Analysis of BBU in compact structure-based accelerator and a suppression method

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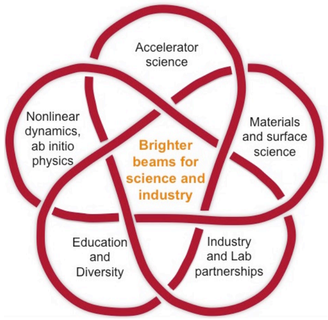
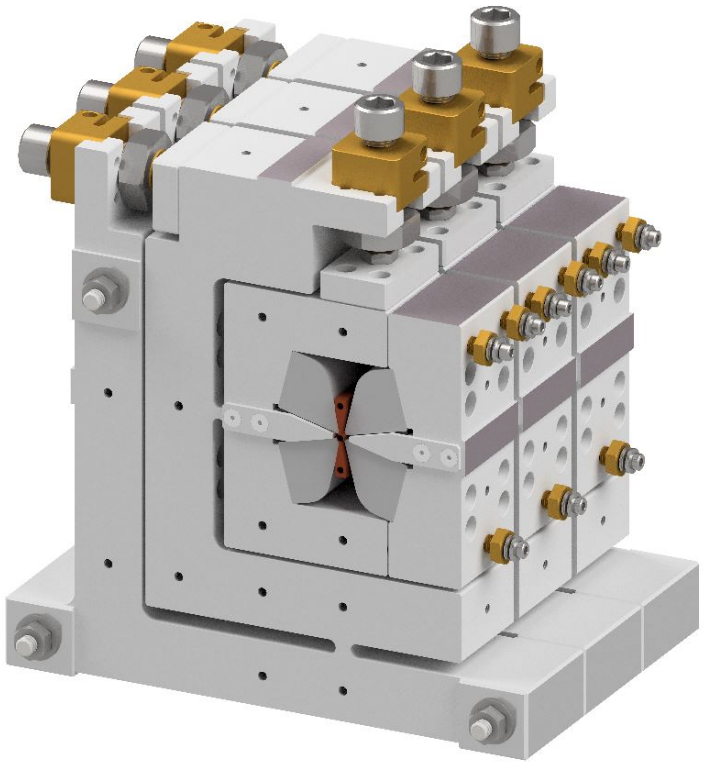


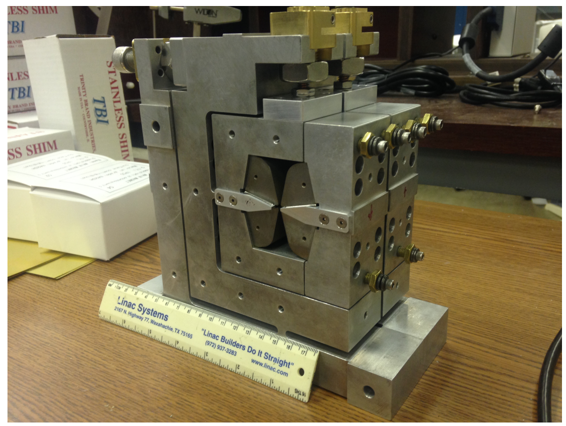
Outline

- Compact structure based wakefield accelerator
- Introduction to dynamics problem
- Adaptive focusing
- BNS damping and stability analysis
- Application to Collinear Wakefield Accelerator

Compact structure based wakefield accelerator



Jointly pursued by Argonne's AWA and APS groups
for a compact FEL facility



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Electroformed Copper Waveguides ICMS # NAV-17-XX

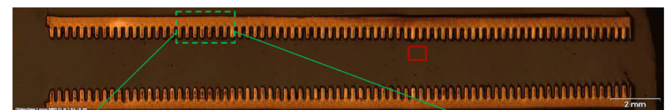

Figure 2 – Short waveguide sample, polished to 0.1 µm roughness, etched 7 seconds ferric nitrate/nitric acid solution.

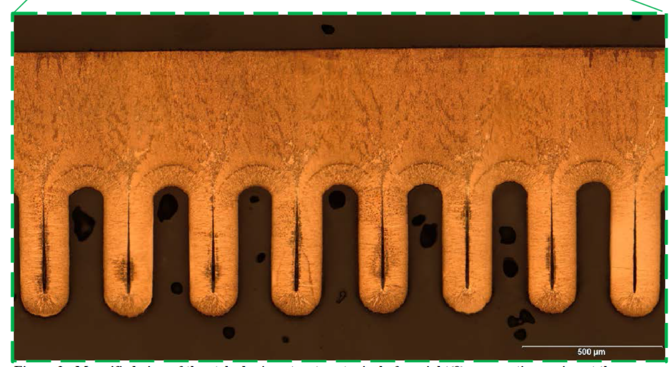

Figure 3 - Magnified view of the etched microstructure typical of an eight(8) corrugation region at the area boxed in green of the 'short' sample Figure 2.

Introduction

- Initial study
- A.W. Chao, B. Richter and C.Y. Yao, Nucl. Instr. and Meth. A, 178 (1980)

- Extensive analysis and full BBU theory for smooth focusing

J.R. Delayen

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **6**, 084402 (2003)

Cumulative beam breakup in linear accelerators with arbitrary beam current profile

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PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **7**, 074402 (2004)

Cumulative beam breakup in linear accelerators with random displacement of cavities and focusing elements

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PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME **6**, 084402 (2003)

Cumulative beam breakup in linear accelerators with arbitrary beam current profile

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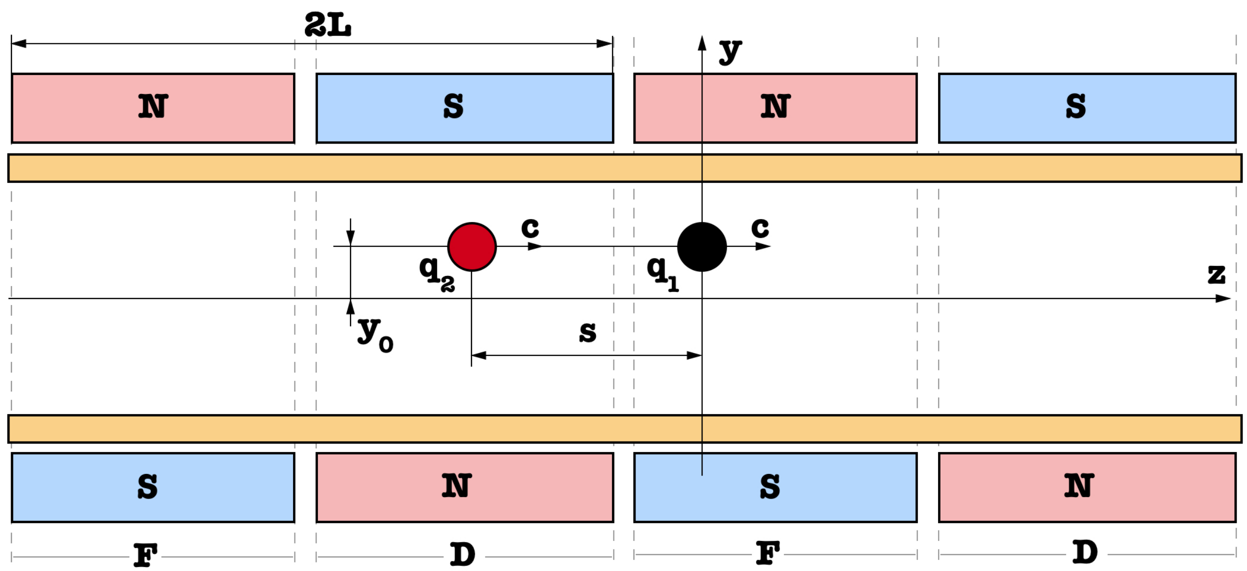
(Received 17 June 2003; published 27 August 2003)

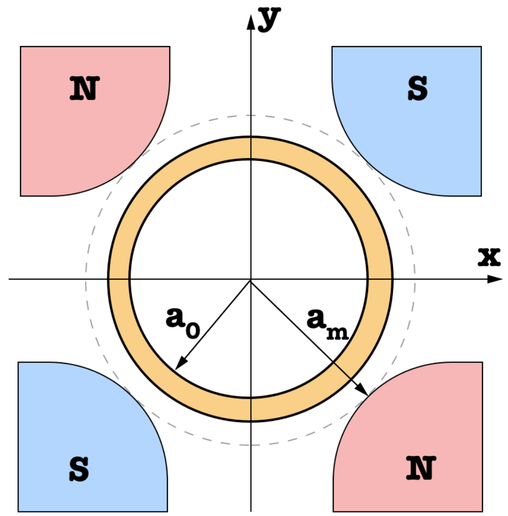
Introduction

Can we apply existing theory directly to analyse CWA?

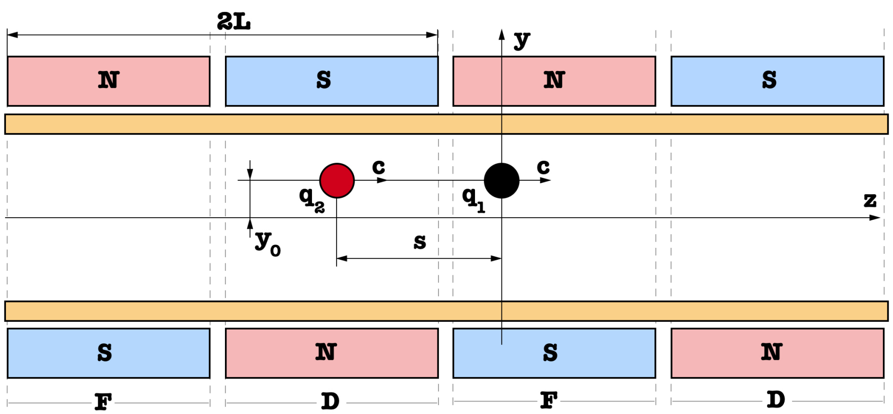
- Chao – limited to small wake amplitudes.
- Delayen – limited to smooth focusing.

The model





The model



Equations of motion

$$\begin{cases} \dfrac{d}{dz}\left(\dfrac{\gamma_1(z)}{\gamma_0}\dfrac{dy_1}{dz}\right) + K(z)y_1 = 0 \\ \dfrac{d}{dz}\left(\dfrac{\gamma_2(z)}{\gamma_0}\dfrac{dy_2}{dz}\right) + K(z)y_2 = w(s)y_1 \end{cases}$$

Adaptive focusing

$$\begin{cases} \dfrac{d}{dz}\left(\dfrac{\gamma_1(z)}{\gamma_0}\dfrac{dy_1}{dz}\right) + K(z)y_1 = 0 \\ \dfrac{d}{dz}\left(\dfrac{\gamma_2(z)}{\gamma_0}\dfrac{dy_2}{dz}\right) + K(z)y_2 = w(s)y_1 \end{cases}$$

- How we keep constant phase advance?
- How to tweak magnetic system to match the energy of the first particle?
- We know that to mitigate the wakefield we need energy spread.

Adaptive energy chirp

$$\frac{d\gamma_1}{dz} \approx \frac{eE_z(0)}{m_e c^2}$$

α

$$\frac{d\gamma_2}{dz} \approx \frac{eE_z(s)}{m_e c^2}$$

$$\gamma_1(z) = \gamma_0(1 - \alpha z)$$

$$\gamma_2(z) = \gamma_0\left(1 - f(s) - \alpha z \frac{E_z(s)}{E_z(0)}\right)$$

Adaptive energy chirp

$$\frac{E_z(s)}{E_z(0)} = 1 - f(s)$$

$$\gamma_2(z) = \gamma_0 \left(1 - f(s) - \alpha z \frac{E_z(s)}{E_z(0)} \right)$$

$$\gamma_2(z) = \gamma_0 \left(1 - f(s)\right)\left(1 - \alpha z\right)$$

Adaptive energy chirp

$$\begin{cases} (1-\alpha z)\dfrac{d^2 y_1}{dz^2} - \alpha\dfrac{dy_1}{dz} + K(z)y_1 = 0 \\ (1-\alpha z)\dfrac{d^2 y_2}{dz^2} - \alpha\dfrac{dy_2}{dz} + \dfrac{K(z)}{1-f(s)}y_2 = \dfrac{w(s)}{1-f(s)}y_1 \end{cases}$$

we introduce new variables

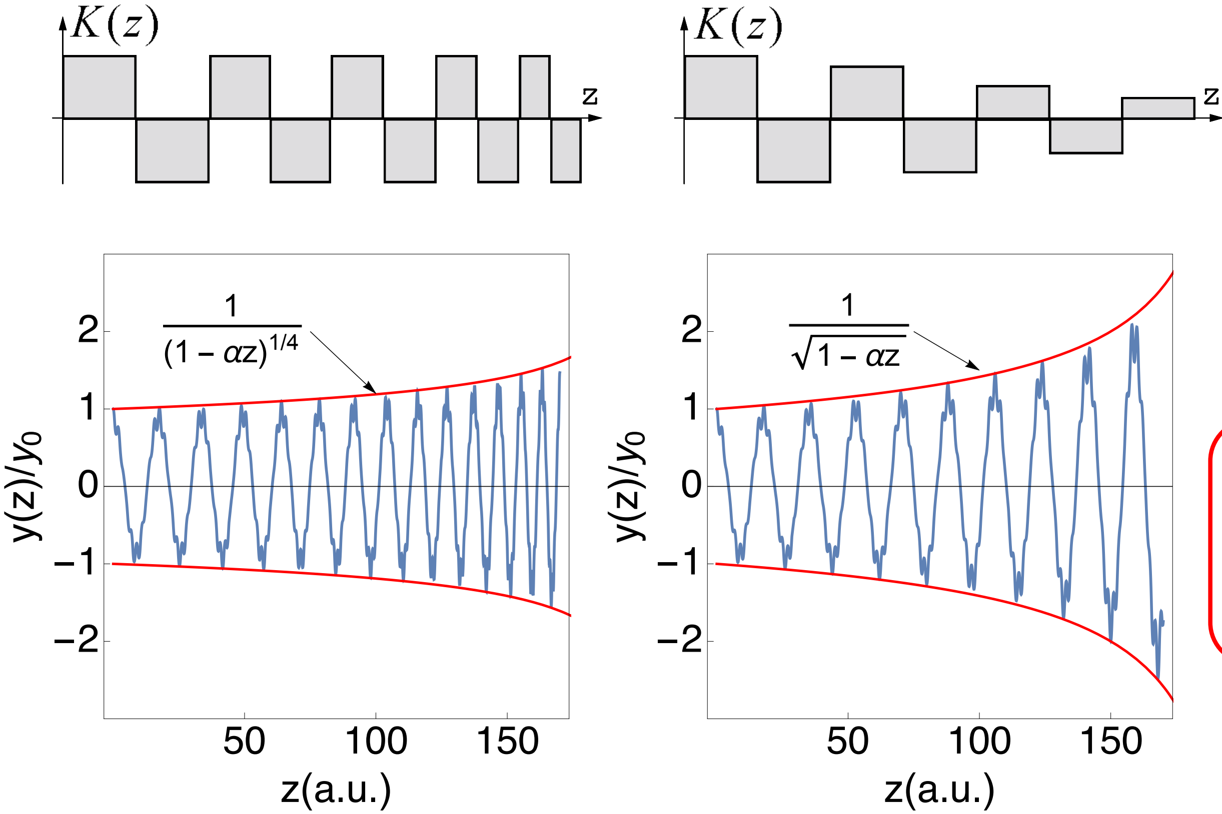
$$y_{1,2}(z) = \frac{g_{1,2}(u)}{\sqrt{u}}$$

$$u = \sqrt{1-\alpha z}$$

$$u > 0.1$$

$$\begin{cases} \dfrac{d^2 g_1}{du^2} + K(z(u))g_1 = 0 \\ \dfrac{d^2 g_2}{du^2} + \dfrac{K(z(u))}{1-f(s)}g_2 = \dfrac{w(s)}{1-f(s)}g_1 \end{cases}$$

Adaptive lens length L



$$K(z(u))$$

Periodic function of u

$$L = L_0\sqrt{1-\alpha z}$$

Adaptive focusing summary

- Adaptive chip

$$\frac{E_z(s)}{E_z(0)} = 1 - f(s)$$

- Adaptive lens length variation

$$L = L_0 \sqrt{1 - \alpha z}$$

Wakefield cancelation

$$\begin{cases} \dfrac{d^2 x_1}{dt^2} + K(t) x_1 = 0 \\ \dfrac{d^2 x_2}{dt^2} + \dfrac{K(t)}{1 - f(s)} x_2 = \dfrac{w(s)}{1 - f(s)} x_1 \end{cases}$$

Wakefield cancelation

$$\begin{pmatrix} x_1(t) \\ p_1(t) \end{pmatrix} = \mathbf{X}_1^t \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

$$\begin{pmatrix} x_2(t) \\ p_2(t) \end{pmatrix} = \mathbf{X}_2^t \begin{pmatrix} x_0 \\ p_0 \end{pmatrix} + \frac{w(s)}{1 - f(s)} \mathbf{X}_2^t \int_0^t \left(\mathbf{X}_2^\tau \right)^{-1} \begin{pmatrix} 0 \\ p_1(\tau) \end{pmatrix} d\tau$$

Vladimir I. Arnol'd, Ordinary Differential Equations,
Springer-Verlag, Berlin (1992)

Wakefield cancelation

$$\begin{pmatrix} x_1(t) \\ p_1(t) \end{pmatrix} = \mathbf{X}_1^t \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

$$\begin{pmatrix} x_2(t) \\ p_2(t) \end{pmatrix} = \mathbf{T}^t \begin{pmatrix} x_0 \\ p_0 \end{pmatrix}$$

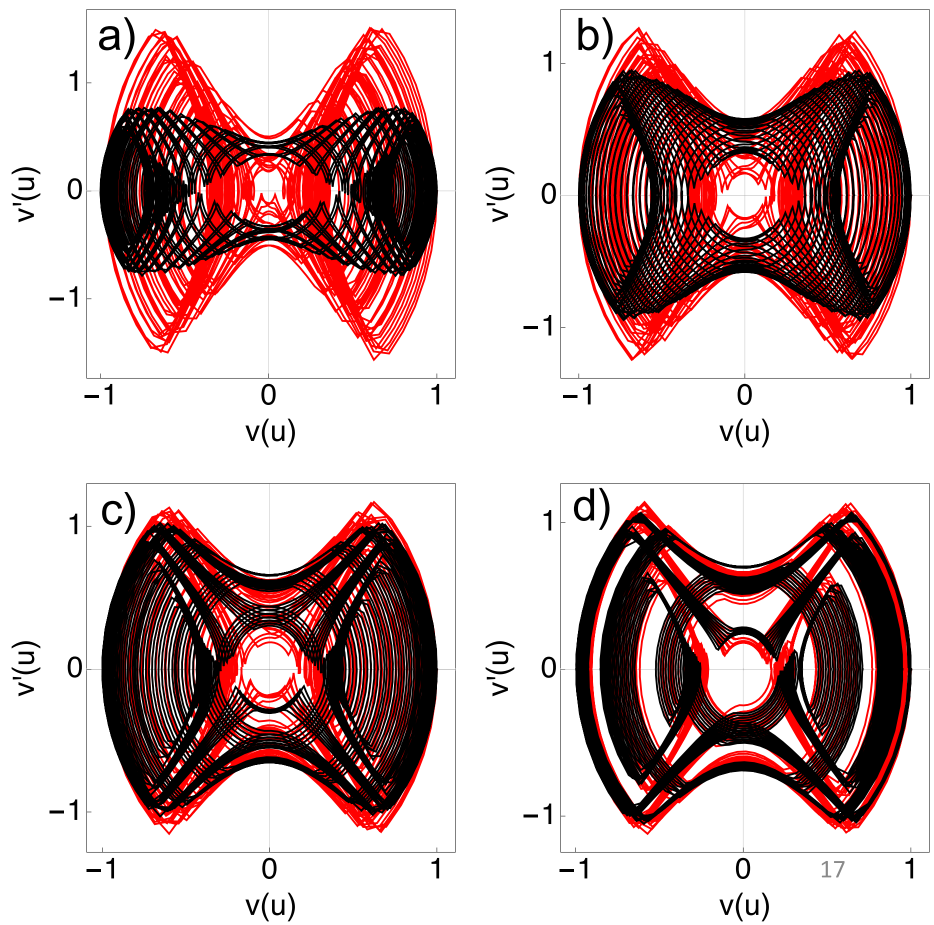
$$\mathbf{T}^t = \mathbf{X}_2^t + \mathbf{X}_2^t \int_0^t \left(\mathbf{X}_2^\tau\right)^{-1} \mathbf{W} \mathbf{X}_1^\tau d\tau$$

$$\mathbf{W} = \begin{pmatrix} 0 & 0 \\ \dfrac{w(s)}{1-f(s)} & 0 \end{pmatrix}$$

Wakefield cancelation

General idea of BNS damping[1]
for determining f(s)

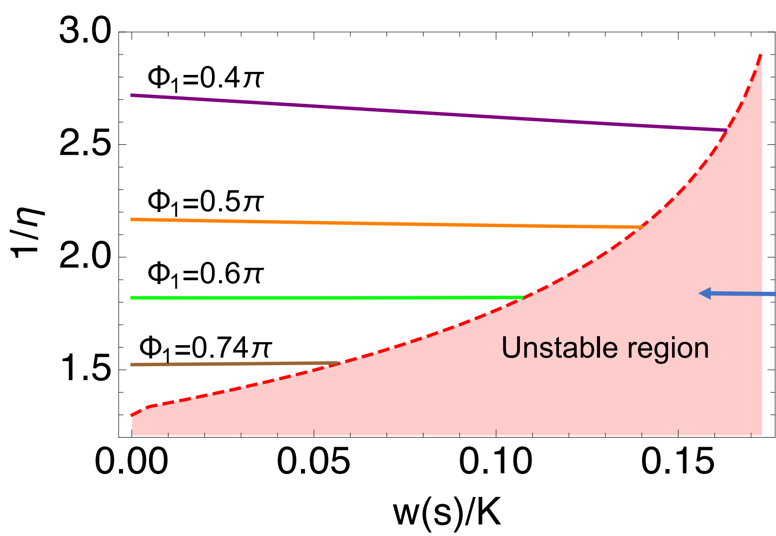
$$\mathrm{Tr}\left[\mathbf{X}_1^{2L}\right] = \mathrm{Tr}\left[\mathbf{T}^{2L}\right]$$

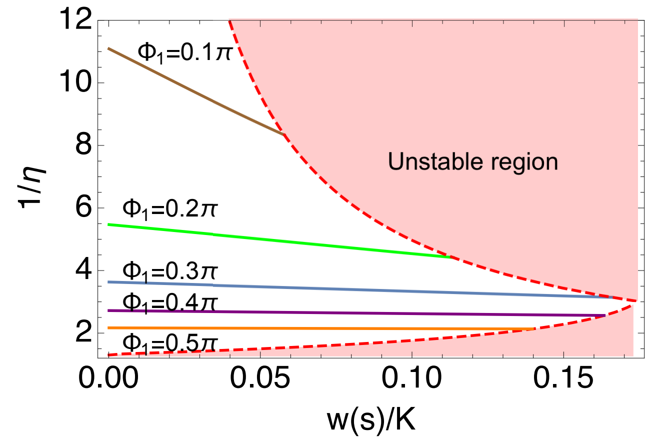


[1] V. Balakin, S. Novokhatsky, V. Smirnov, Transverse Beam Dynamics, Proc. 12th Int. Conf. on High Energy Acc., Batavia, Illinois, (1983) p.119- 120

Wakefield cancelation. Stability.

$$f(s) = \frac{1}{\eta} \frac{w(s)}{\max|K|}$$





$$\left|\mathrm{Tr}\left[\mathbf{X}_2^{2L}\right]\right| > 2$$

Stability condition for Collinear Wakefiled Accelerator

$$f(s) \approx \frac{1}{\eta} \frac{\int_0^s w(s-s_0)q(s_0)ds_0}{\max|K|}$$

$$E_z(s) = 2\kappa_{||}\int_0^s w_{||}(s-s_0)q(s_0)ds_0$$

$$\frac{E_z(s)}{E_z(0)} = 1 - f(s)$$

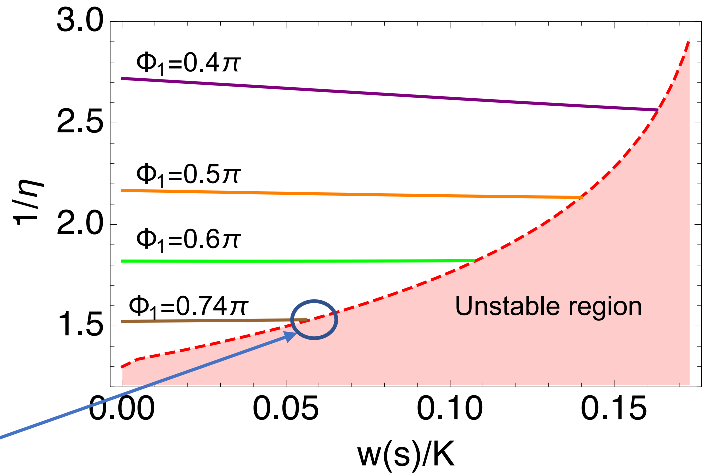
Stability condition for Collinear Wakefiled Accelerator

$$\left|\frac{\Delta\gamma}{\gamma}\right| > \frac{1}{\eta}\frac{a_m}{cB_0}\frac{\kappa_\perp}{2\kappa_\parallel}\frac{\max\left|E_+\right|}{k_0}\frac{\sqrt{R^2-1}}{R}$$

Stability condition for Collinear Wakefiled Accelerator

$$\Phi_1 = 0.74\pi$$

$$\frac{1}{\eta} = 1.53$$



$$0.086 > \left|\frac{\Delta\gamma}{\gamma}\right| > 2\frac{1.53}{k_0 a_0}\frac{\max|E_+|}{cB_0}$$

Summary

- Adaptive chip

$$\frac{E_z(s)}{E_z(0)} = 1 - f(s)$$

- Adaptive lens length variation

$$L = L_0 \sqrt{1 - \alpha z}$$

- Restrictions on chirp

$$0.086 > \left| \frac{\Delta\gamma}{\gamma} \right| > 2 \frac{1.53}{k_0 a_0} \frac{\max |E_+|}{c B_0}$$

Numeric example

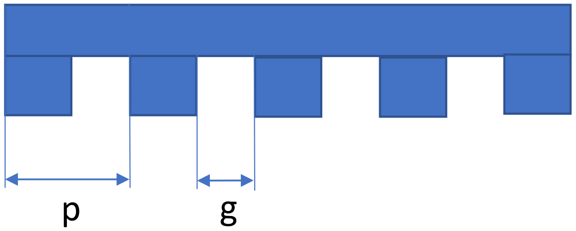
$$0.086 > \left|\frac{\Delta\gamma}{\gamma}\right| > 2\frac{1.53}{k_0 a_0}\frac{\max|E_+|}{cB_0}$$

$B = 1T$

$cB \approx 300MV/m$

$$\max|E_+| = 8.43 k_0 a_0 \, [MV/m]$$

Numeric example



$$\max|E_+| = 8.43 k_0 a_0 \; [MV/m]$$

100 MV/m → $k_0 a_0 \approx 12$

$$k_0 \approx \sqrt{\frac{p}{g}} \sqrt{\frac{2}{a_0 \Delta}} \qquad \sqrt{\frac{p}{g}} = \sqrt{2}$$

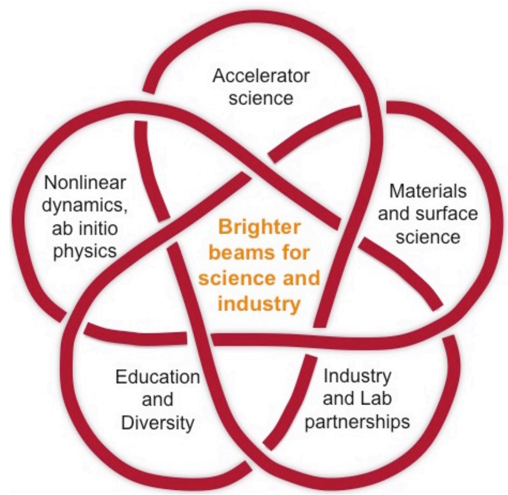
$$\Delta \approx \frac{a_0}{36}$$

$$a_0 = 1mm$$

$$\Delta = 28\mu m$$

Acknowledgments

- Center for Bright Beams and NSF



Full simulation of two particle motion

$$\frac{d}{dt}\left[\frac{\gamma_1(t)}{c^2\gamma_0}\frac{dz_1}{dt}\right] = -\alpha_1,$$
$$\frac{d}{dt}\left[\frac{\gamma_1(t)}{c^2\gamma_0}\frac{dy_1}{dt}\right] + K(z_1)y_1 = 0,$$
$$\frac{d}{dt}\left[\frac{\gamma_2(t)}{c^2\gamma_0}\frac{dz_2}{dt}\right] = -\alpha_2,$$
$$\frac{d}{dt}\left[\frac{\gamma_2(t)}{c^2\gamma_0}\frac{dy_2}{dt}\right] + \frac{K(z_2)}{1-f(s)}y_2 = \frac{w(s)}{1-f(s)}y_1$$