Saturation of the beam-hosing instability in quasi-linear plasma-wakefield accelerators

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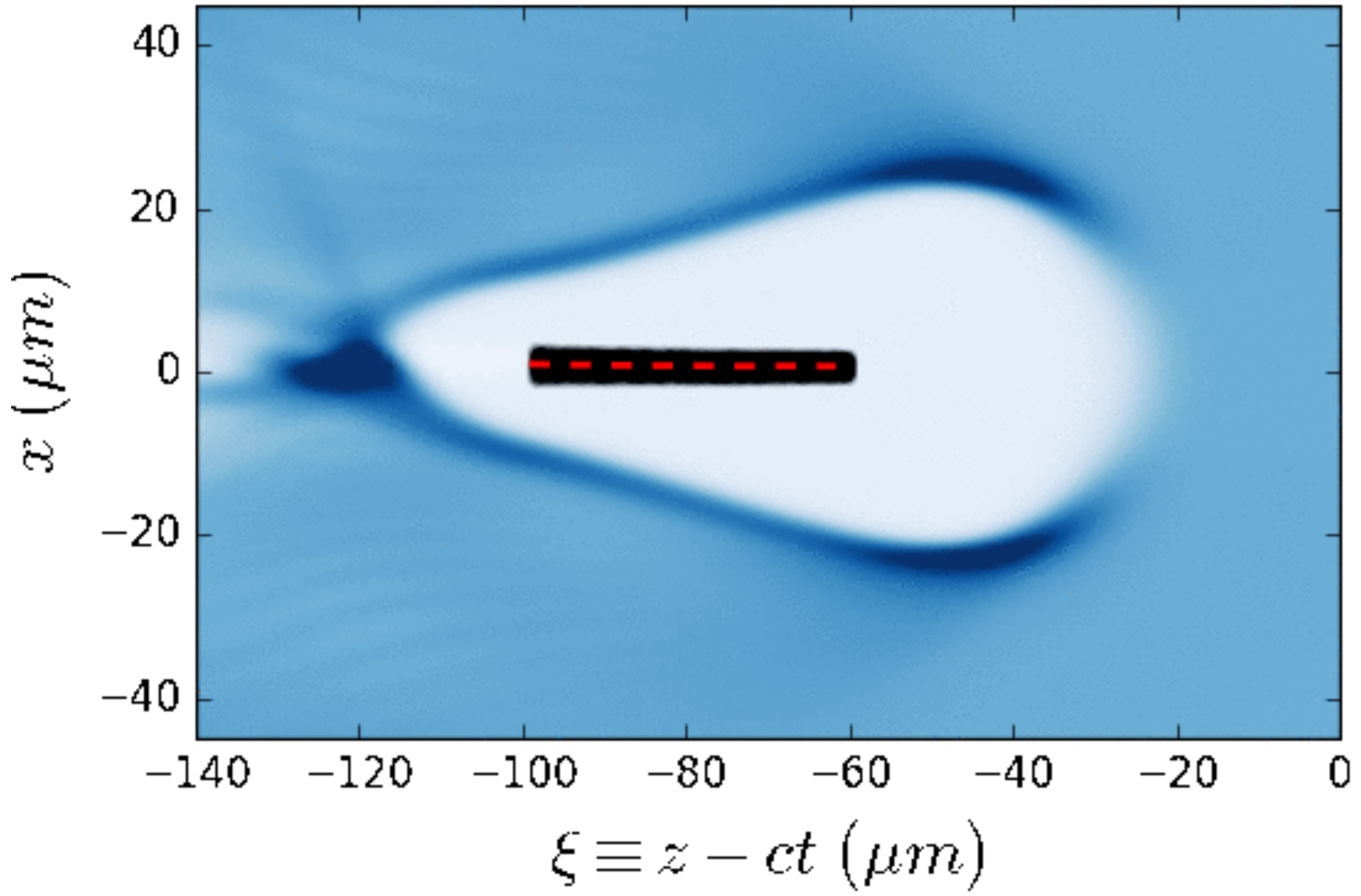
Outline

- Context: the beam-hosing instability
- Saturation of the instability with a linear betatron chirp
- Betatron chirp in the quasi-linear regime

Beam-hosing instability



For long-distance laser/plasma wakefield acceleration,
the beam has **unstable betatron oscillations**

Potential issue for **preservation of emittance**
in e.g. prospective plasma-based colliders.

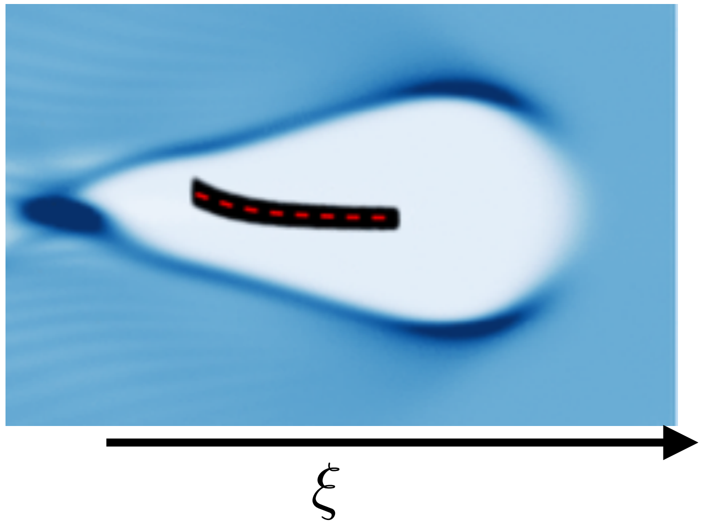
Equation of beam-hosing instability

Equation of the oscillations for a flat-top bunch

$$\partial_z^2 x_c(\xi, z) + k_\beta^2 \, x_c(\xi, z) = k_c^2 \int_\xi^0 \sin(\kappa_p(\xi' - \xi)) x_c(\xi', z) \; \kappa_p d\xi'$$

(Acceleration effects neglected)

x_c: off-axis position of the bunch, at a given slice
z : propagation distance
ξ : head-to-tail slice coordinate along the bunch



Equation of beam-hosing instability

Equation of the oscillations for a flat-top bunch

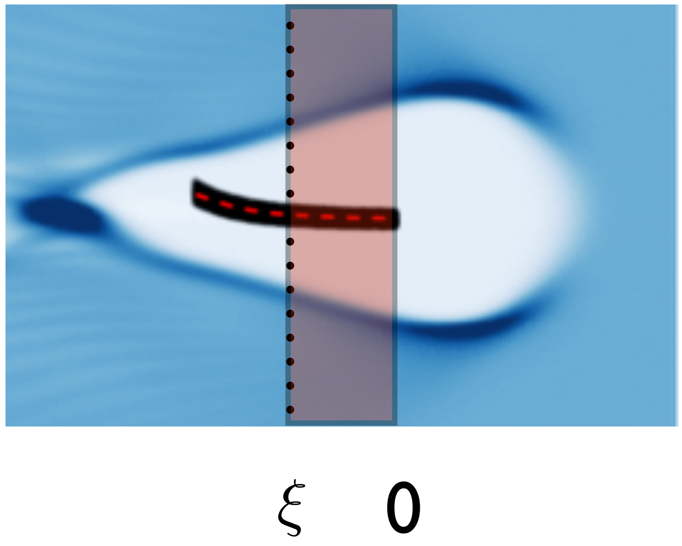
$$\partial_z^2 x_c(\xi,z) + k_\beta^2\, x_c(\xi,z) = k_c^2 \int_\xi^0 \sin(\kappa_p(\xi'-\xi))x_c(\xi',z)\ \kappa_p d\xi'$$

Betatron oscillations in an unperturbed bubble/wakefield

Perturbation of the bubble/wakefield by the part of the bunch that is **ahead of** ξ

If k_β is independent of ξ (constant along the bunch): the right-hand side (due to the bunch which is ahead) oscillates at the **resonant frequency** of the left-hand side

→ Growing instability



Equation of beam-hosing instability

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The above equation is valid for:

- **LWFA** and **PWFA**
- in the **blow-out** and **quasi-linear regime** (under certain conditions)

with different expressions for $k_\beta, \ \kappa_p, \ k_c$

Bubble regime

$$\kappa_p = \frac{r_{adiab}}{r_b} \frac{k_p}{\sqrt{2(1+\psi_0)}}$$

$$k_\beta = \frac{k_p}{\sqrt{2\gamma}} \qquad k_c^2 = \frac{n_b}{n_p} \frac{k_p^2}{2\gamma}$$

Huang, PRL, 2007

Quasi-linear regime

$$\kappa_p = k_p \qquad k_c^2 = \frac{k_p^2}{2\gamma}$$

$$k_\beta^2 = \frac{k_p^2}{2\gamma} \left(\frac{n_d k_p L_d}{n_p} \underset{\underset{\text{driver}}{\uparrow}}{\sin(k_p(L - \xi))} + \frac{n_b}{n_p} \int_\xi^0 \underset{\underset{\text{beam loading}}{\uparrow}}{\sin(k_p(\xi' - \xi)) k_p d\xi'} \right)$$

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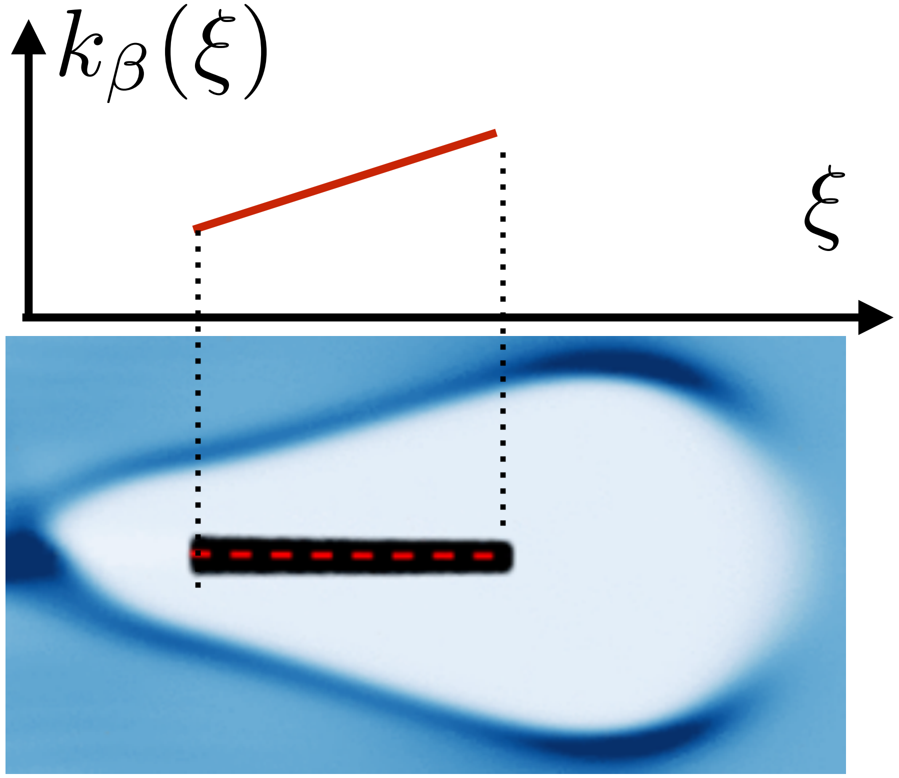
Using a betatron variation to reduce beam-hosing

$$\partial_z^2 x_c(\xi,z) + k_\beta(\xi)^2\, x_c(\xi,z) = k_c^2 \int_\xi^0 \sin(\kappa_p(\xi'-\xi))x_c(\xi',z)\kappa_p d\xi'$$

Introducing a **head-to-tail variation** in betatron frequency should **mitigate** the instability. (e.g. *Balakin et al., 1983*)



In the blow-out regime, the betatron variation can be generated with **an energy chirp**.
(e.g. *Mehring et al., PRL, 2017*)

Betatron variation: auto-phasing condition

$$\partial_z^2 x_c(\xi, z) + k_\beta(\xi)^2 \; x_c(\xi, z) = k_c^2 \int_\xi^0 \sin(\kappa_p(\xi' - \xi)) x_c(\xi', z) \kappa_p d\xi'$$

Auto-phasing condition

For betatron oscillations **with a constant amplitude and frequency:**

$$k_\beta(\xi)^2 - k_\beta(0)^2 = k_c^2 \int_\xi^0 \sin(\kappa_p(\xi' - \xi)) \kappa_p d\xi'$$

$$k_\beta(\xi) \approx k_{\beta,0} + \frac{k_c^2 \kappa_p^2}{2k_{\beta,0}} \xi^2$$

Requires quadratic variation ; difficult in practice

However, this condition is somewhat **restrictive** because it searches **exclusively** for conditions in which the oscillations have a **constant** amplitude.

Betatron variation: linear chirp

$$\partial_z^2 x_c(\xi, z) + k_\beta(\xi)^2 \; x_c(\xi, z) = k_c^2 \int_\xi^0 \sin(\kappa_p(\xi' - \xi)) x_c(\xi', z) \kappa_p d\xi'$$

Instead: linear chirp

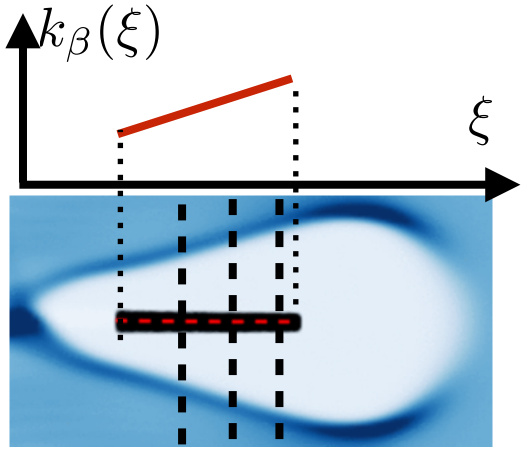
$$k_\beta(\xi) = k_{\beta,0} + (\partial_\xi k_\beta)\xi$$

→ Several analytical solutions in different regimes **relevant for conventional accelerators** show mitigation as a function of $(\partial_\xi k_\beta)$

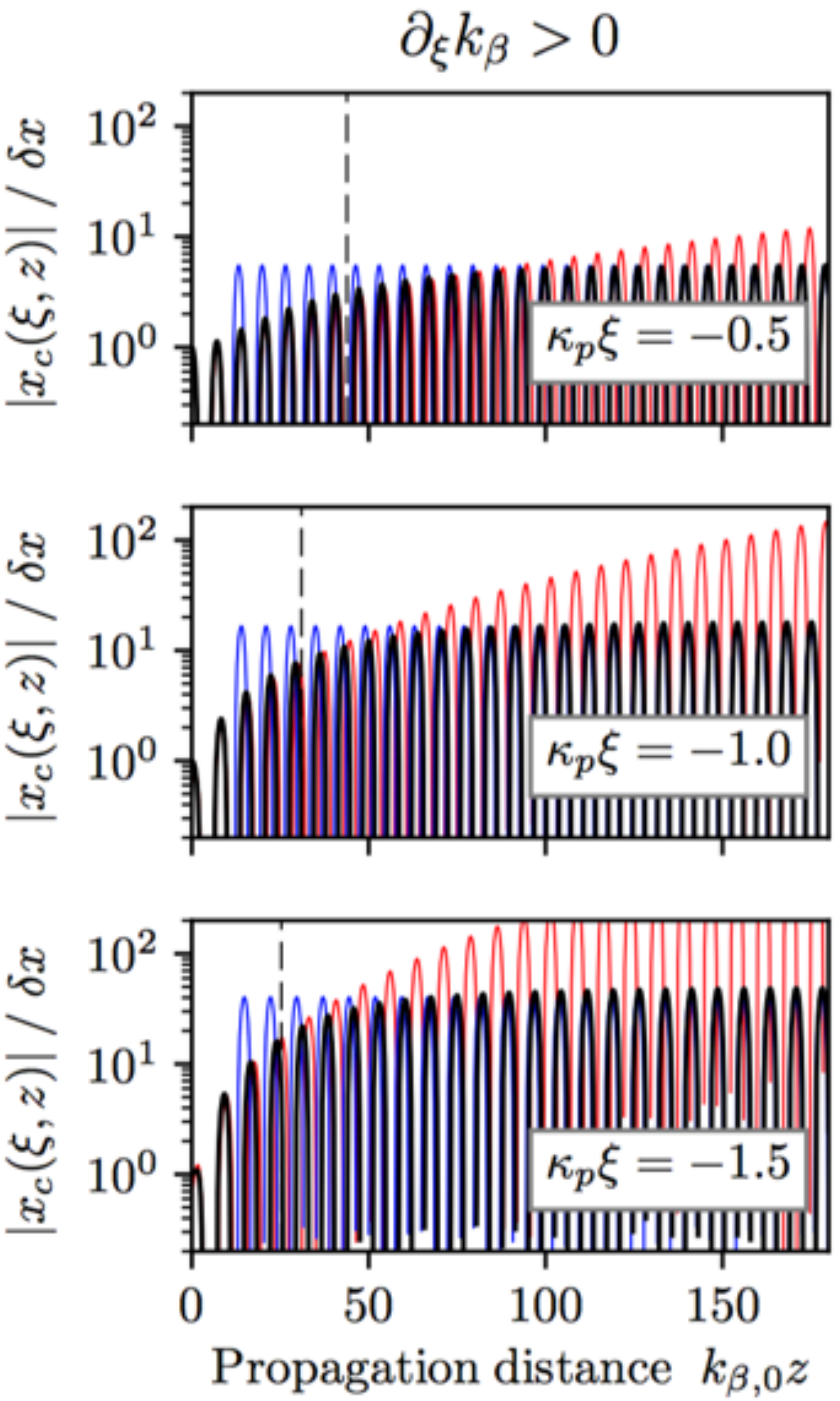
- *Whittum, J. Phys. A, 1997*
- *Stupakov, SLAC report, 1997*
- *Chernin & Mondeli, Particle Accelerators, 1989*

→ Recently: analytical solution in regime **relevant for plasma accelerator**
Lehe et al., submitted for publication
Valid for small chirp
Interestingly: depends on the **sign** of the chirp

Positive chirp: saturation



$$L_{sat} = \left(\frac{k_c^2 \kappa_p^2}{k_{\beta,0} |\partial_\xi k_\beta|^3 |\xi|} \right)^{1/2}$$



Standard beam-hosing (no chirp)

$$x_c(\xi, z) = \delta x \, \frac{\cos\left(k_\beta z - \frac{3}{4} N(\xi, z) + \frac{\pi}{12}\right)}{(6\pi)^{1/2} N(\xi, z)^{1/2}} \, e^{\frac{3\sqrt{3}}{4} N(\xi, z)}$$

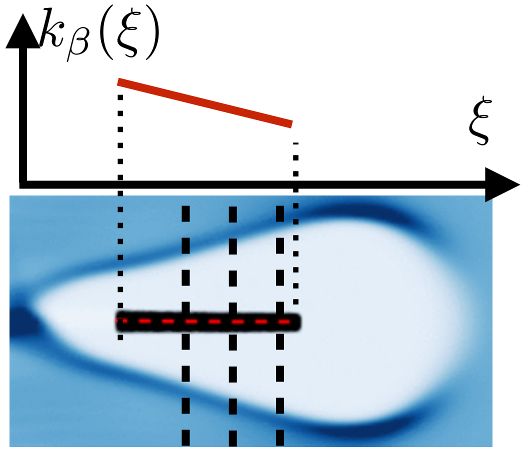
$$N(\xi, z) = \left(\frac{k_c^2 \kappa_p^2 |\xi|^2 z}{k_{\beta,0}} \right)^{1/3}$$

Asymptotic saturated solution

$$x_c(\xi, z) = \delta x \, \frac{\cos[\, k_\beta(\xi) z - \varphi(z) \,]}{(8\pi^2)^{1/4} N_{sat}(\xi)^{1/2}} \, e^{\sqrt{2} N_{sat}(\xi)}$$

$$N_{sat}(\xi) = \left(\frac{k_c^2 \kappa_p^2 |\xi|}{k_{\beta,0} \, (\partial_\xi k_\beta)} \right)^{1/2}$$

Negative chirp: saturation and slow decay



$$L_{sat} = \left(\frac{k_c^2 \kappa_p^2}{k_{\beta,0} |\partial_\xi k_\beta|^3 |\xi|} \right)^{1/2}$$

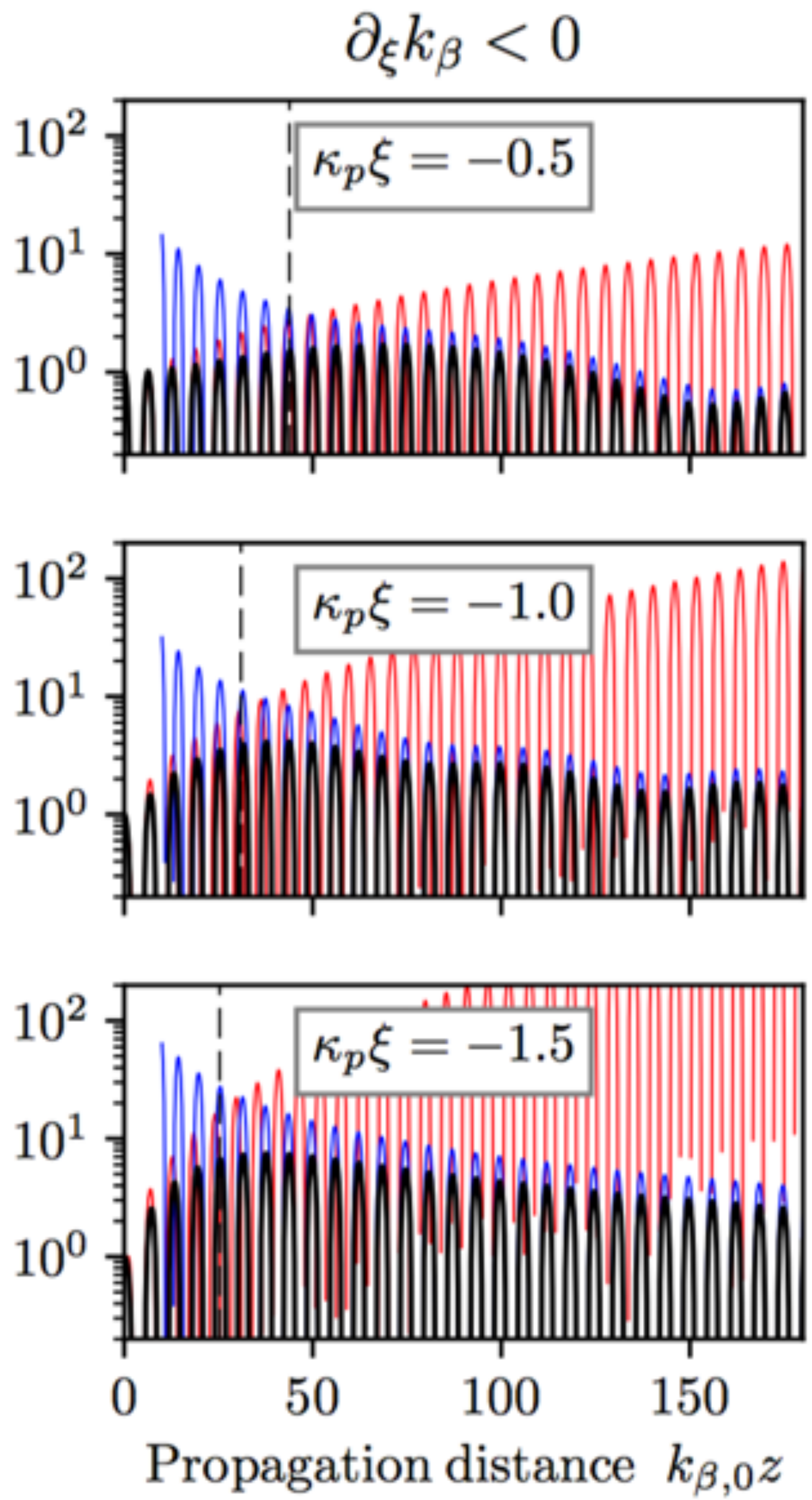
Standard beam-hosing (no chirp)

$$x_c(\xi,z) = \delta x \, \frac{\cos\left(k_\beta z - \frac{3}{4} N(\xi,z) + \frac{\pi}{12}\right)}{(6\pi)^{1/2} N(\xi,z)^{1/2}} \, e^{\frac{3\sqrt{3}}{4} N(\xi,z)}$$

Asymptotic solution

$$x_c(\xi,z) = -\delta x \, \frac{\sin(k_{\beta,0} z)}{(32\pi^2)^{1/4} N_{sat}(\xi)^{-1/2}} \, \frac{e^{\sqrt{2} N_{sat}(\xi)}}{|(\partial_\xi k_\beta)\, z \, \xi|}$$

$$+ \, \delta x \, \frac{\cos[\, k_\beta(\xi) z - \varphi(z) \,]}{(\pi^2/2)^{1/4} N_{sat}(\xi)^{1/2}} \cos\left(\sqrt{2} N_{sat}(\xi) - \frac{\pi}{4}\right)$$



Summary

- The beam-hosing instability can severely **degrade** the emittance.
- A **positive chirp** in betatron frequency causes the instability to saturate.
- A **negative chirp** in betatron frequency causes the instability to decay.
- In both case (positive and negative), the instability is much **less severe** than predicted by **standard scaling**, which assumes constant betatron frequency.

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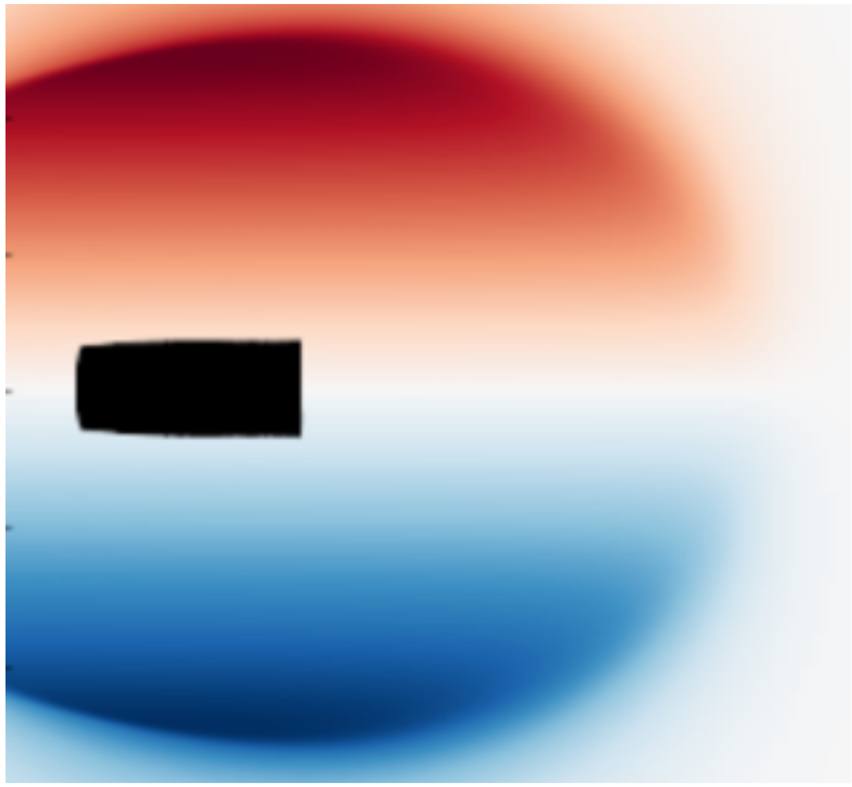
Generating a betatron chirp in different regimes

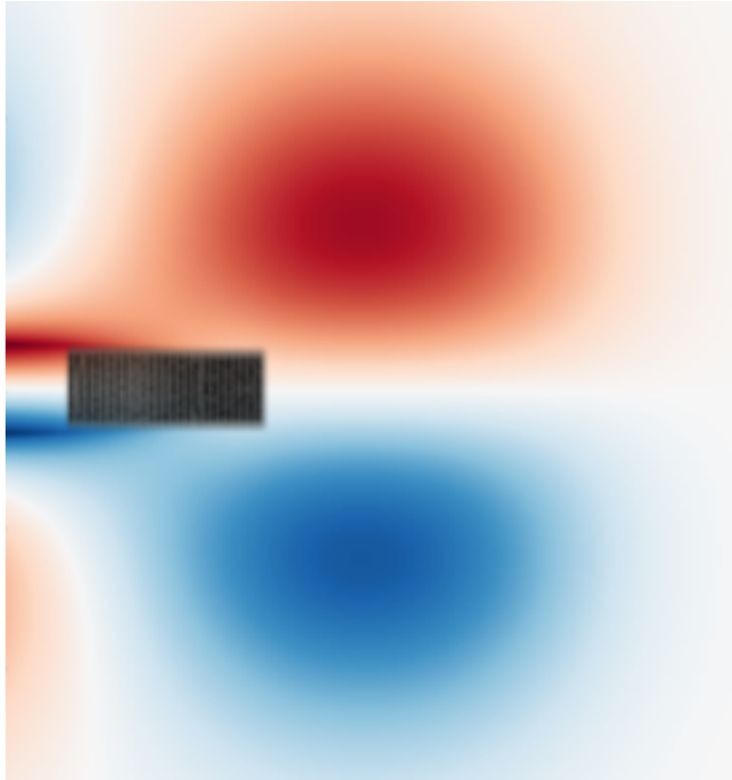
$$k_\beta = K/\sqrt{\gamma} \qquad F_{foc} = -mc^2K^2r$$

Blow-out regime



Focusing force is independent of ξ
Chirp requires a (correlated) energy spread

Quasi-linear regime



Focusing force naturally depends on ξ
No energy spread required

→ Confirm saturation in PIC simulations?

The code FBPIC (Fourier-Bessel PIC)

- **Spectral quasi-cylindrical Particle-In-Cell algorithm (azimuthal mode decomposition)**
- **Runs on GPU and (multi-core) CPU**
- **Open-source: github.com/fbpic/fbpic**
  Documentation: fbpic.github.io

Several useful features for plasma acceleration:

- Intrinsic mitigation of Numerical Cherenkov Radiation (NCR)
- Supports the boosted-frame technique
- Calculation of initial space-charge fields
- Field ionization physics (ADK model)

Using a quasi-cylindrical code for beam-hosing

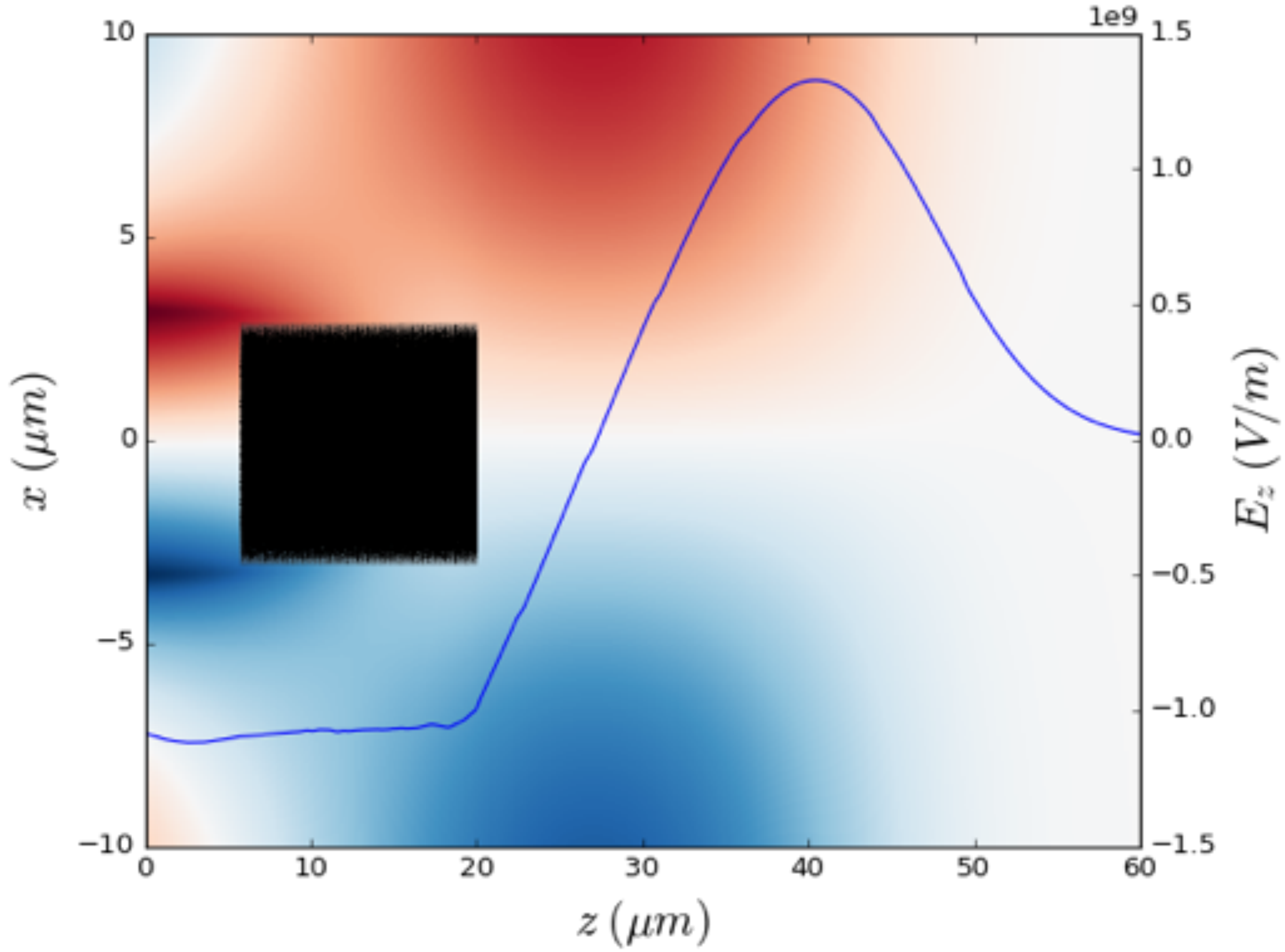
- The beam-hosing instability is **not cylindrically symmetric**.
- This asymmetry can be decomposed into **azimuthal modes** (m=0: cylindrical symmetric, m=1: dipole mode, m=2: quadrupole mode, etc.)
- **If** the centroid offset is **small compared** to the beam radius, the beam hosing instability excites predominantly the mode m=1. (i.e. modes m>1 are negligible)
- FBPIC simulates modes m=0 and m=1, and thus captures the beam-hosing instability in this case (and is much faster than a full 3D Cartesian code).

Simulation results

Simulation setup

- **Witness bunch:**
  No energy spread
  Triangular longitudinally (flattens the Ez field)
  matched K-V distribution

- **Driver:**
  either laser or bunch,
  in the linear regime



Parameters

$n_p = 2 \times 10^{17}\ \mathrm{cm}^{-3}$ $\gamma_{bunch} = 200$ $r_{bunch} = 3\ \mu\mathrm{m}$ $\ell_{bunch} = 15\ \mu\mathrm{m}$

- Laser-driven: $a_0 = 0.4$ $w_0 = 24\ \mu\mathrm{m}$ $\tau = 20\ \mathrm{fs}$
- Beam-driven: $n_d = 0.7\, n_p$ $r_d = 4\ \mu\mathrm{m}$ $\ell_d = 3\ \mu\mathrm{m}$

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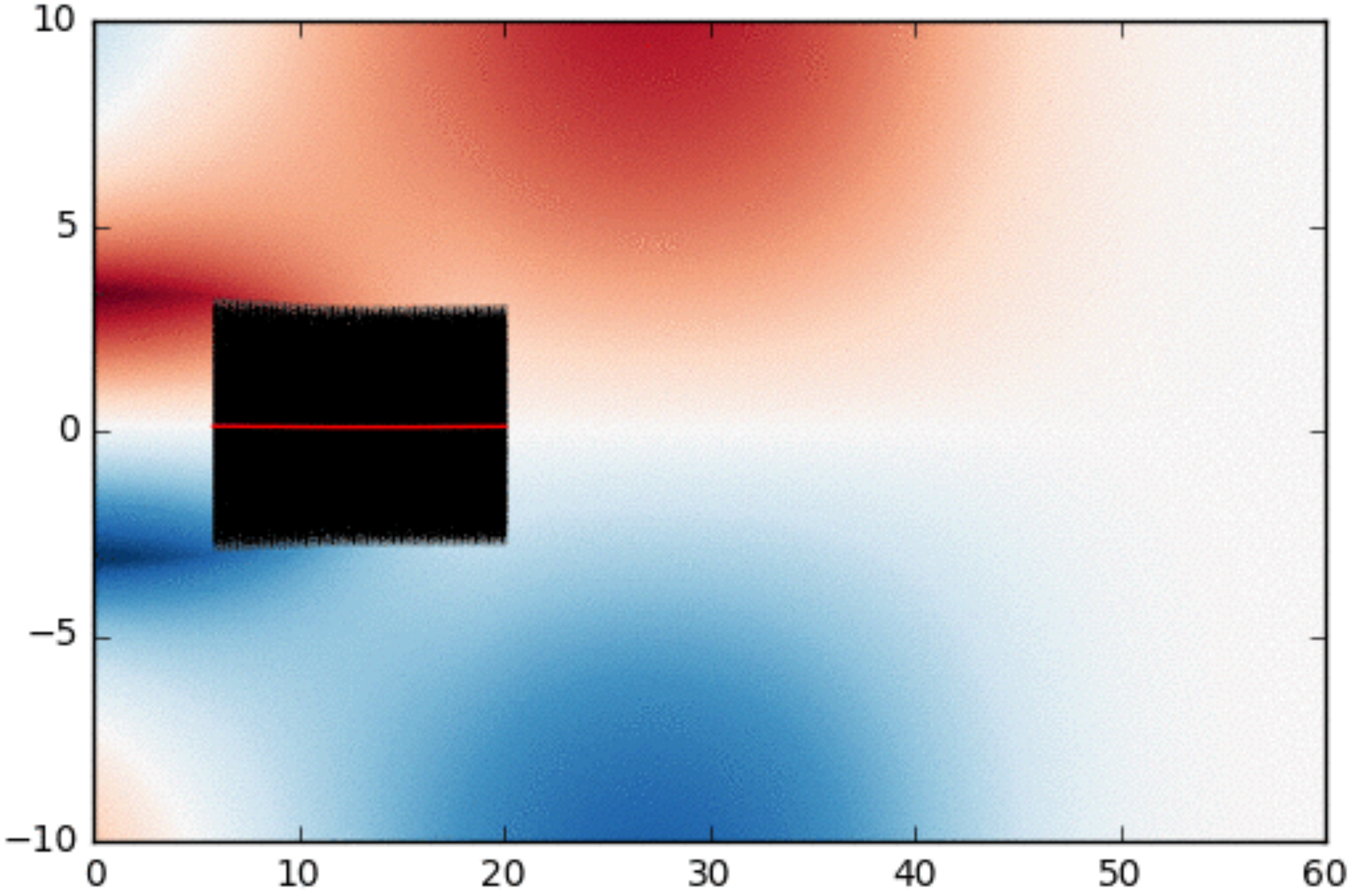

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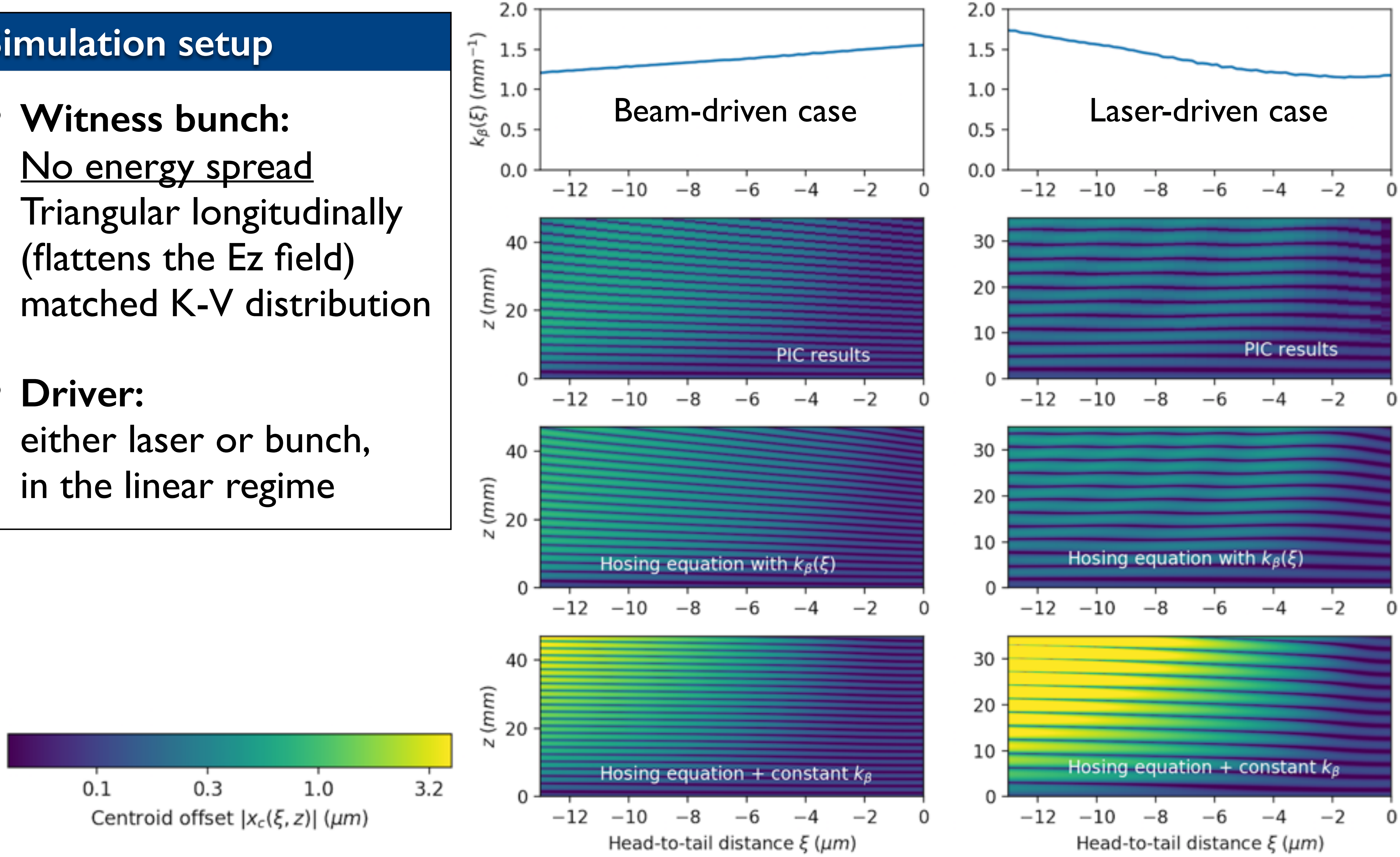

Conclusion

- New **analytical formula**, shows that the beam-hosing instability **saturates** in the presence of a **betatron frequency chirp** along the witness beam

- In the quasi-linear regime, betatron chirp **occurs naturally**, even for a monoenergetic bunch.

- Thus, the beam-hosing instability in the quasi-linear regime is **much less severe** than predicted by standard hosing scalings.

Thank you for your attention

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