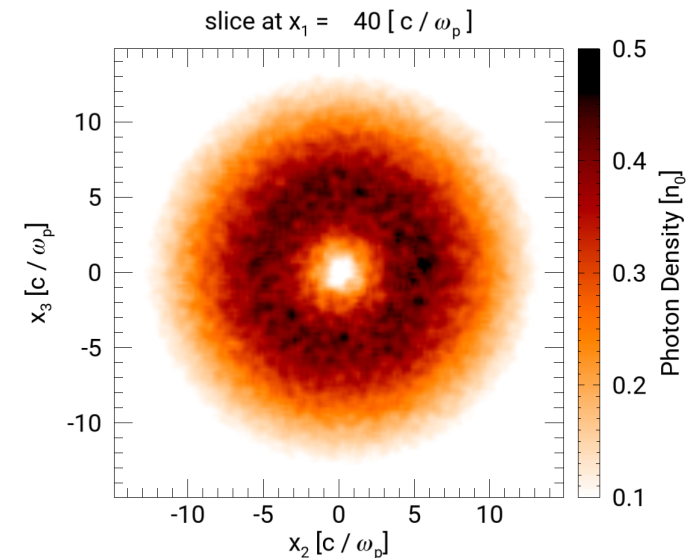
## After interaction



## Profiles



If the **beam density profile** is not disrupted, the produced **photon beam** shows a characteristic hole, where the **B field** is null





## Particles trajectories are close to straight lines

the transverse density profile is constant

$$\rho(x) = \frac{e^{-x^2/2\sigma_o^2}}{\sqrt{2\pi}\sigma_o}$$

## The model accounts for QED processes

The time average probability for photon emission

$$g(\xi, x) = \int_{-\infty}^{\infty} \frac{d^2 P}{dt d\xi} dt$$

Differential probability rate for pair creation ( $\chi_e \ll 1$ )

$$W_p \simeq \frac{3\pi\alpha}{50} E(x, t) e^{-8/3\xi\chi_e}$$

## Photon emission process

Average intensity

$$I = n_o c \epsilon_e \int_{-\infty}^{\infty} \frac{\int_0^1 \xi g d\xi}{\int_0^1 g d\xi} \rho dx$$

Energy spectrum

$$S(\xi) = \int_{-\infty}^{\infty} \xi g(\xi, x) \rho(x) dx$$

Probability a photon generated at  $t'$  will decay in a pair along the remaining interaction time of the two beams

$$\mathcal{P}_{\gamma \rightarrow p}(\xi, t', x) = \int_{t'}^{\infty} W_p dt = \frac{\sqrt{\pi}}{2} \tau \left[ 1 - \operatorname{erf} \left( \frac{t'}{\tau} \right) \right] W_p|_{t=0}$$

Characteristic time

$$\tau = \sigma_z / \sqrt{2 + 16/3\xi\chi_e(x)}$$

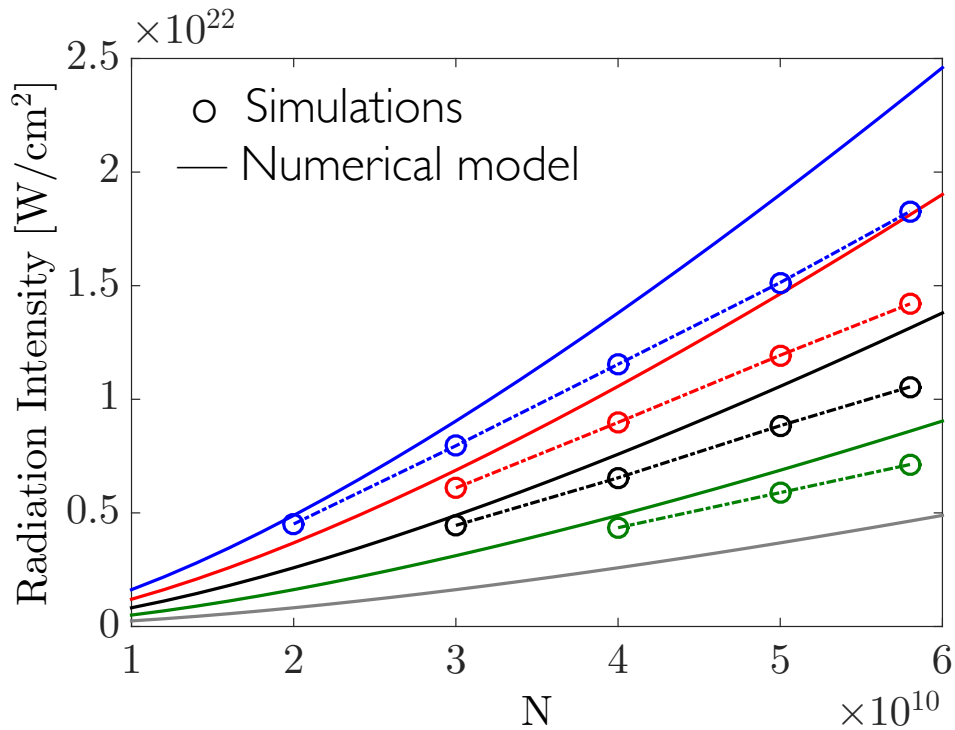
Time delay

Max. probability to decay

## Photon decay process generates pairs

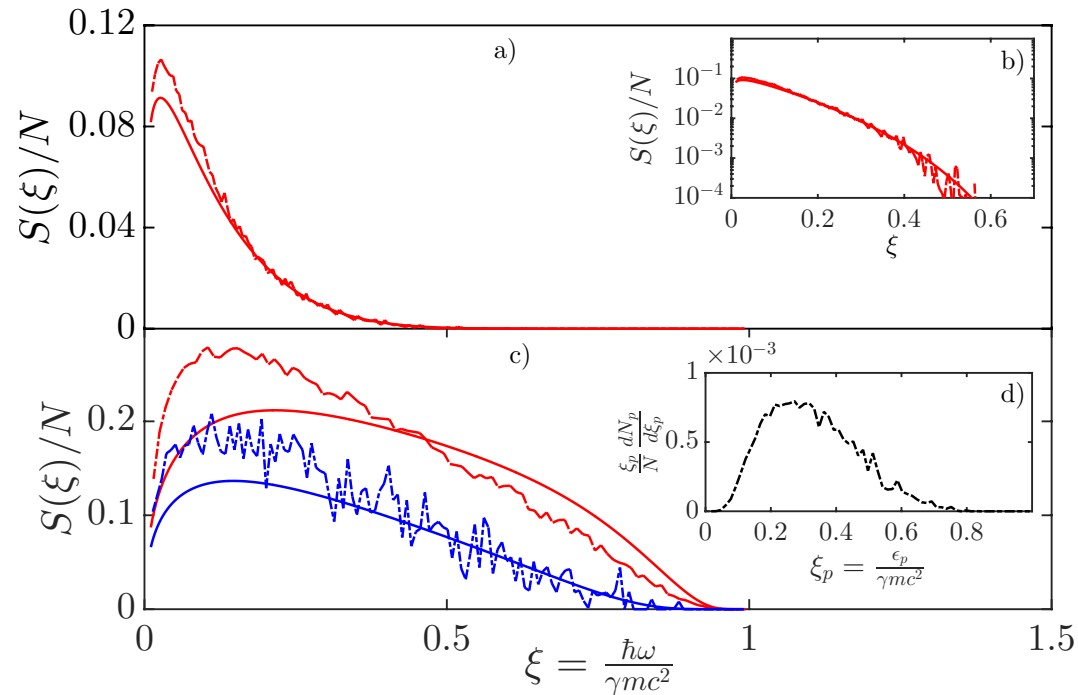
$$N_p = N \int_x \rho \int_{-\infty}^{\infty} \int_0^1 \frac{d^2 P}{dt' d\xi} \mathcal{P}_{\gamma \rightarrow p} d\xi dt' dx$$

# Prediction of beamstrahlung radiation



2D simulations for beam energies of 30, 25, 20, 15, 10 GeV

Beam dimensions  
 $\sigma_0 = 1\mu\text{m}$   $\sigma_z = 3\mu\text{m}$



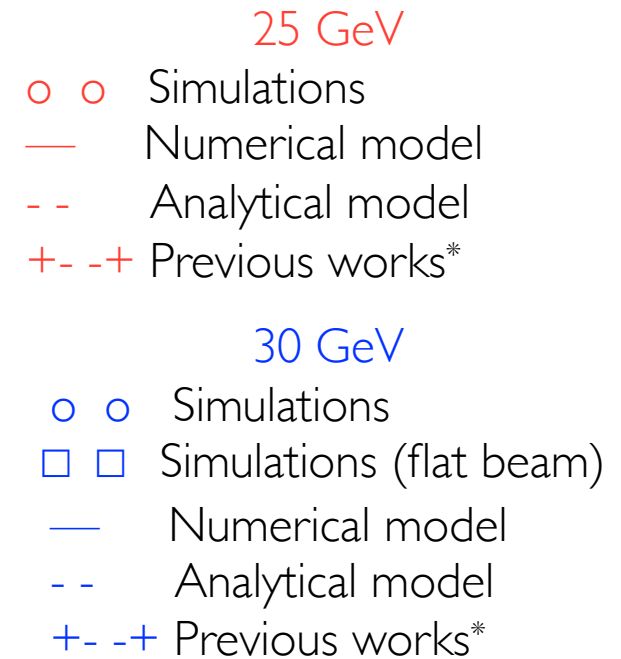
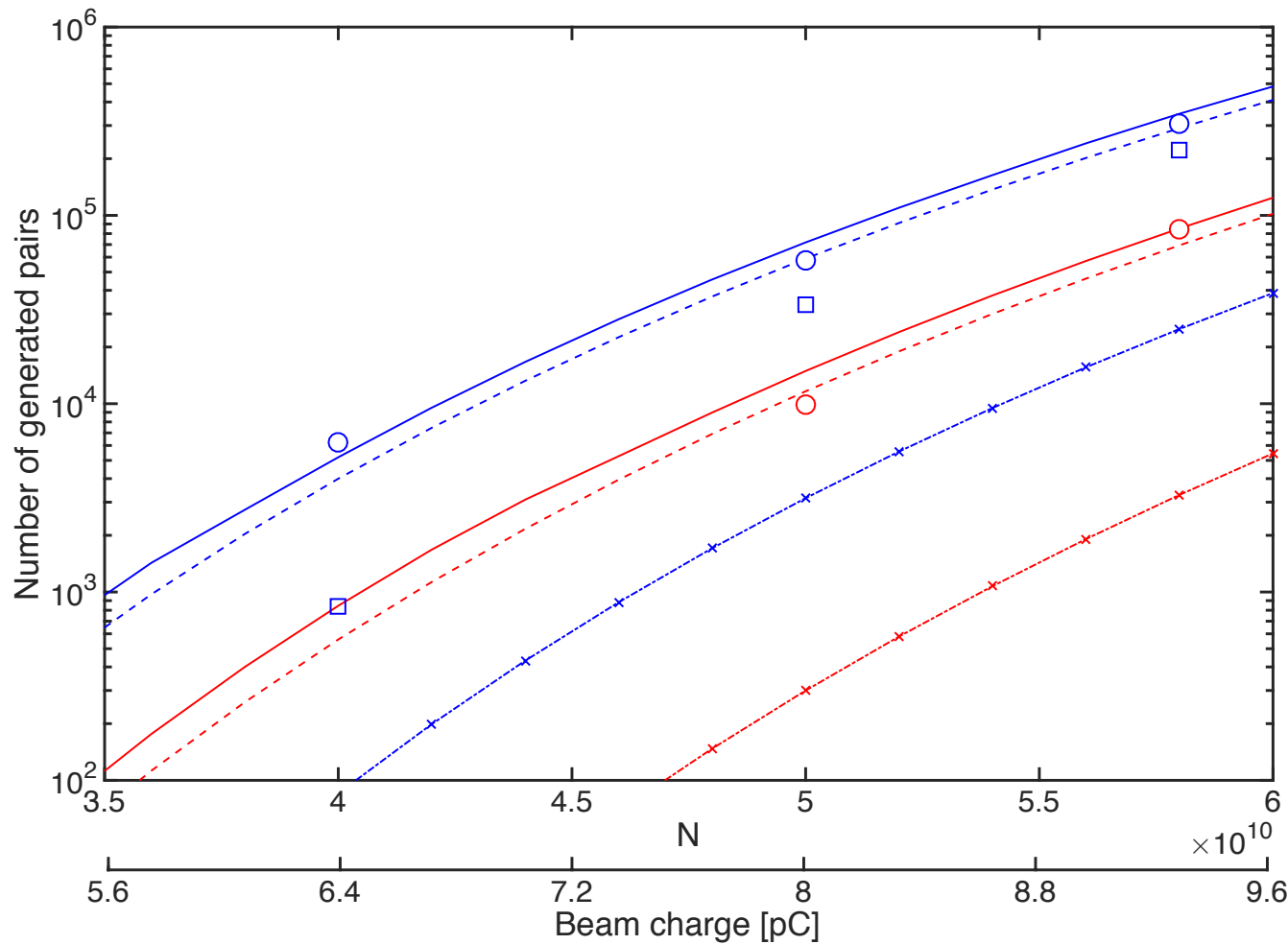
Simulation

Model

- i) 2D simulation, 5 GeV,  $2 \times 10^{10}$  particles
- ii) 2D simulation, 30 GeV,  $5.8 \times 10^{10}$  particles
- iii) 3D simulation, 30 GeV,  $5.8 \times 10^{10}$  particles

**The model overestimates the radiation intensity and energy spectrum, when a relevant number of pairs is produced during the interaction**

# Secondary pairs from 2D-3D simulations



Beam dimensions  
 $\sigma_0 = 1\mu\text{m}$   $\sigma_z = 3\mu\text{m}$   
( $R \sim 2.2$ )

**Our model agrees with QED-PIC simulations and predicts higher number of pairs with gaussian beams than uniform beams**

\* P. Chen, V. Telnov at Phys. Rev. Lett 63, 1796 (1989)  
F. Del Gaudio, et al., to be submitted

## Comparing FACET-II Electron Beam and PW laser

SLAC

	FACET-II	100GeV
Energy [GeV]	10	100
Beam peak current [kA]	200	1000
Beam size $\sigma_r, \sigma_z$ [nm]	40, 500	10, 10
Number of electrons	$10^{10}$	$10^{10}$
Beam density [cm <sup>-3</sup> ]	$10^{24}$	$7 \cdot 10^{25}$
Radial electric field [V/m]	$3 \cdot 10^{14}$	$6 \cdot 10^{15}$
Laser intensity equivalent to electric field [W/cm <sup>2</sup> ]	$10^{22}$	$5 \cdot 10^{24}$
Laser power equivalent focussed into 2 $\mu$ m [PW]	1.5	600
$\chi = 2\gamma E/E_s$ seen by 300 MeV beam	0.3	
$\chi$ seen by beam in 100TW laser focussed to 2 $\mu$ m	2.3	
$\chi$ seen by 100 GeV beam		1800
$a \chi^{2/3}$		1
Beam power [ $I_{\text{peak}} E$ ] [PW]	2	100
Beam power density [W/cm <sup>2</sup> ]	$4 \cdot 10^{25}$	$3 \cdot 10^{28}$

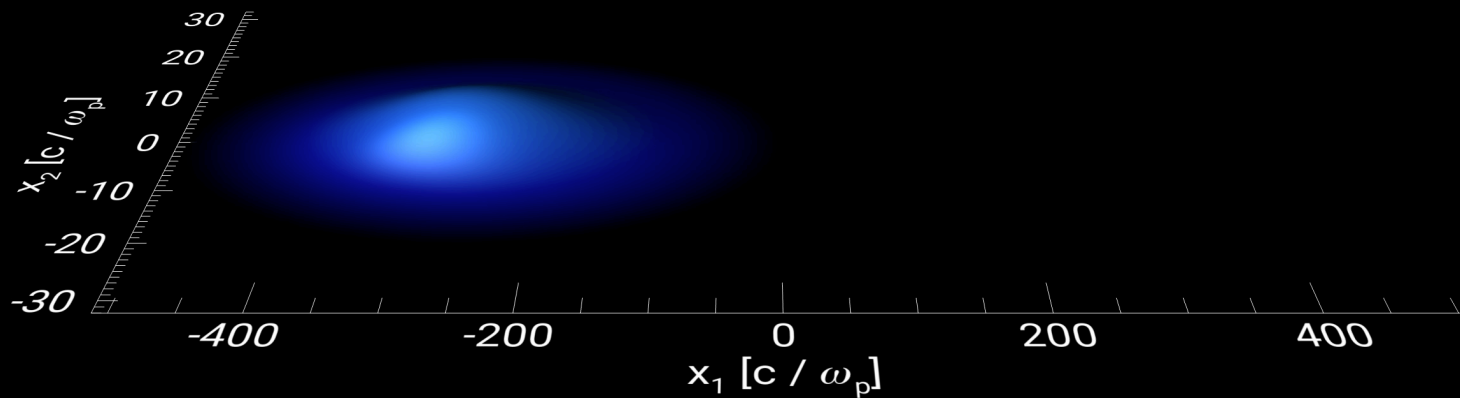
~100Hz rep. rate / ~fs duration / 10's nm focus / radial polarization

## Electron density (10 GeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 0.05

$\chi = 8$

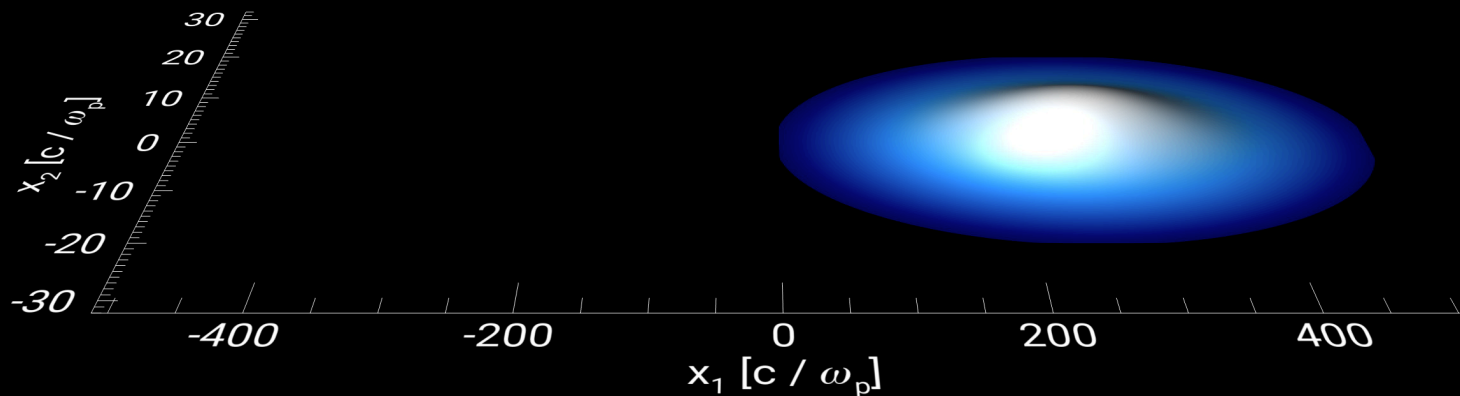


## Electron density (300 MeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 15

$\chi = 0.3$

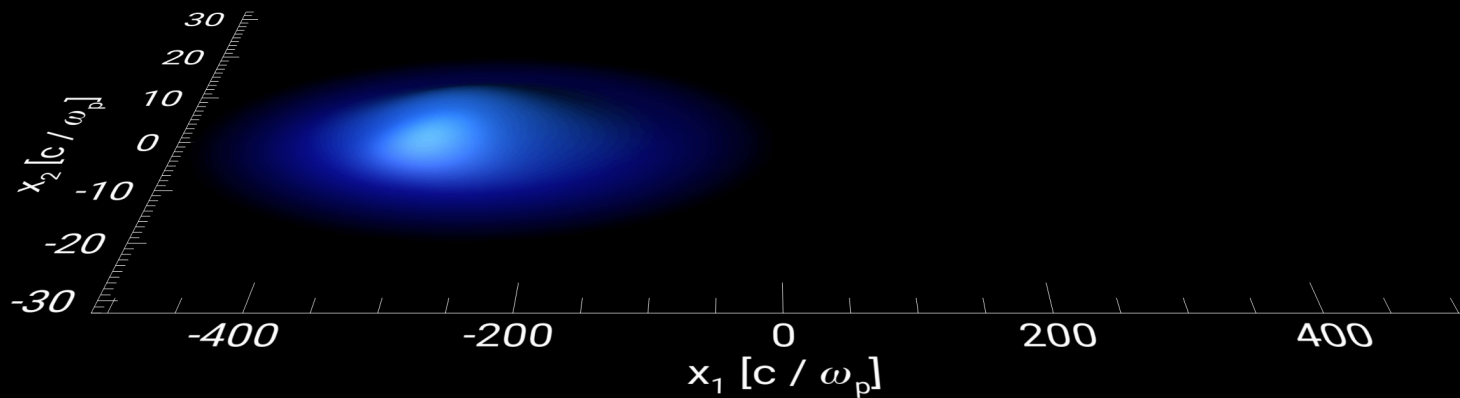


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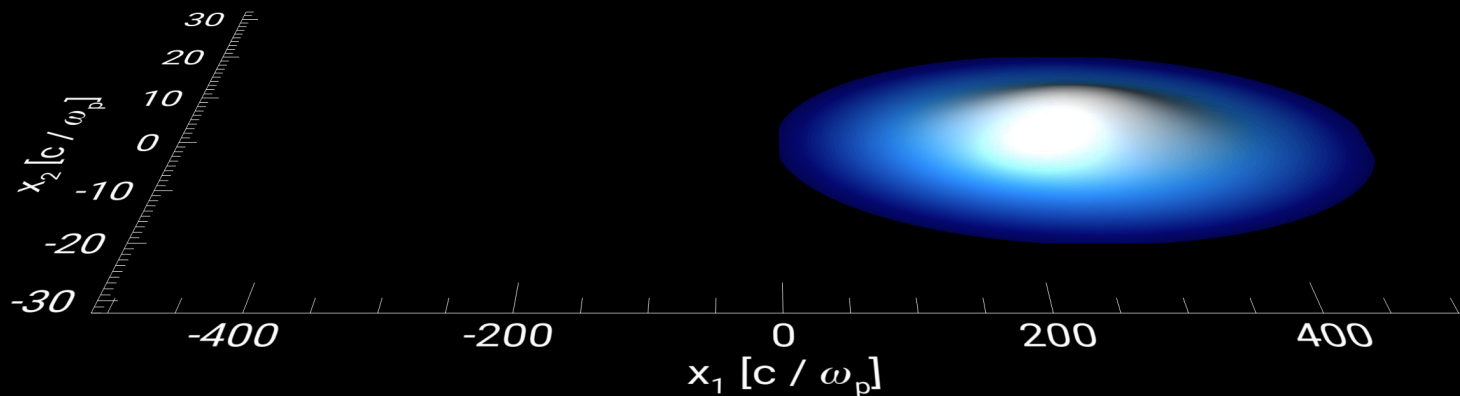


## Electron density (300 MeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 15

$\chi = 0.3$

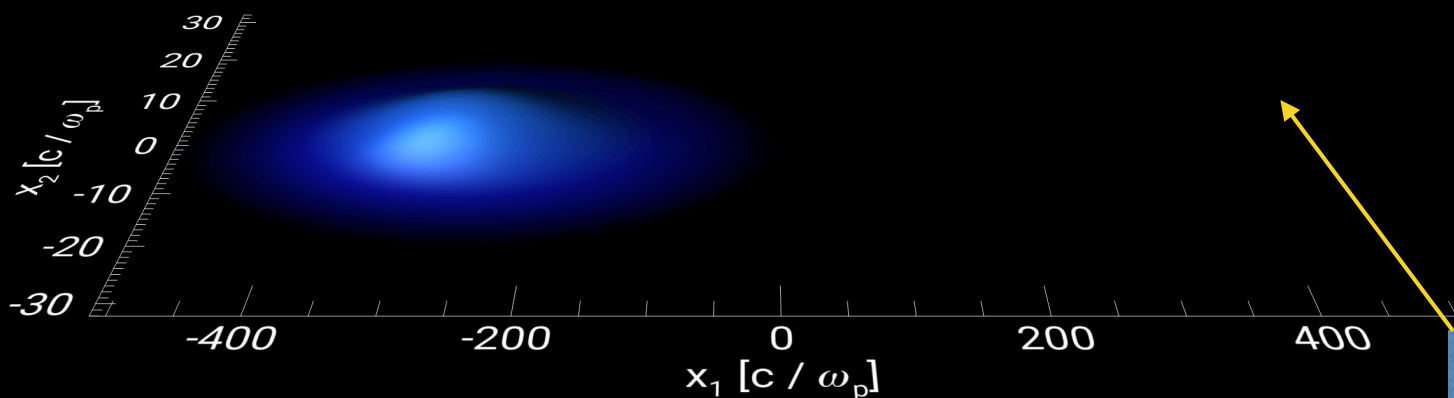


## Electron density (10 GeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 0.05

$\chi = 8$



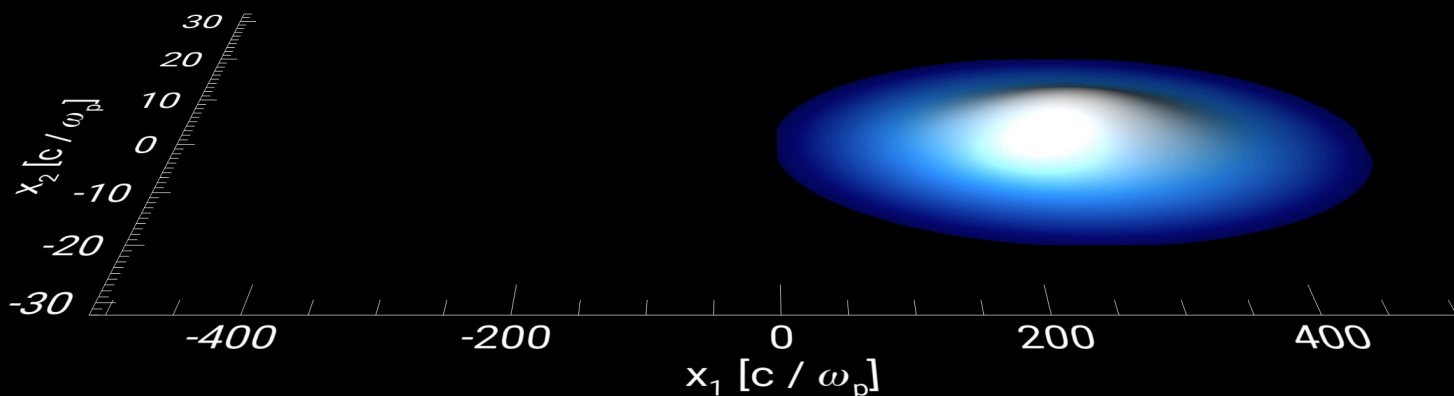
New pairs  
being created  
(5 pairs per e-)

## Electron density (300 MeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 15

$\chi = 0.3$

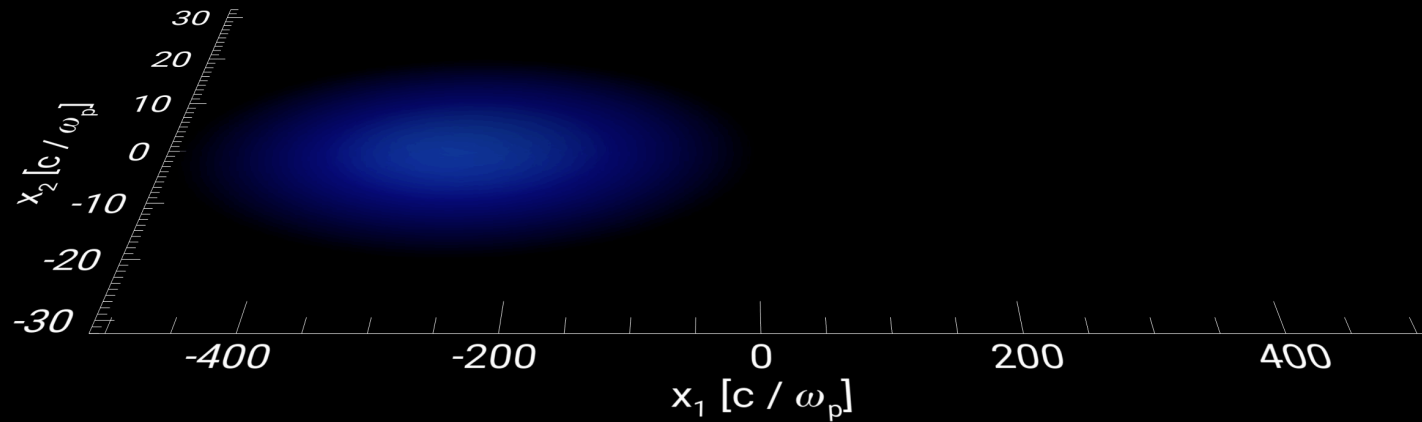


## Electron density (10 GeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 0.05

$\chi = 8$

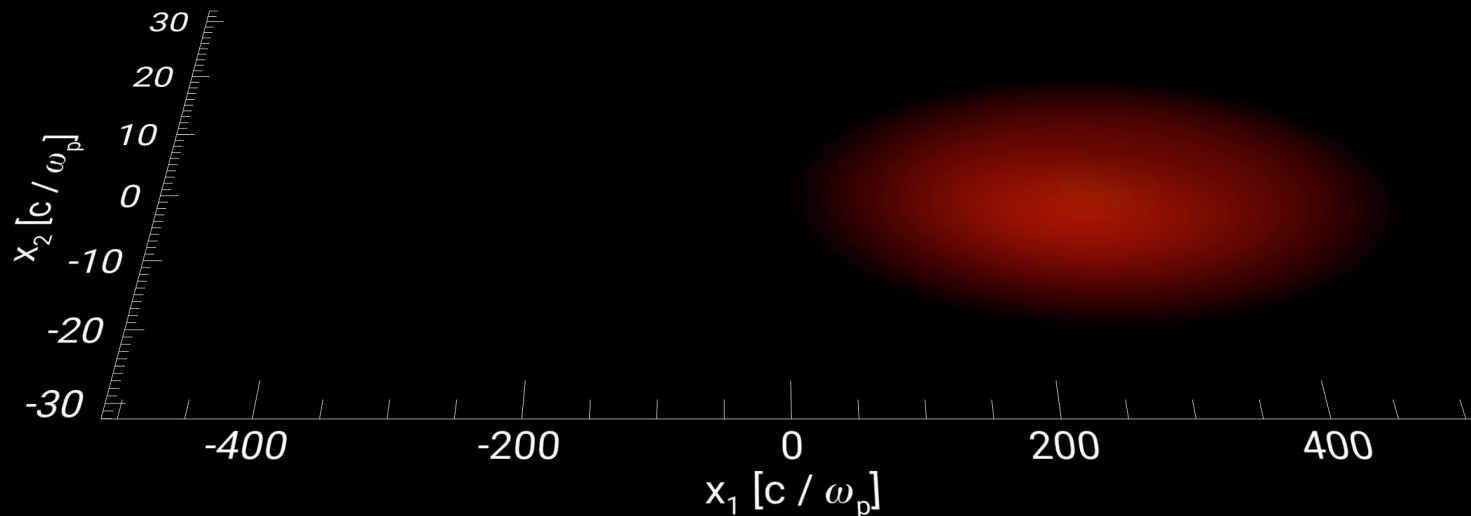


## Positron density (300 MeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 15

$\chi = 0.3$



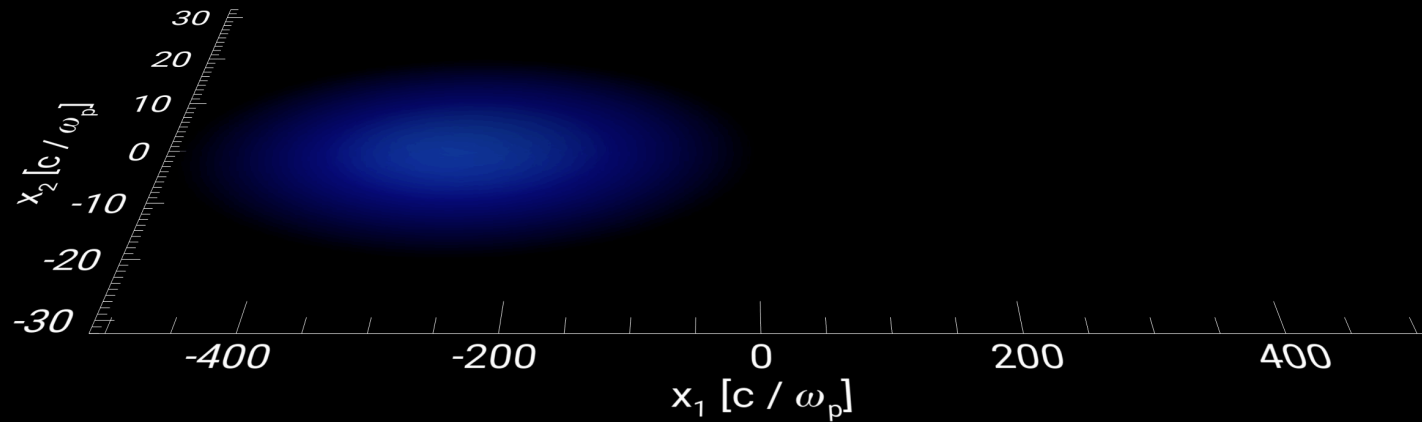


## Electron density (10 GeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 0.05

$\chi = 8$

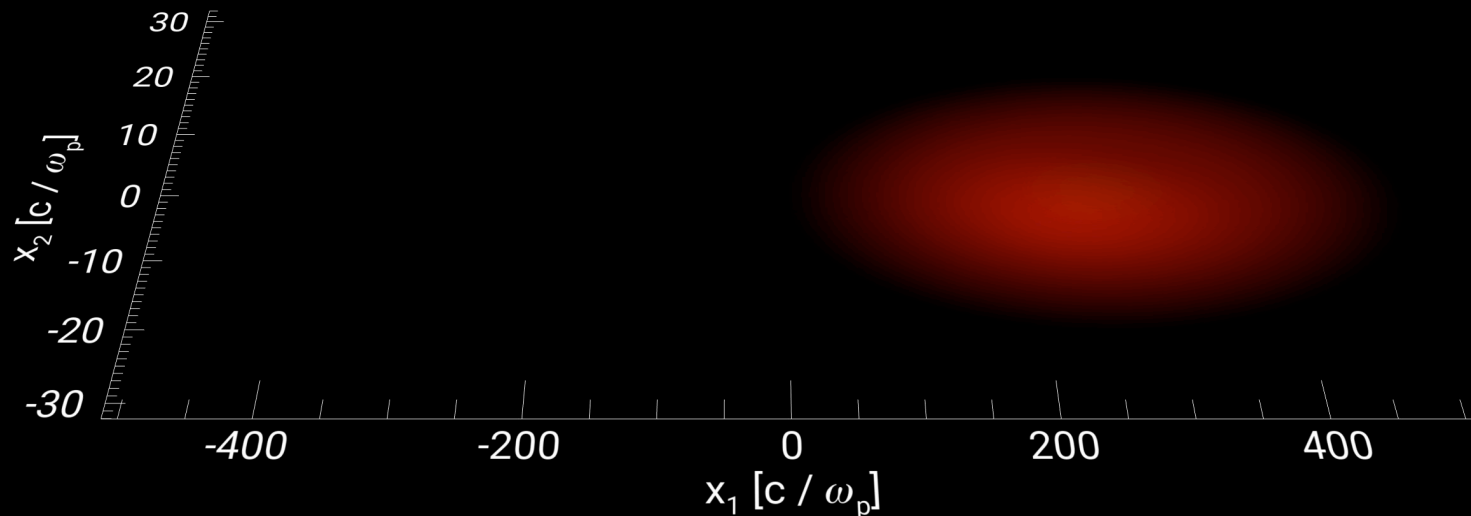


## Positron density (300 MeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 15

$\chi = 0.3$

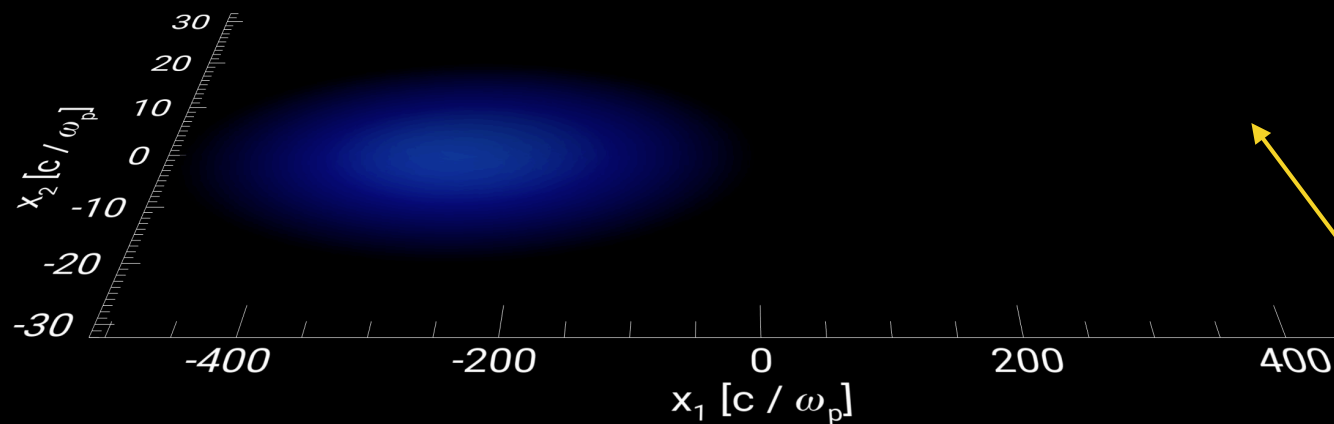


## Electron density (10 GeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 0.05

$\chi = 8$

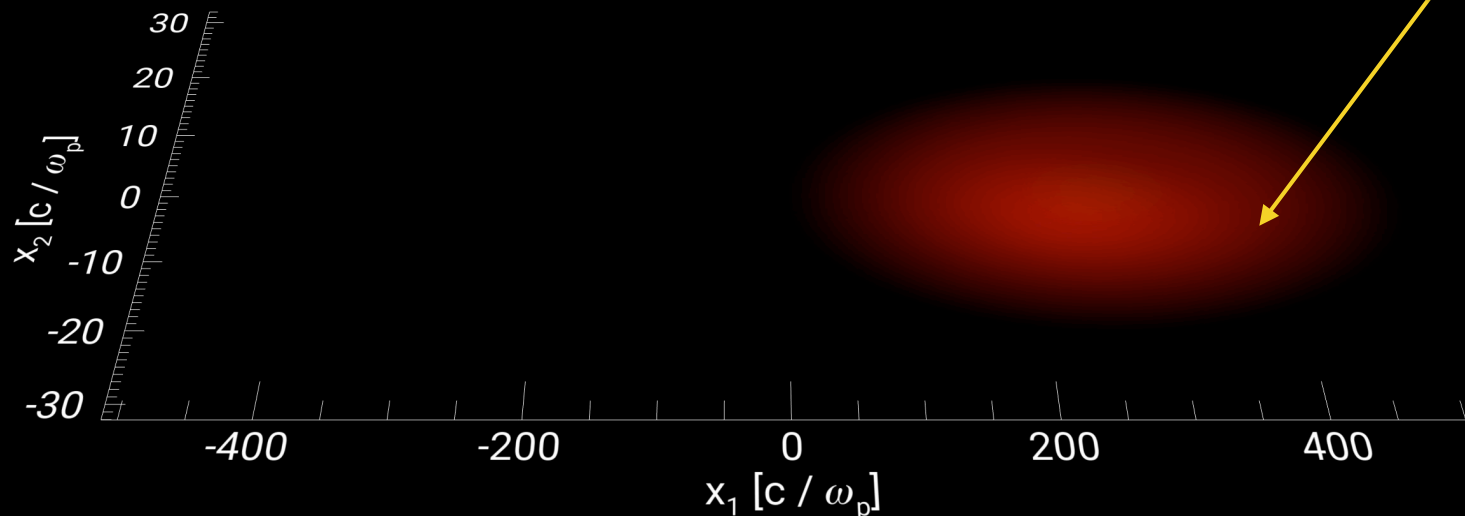


## Positron density (300 MeV)

Time = 0.00 [1 /  $\omega_p$ ]

D = 15

$\chi = 0.3$

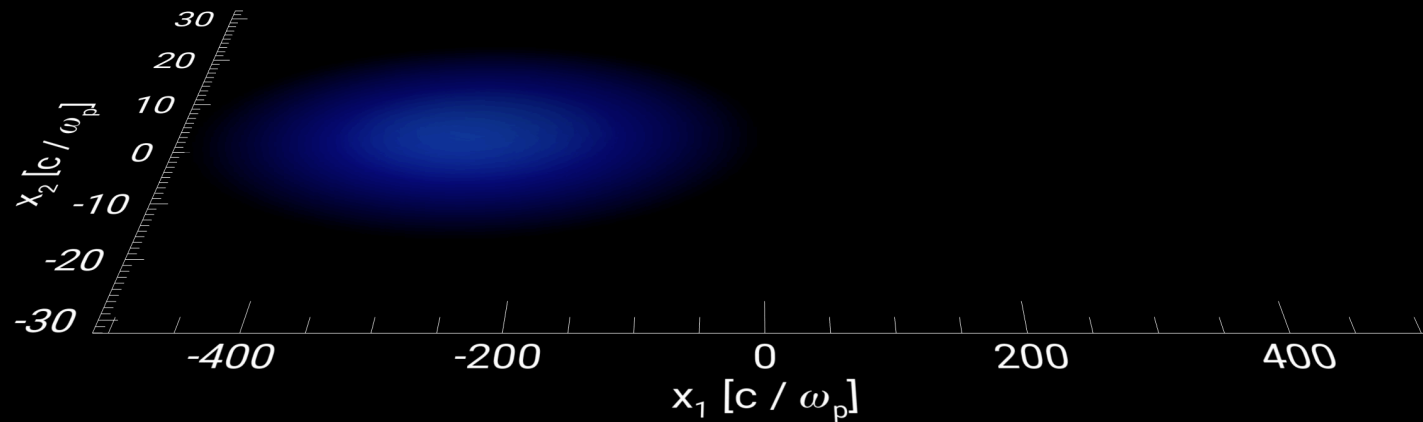


New pairs  
being created  
(5 pairs per e<sup>-</sup>)

# (FACET II) misaligned by $\sigma$ : kink instability

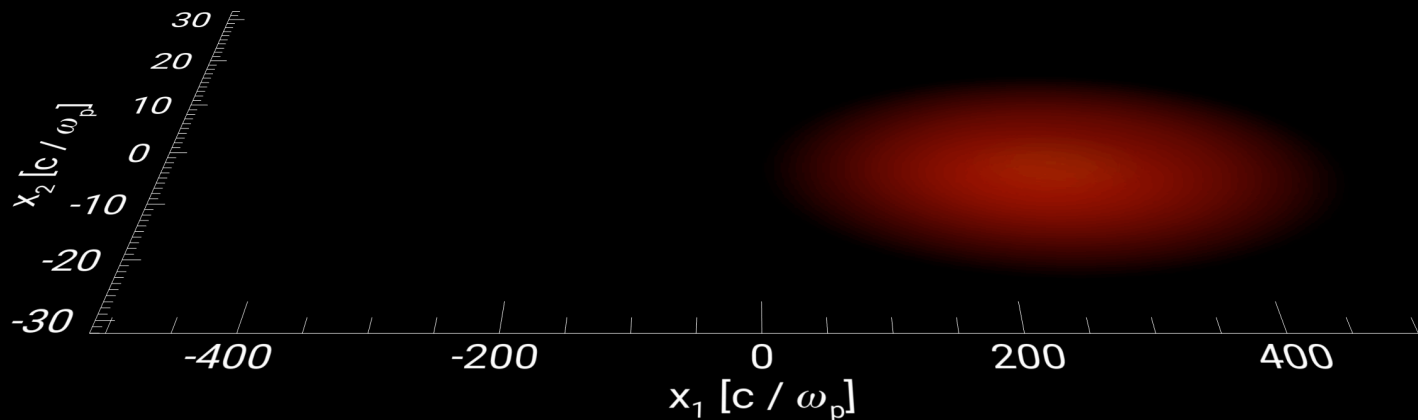
**Electron density (10 GeV)**

Time = 0.00 [1 /  $\omega_p$ ]



**Positron density (300 MeV)**

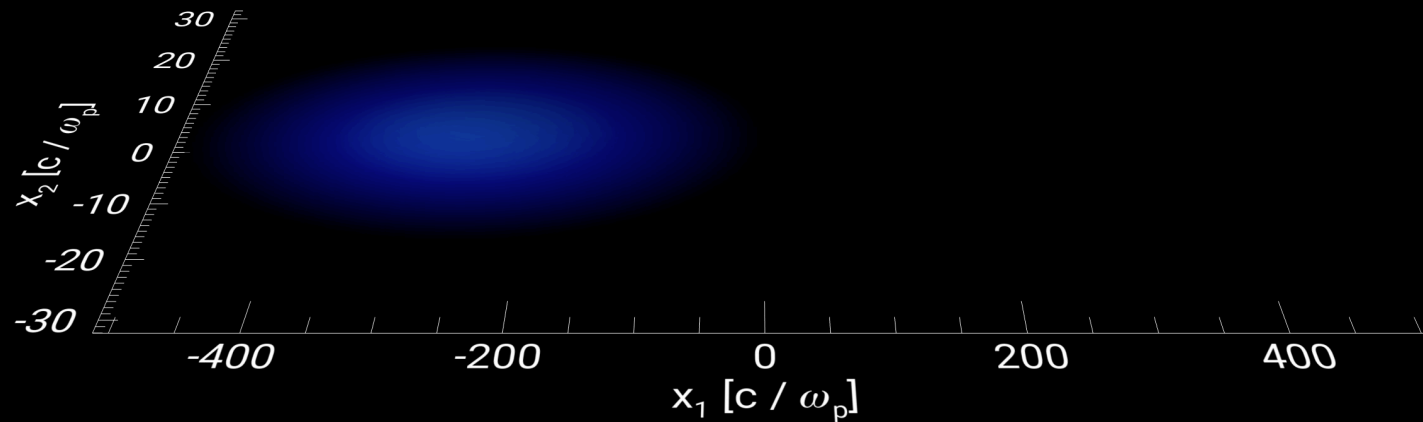
Time = 0.00 [1 /  $\omega_p$ ]



# (FACET II) misaligned by $\sigma$ : kink instability

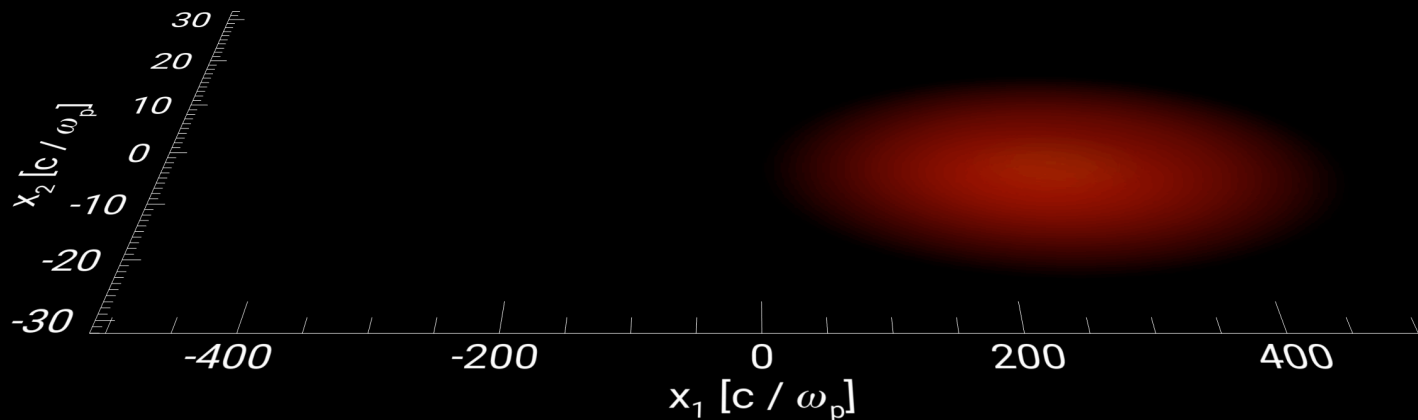
**Electron density (10 GeV)**

Time = 0.00 [1 /  $\omega_p$ ]



**Positron density (300 MeV)**

Time = 0.00 [1 /  $\omega_p$ ]



## **New Tools to tackle a variety of extreme plasma physics problems**

- ▶ Classical Radiation Reaction higher energy particles radiate more
- ▶ QED module (Non-linear Compton scattering, Breit-Wheeler, ....)
- ▶ Vacuum polarization solver

## **These tools have used to simulate various scenarios with intense lasers**

- ▶ Counter propagating electron beam - laser
- ▶ Counter propagating laser - laser : QED cascades
- ▶ Counter propagating optical laser - X-rays laser : vacuum birefringence

## **QED-PIC simulations can be envisaged to simulations beam-beam physics**

- ▶ High disruption regime can be simulated self-consistently
- ▶ New QED cross-sections need to be developed and added to fully study beam-beam physics for SFQED