Measuring very low emittances using betatron radiation

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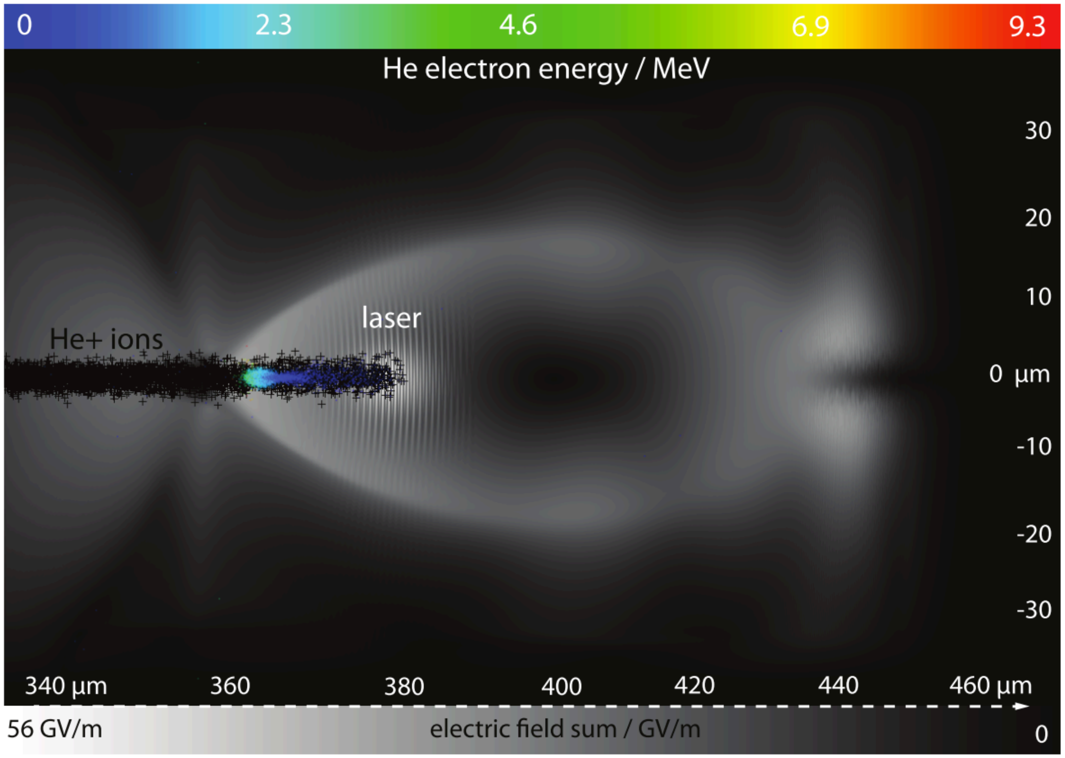
FACET-II Science Workshop

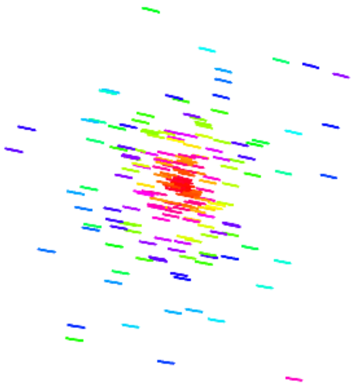
Plasma photocathode injection

- "Trojan horse"
  - High and low ionization threshold gases
  - Blowout bubble formation followed by laser pulse to spawn ultracold electrons in bubble
- $\varepsilon_n \sim 100$ nm-rad
- Wakeless plasma at end
  - Constant energy, same radial fields



[Hidding2012]

Betatron oscillations



Unmatched condition. Not to scale.

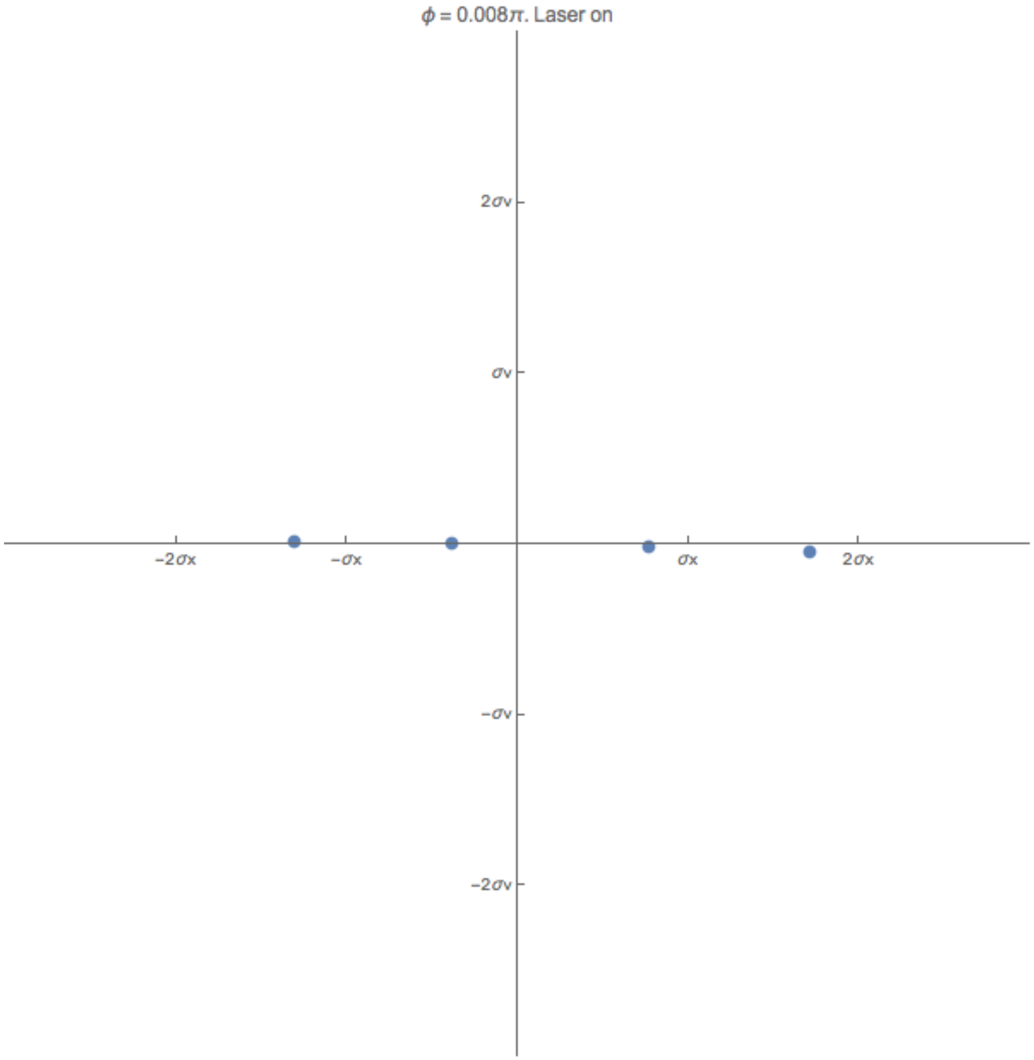
Blowout regime

Linear (radial) field $\vec{E} = \frac{-n\,e\,\vec{r}}{2\,\epsilon_0}$

Harmonic beam oscillations $k_\beta = \frac{k_p}{\sqrt{2\,\gamma}}$

Injection phase space

- Laser releases electrons from cold plasma corresponding to focus profile and approximately zero velocity
- Over the duration of the pulse (~30fs) these electrons begin betatron oscillation while new electrons are still being introduced
- “Paint” transverse phase space with matched beam



Typical parameters

- Plasma: 10^{17}-10^{18} cm^{-3}
  - $\lambda_p = 30\mu\text{m}$
- 1-20 GeV
  - $\lambda_\beta = 2\text{mm}$
- 100 nm-rad
- Fundamental radiation at ~3Å (4keV)

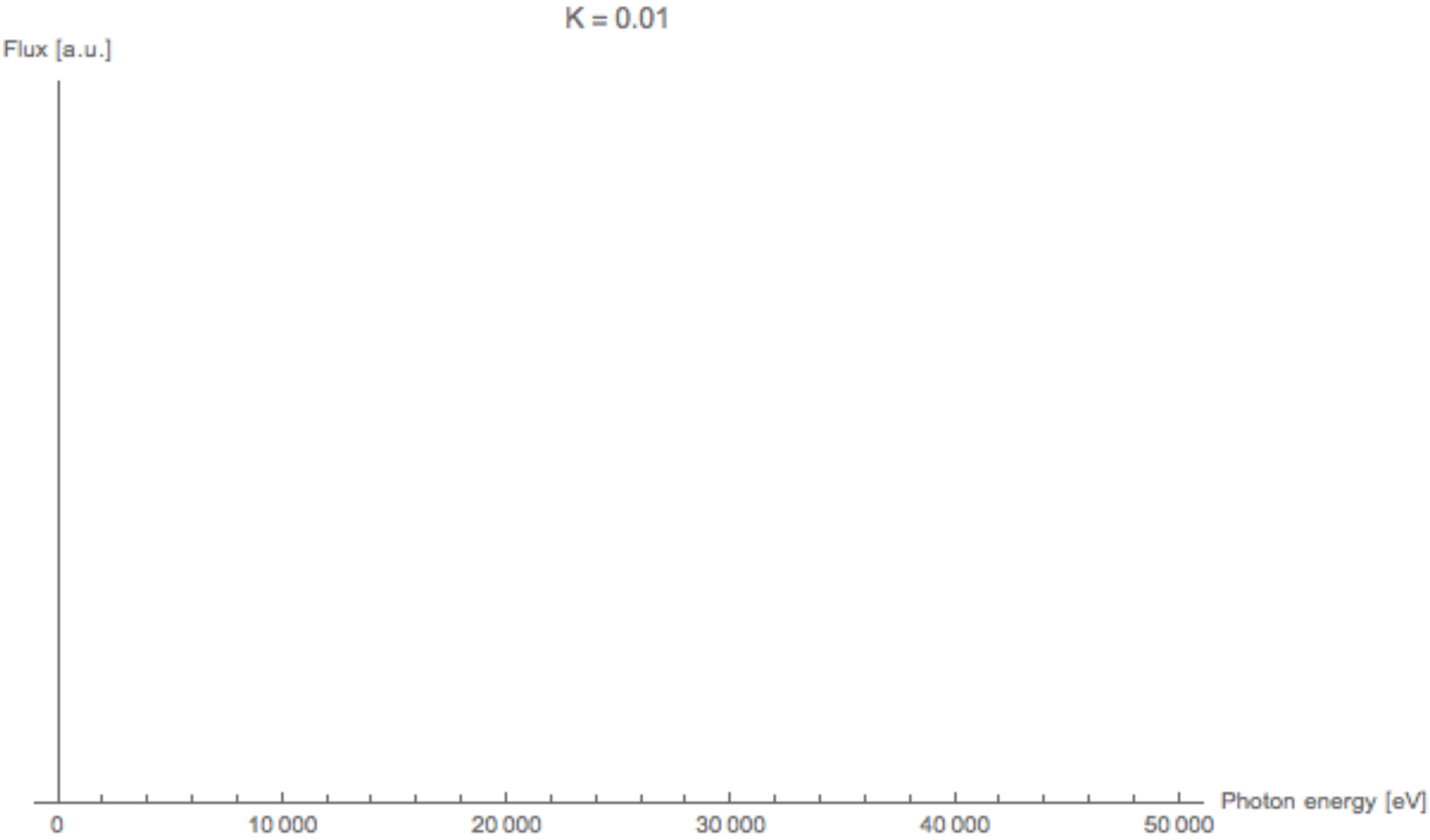
Betatron spectrum

- Plasma wiggler characterized by *K*, the undulator parameter:

$$K = \frac{2\pi\gamma x_0}{\lambda_\beta} \simeq 1.33 \text{ x } 10^{-10}\, \gamma^{0.5}\, n_e{}^{0.5}[\mathrm{cm}^{-3}]\, x_0[\mu\mathrm{m}]$$

- For *K* << 1 (“undulator regime”) this term dominates spectrum. As *K* increases, contributions of harmonics also increase
  - Approaches synchrotron spectrum
- Angular and *K* dependence of fundamental radiation wavelength: $\lambda_r(\theta) = \frac{\lambda_\beta}{2\gamma^2}\left(1 + \frac{1}{2}K^2 + (\gamma\theta)^2\right)$

Spectrum by *K*

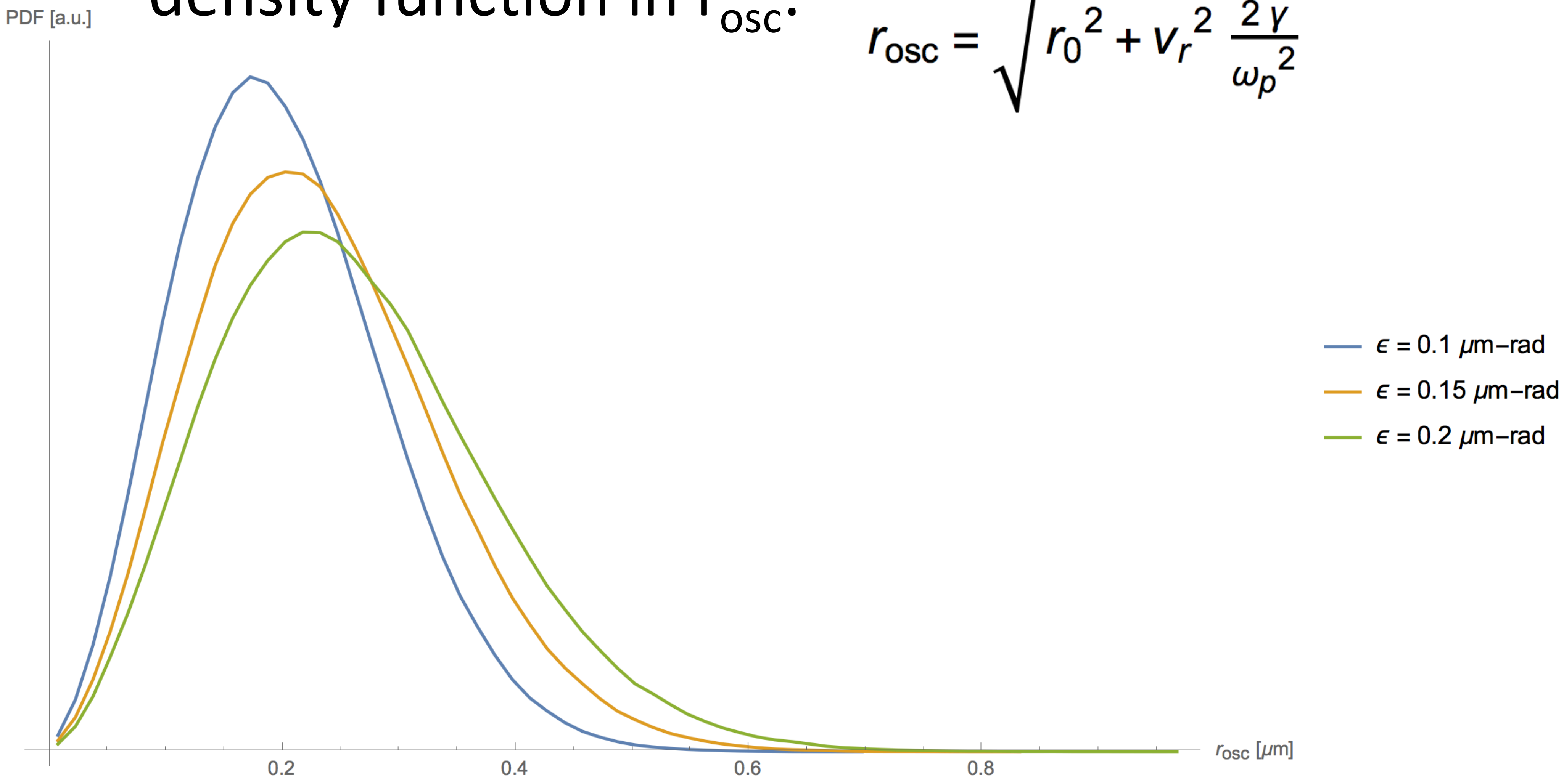


Simulating idealized undulator in SPECTRA (near axis)

Betatron amplitude PDF

- Map 4D transverse phase space to probability density function in r_{osc}:

$$r_{osc} = \sqrt{{r_0}^2 + {v_r}^2 \frac{2\gamma}{{\omega_p}^2}}$$

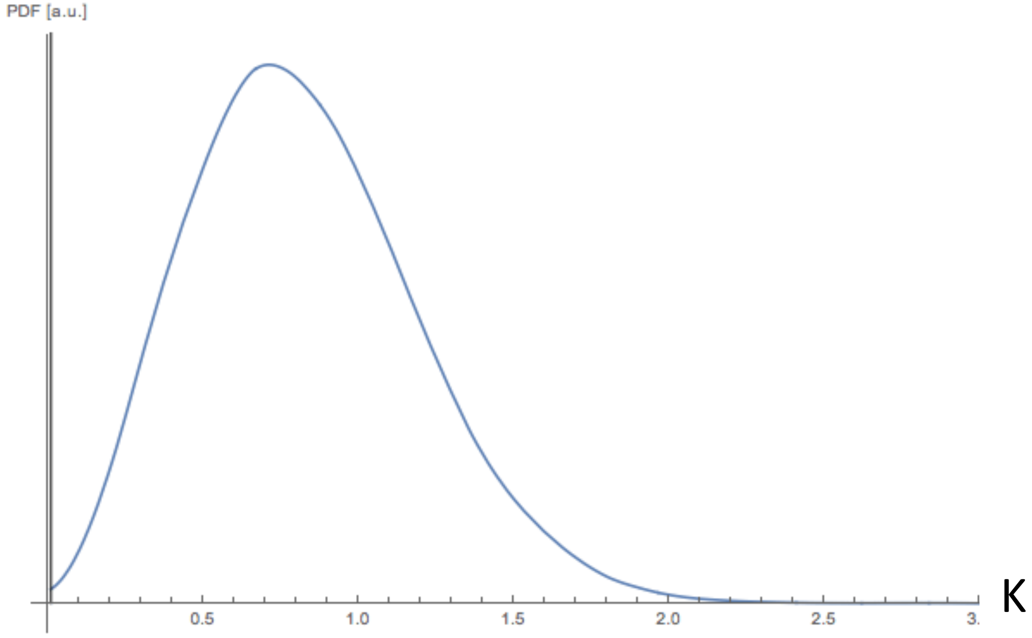


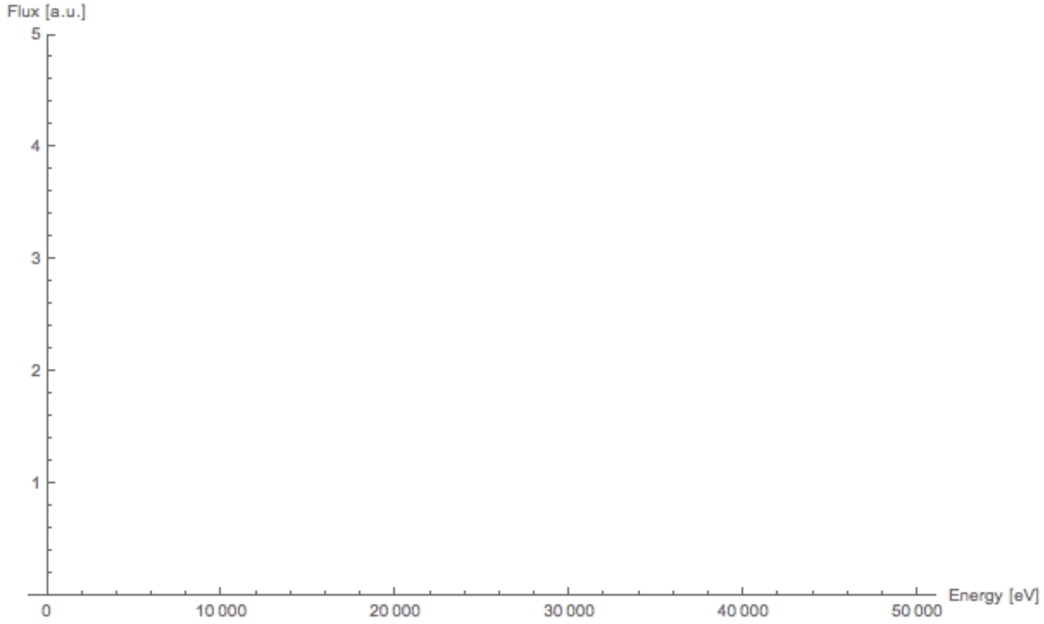
Signal superposition

- Take the r_{osc} PDF, transform into *K* PDF

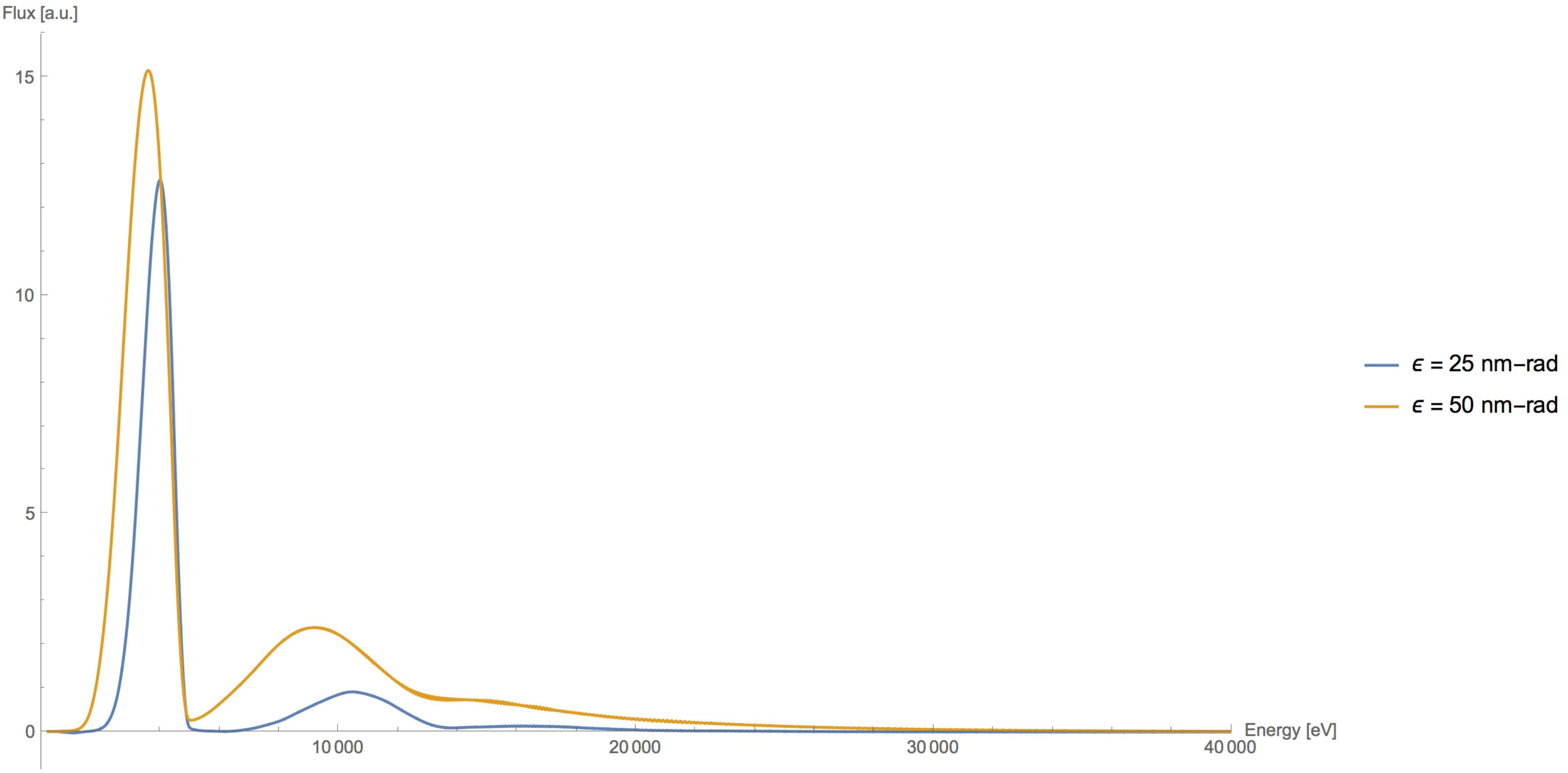
$$K = \frac{2\pi\gamma x_0}{\lambda_\beta} \simeq 1.33 \text{ x } 10^{-10}\,\gamma^{0.5}\, n_e^{0.5}[\mathrm{cm}^{-3}]\, x_0[\mu\mathrm{m}]$$

- Weight each betatron spectrum at *K* according to this PDF and sum





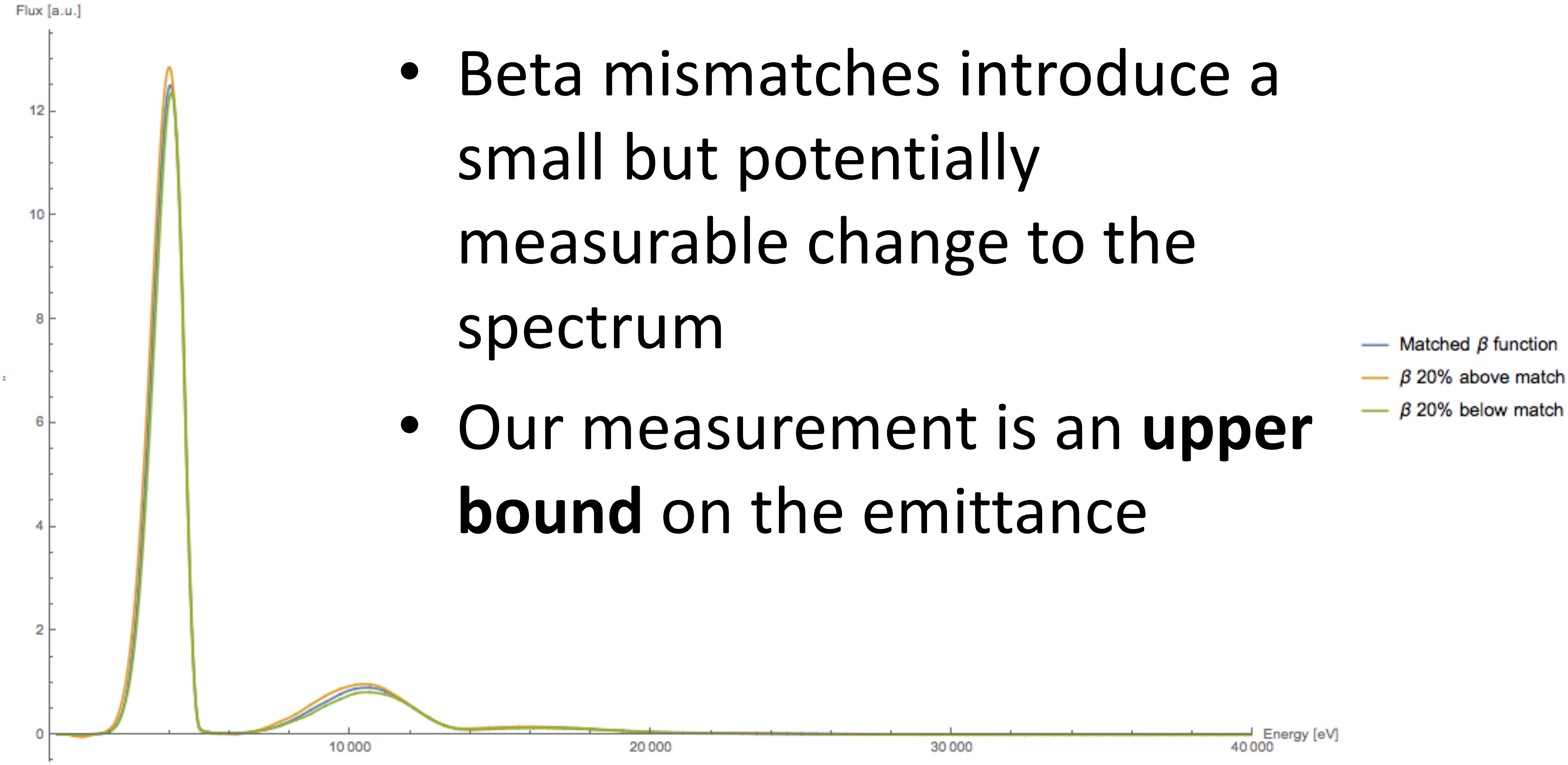
Emittance measurement



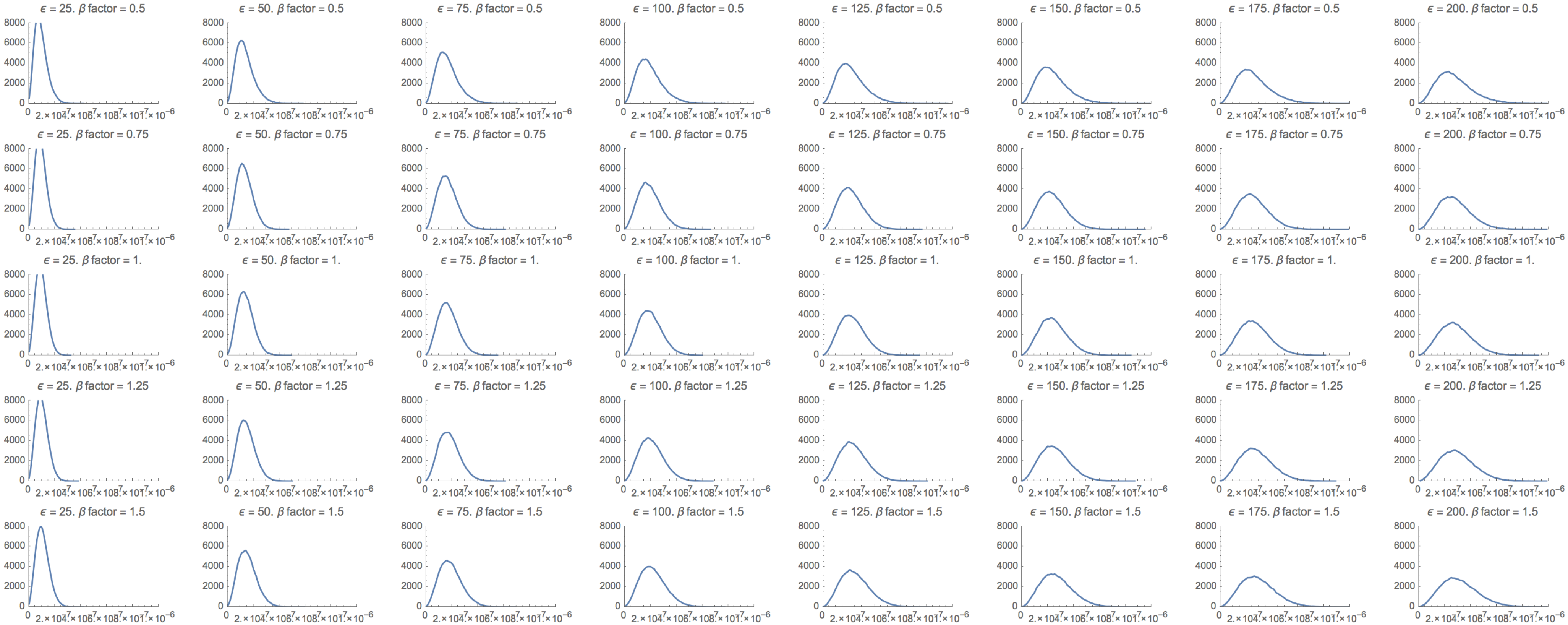
Increasing emittance decreases fundamental frequency and broadens the light from harmonics

Caveat: Beta matching

- Beta mismatches introduce a small but potentially measurable change to the spectrum
- Our measurement is an **upper bound** on the emittance



Beyond emittance

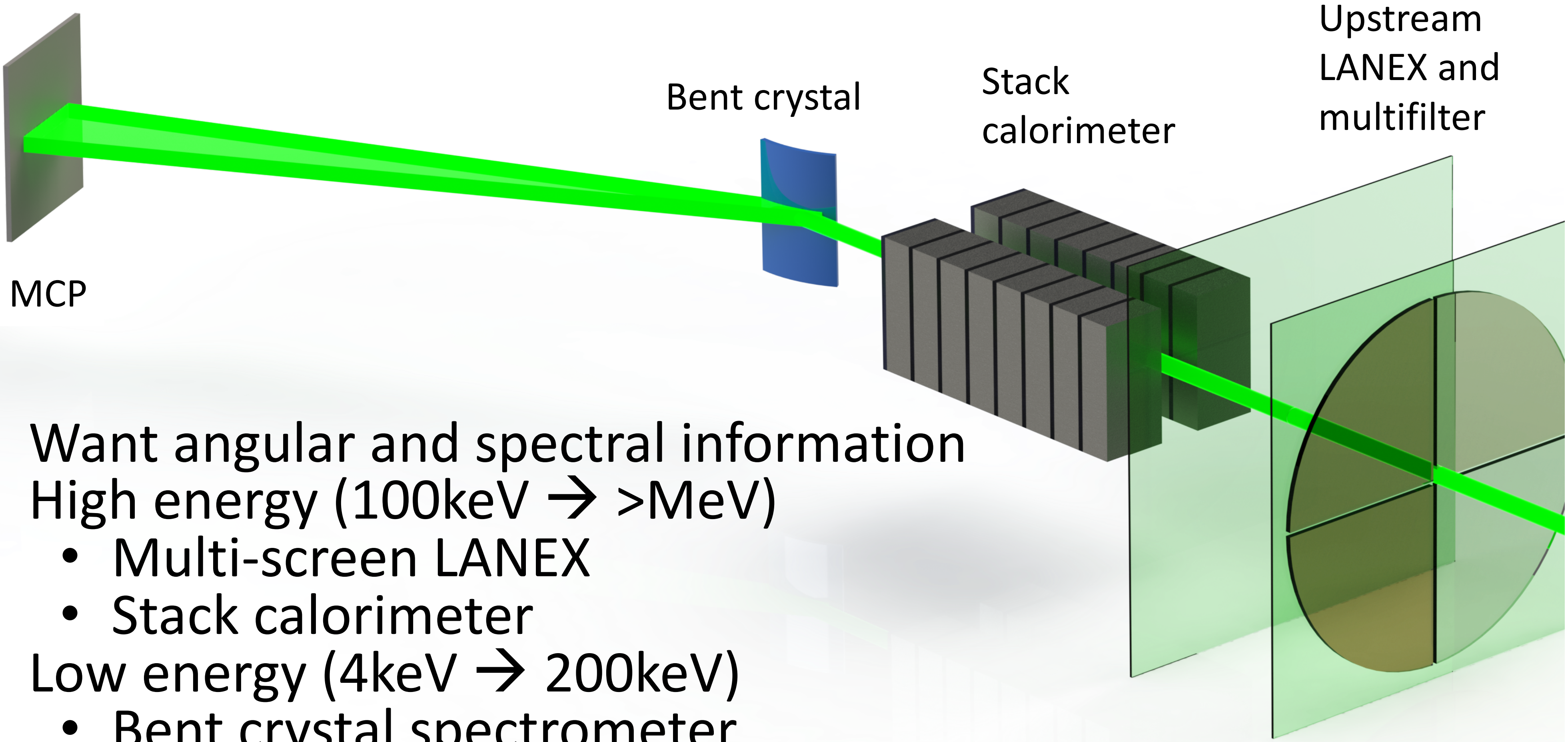


- Ideally, each r_{osc} value can be thought of as identifying a unique basis vector of the spectrum

$$r_{osc} = \sqrt{\left(x_0^2 + y_0^2\right) + \frac{2\gamma}{\omega_p^2}\frac{\left(v_x x_0 + v_y y_0\right)^2}{\left(x_0^2 + y_0^2\right)}}$$

- This simplification would allow a complete reconstruction of the densities of each r_{osc} band

Diagnostic schematic



MCP

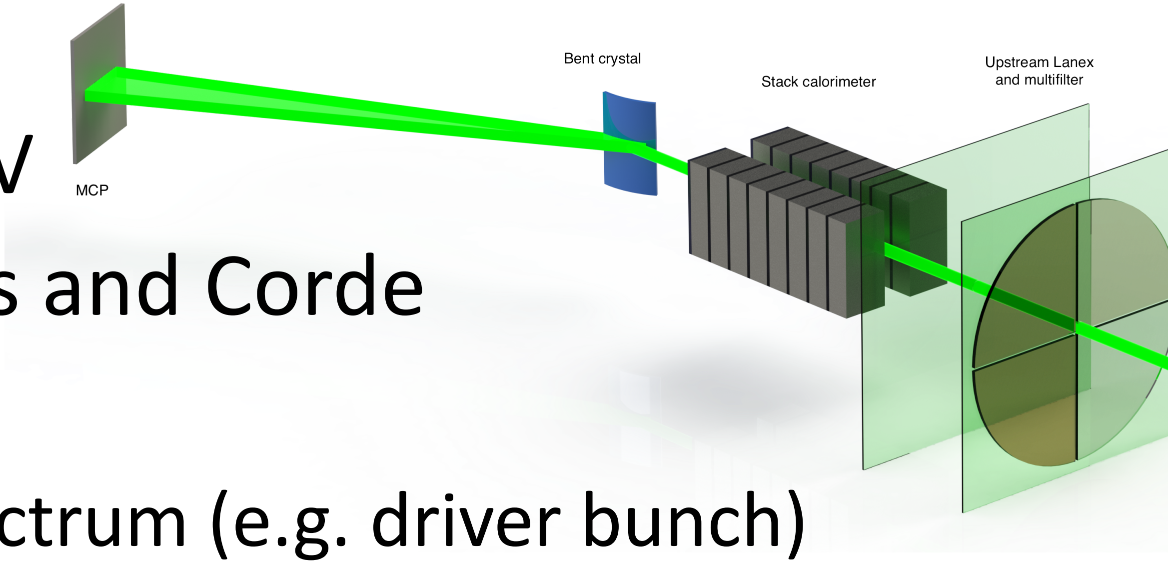
Bent crystal

Stack calorimeter

Upstream LANEX and multifilter

- Want angular and spectral information
- High energy (100keV → >MeV)
  - Multi-screen LANEX
  - Stack calorimeter
- Low energy (4keV → 200keV)
  - Bent crystal spectrometer

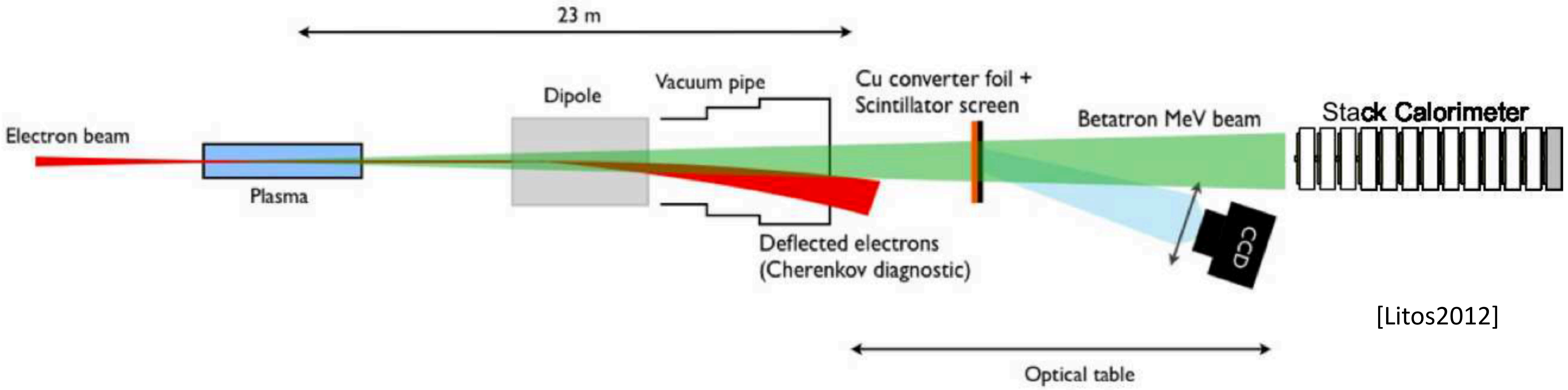
High energy diagnostics



- Detection from 100 keV to >1 MeV
- Headed by Dr. Litos and Corde
- Stack calorimeter
  - Identify high *K* spectrum (e.g. driver bunch) cutoff energy $\epsilon_c[keV] = 5\cdot 10^{-21}\gamma^2 n[\mathrm{cm}^{-3}]x_0[\mu\mathrm{m}]$
- Multi-screen LANEX system
  - Angular information
  - Limited spectral information

Stack calorimeter

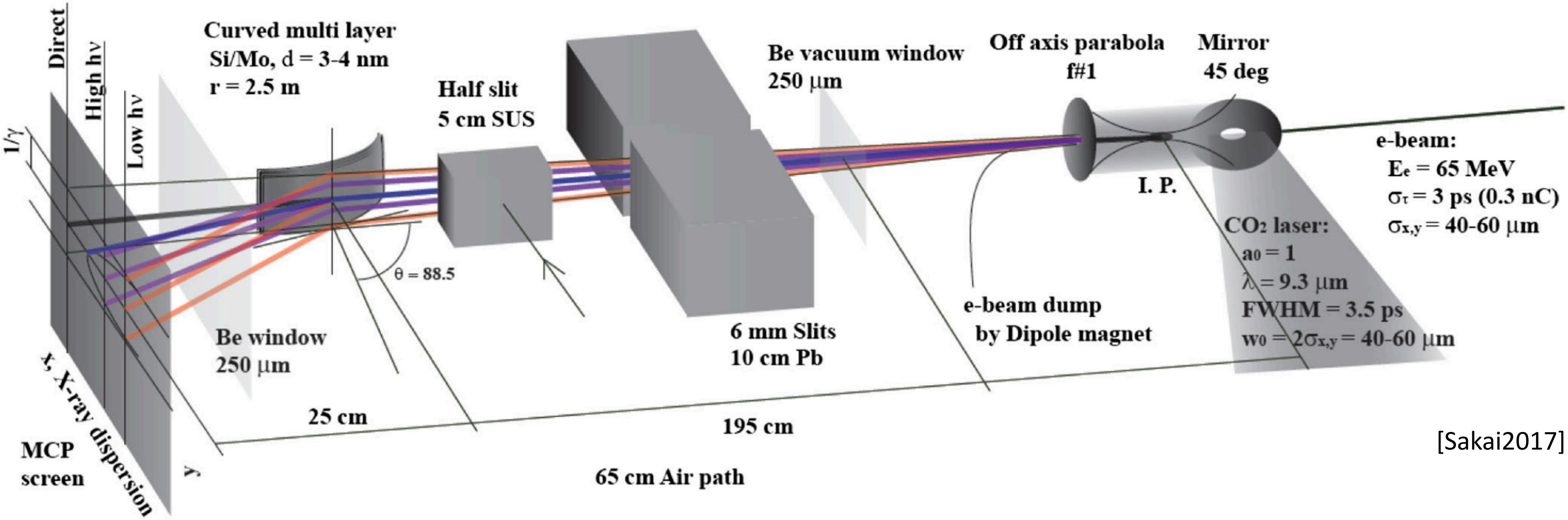
- Precursor from E200. Screen and stack calorimeter to characterize betatron radiation:



[Litos2012]

- Upgrades in new version:
  - Actuated gap for downstream collimation
  - Angular information from 2D photodiode array (resolution limited by EM shower)

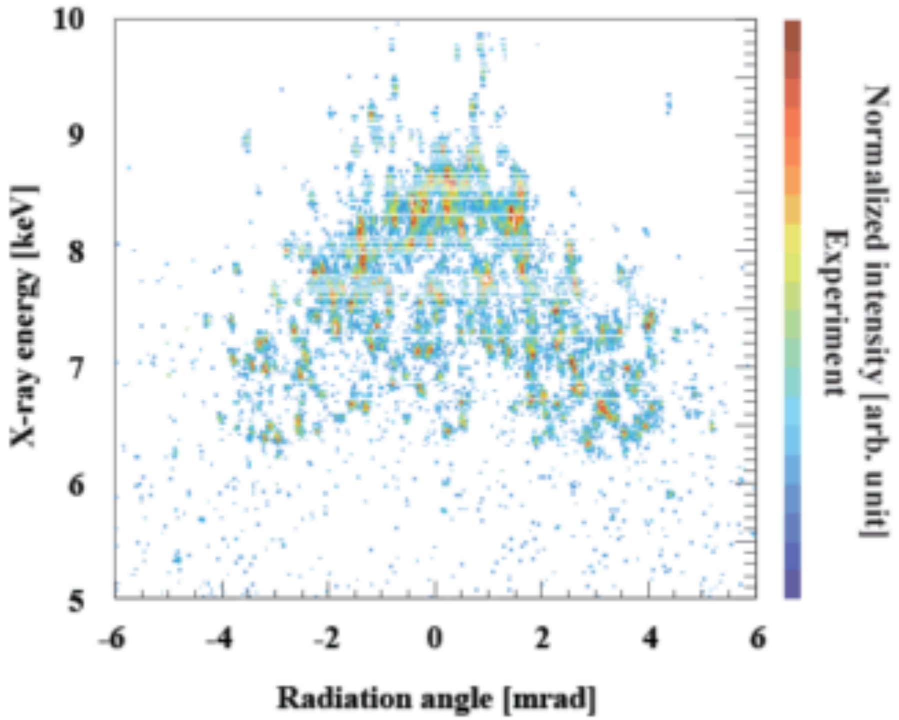
Low energy diagnostics

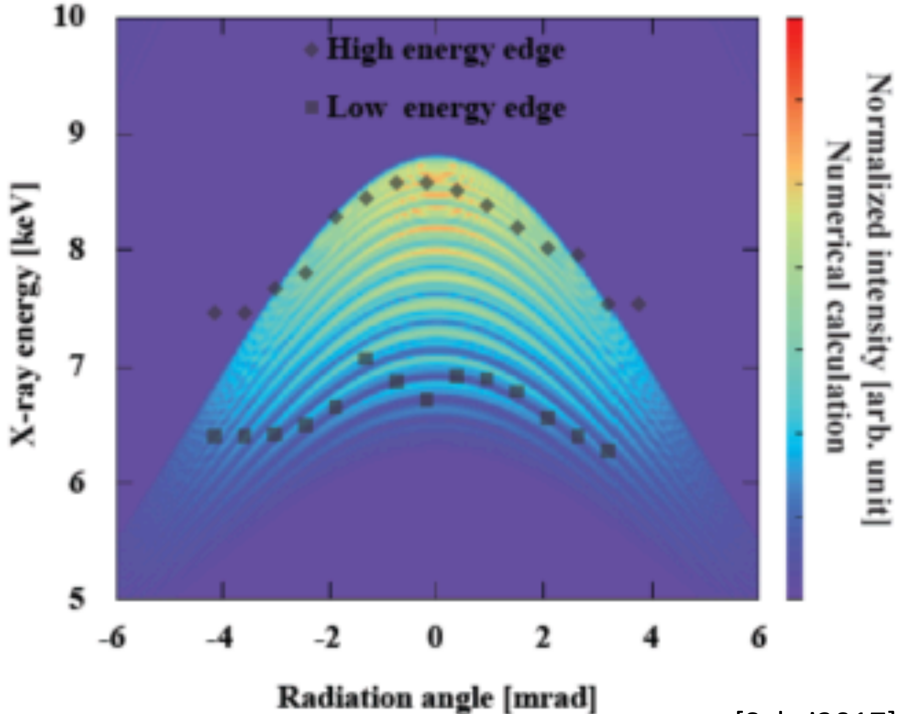


[Sakai2017]

- Double differential bent crystal spectrometer
- Crystal disperses in bend plane but retains angular information in vertical plane

Double differential spectrum





[Sakai2017]

- Results from ICS experiment
- Recall: $\lambda_r(\theta) = \frac{\lambda_\beta}{2\gamma^2}\left(1 + \frac{1}{2}K^2 + (\gamma\theta)^2\right)$

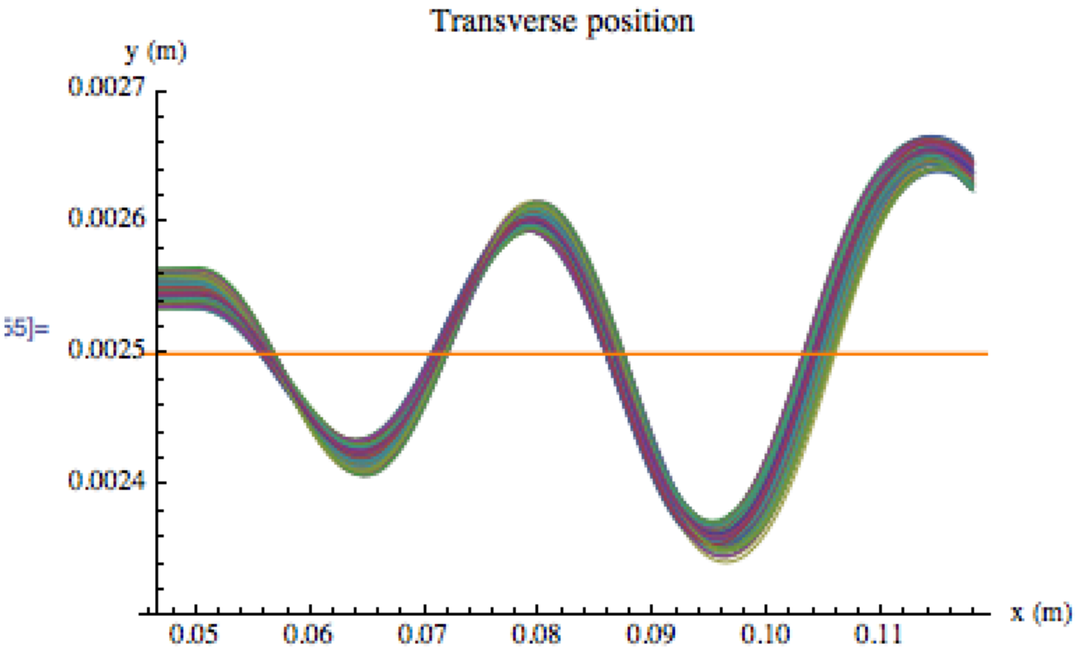
Crystal selection

- Cover 4-200 keV with 3 interchangeable crystals while keeping constant optical paths (i.e. no need to move detector)
  - 55-200 keV with 3.1Å spacing (natural crystal candidate)
  - 15-55 keV with 11.5Å spacing (natural or synthetic)
  - 4-15 keV with 42Å spacing (existing Si/Mo synthetic from BNL experiment)
- Numbers assume the specified spectrum will be spread over ~30mrad of detector

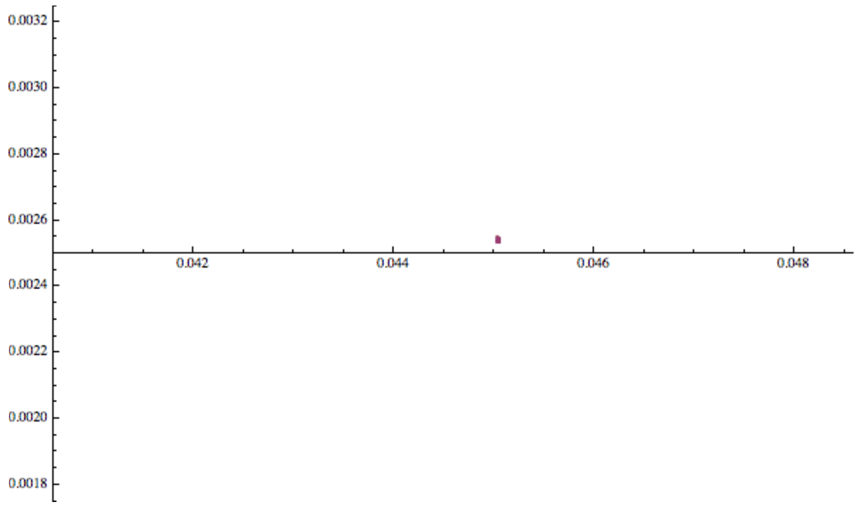
Resonant betatron excitation

- Overlay undulator field on ion channel
- Match undulator and plasma betatron wavelengths for resonant amplitude growth
  - Amplitude (beam centroid): $\frac{qB_0}{2\gamma mck_u^2}\sqrt{1 + k_u^2 z^2}.$
  - Effective field: $B_{\mathrm{eff}} = \frac{B_0}{2}\sqrt{1 + (\mathrm{numPeriods})^2}$
- Already expect to have wakeless tail to PWFA. Just need to add undulator

RBE simulation

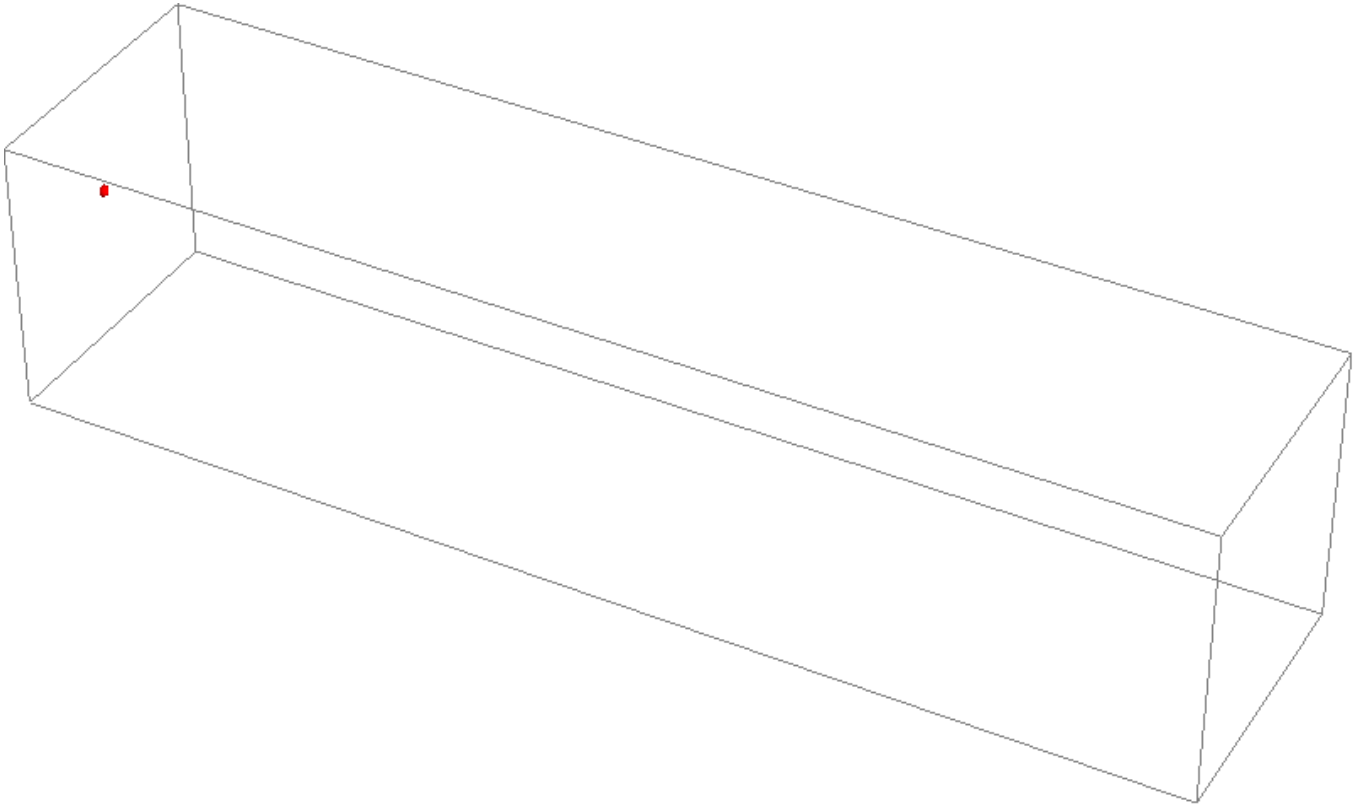


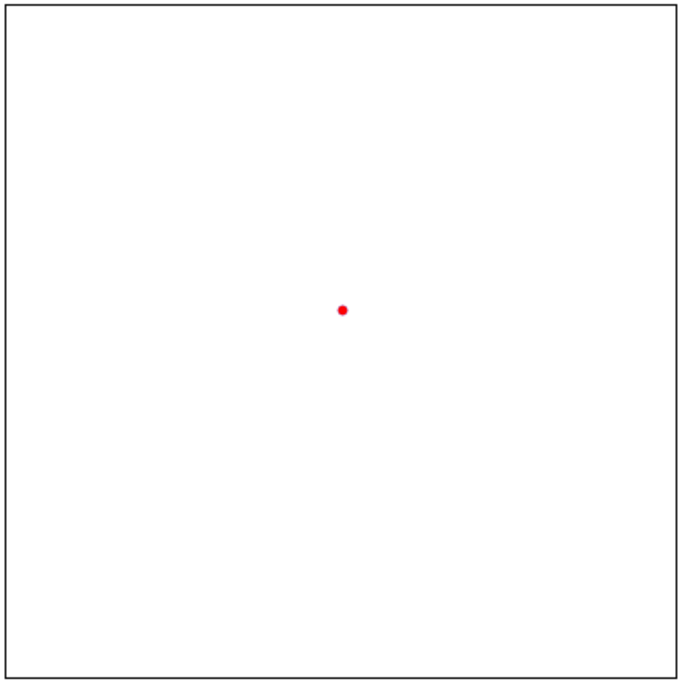
- Linear amplitude growth
- Confine with electric field, boost with magnetic field
- Amplitude limited only by bubble radius



Plasma ion species	Ar+
Plasma density	10^{15} cm^{-3}
Undulator magnetic field amplitude	1 T
Undulator period	3 cm
Drive beam energy	20 GeV
Drive beam charge	3.2 nC
Drive beam dimensions σ_x,σ_z	30 μm, 30 μm
Witness beam energy (negligible charge)	250 MeV

Linear RBE… stepping stone to helical





- Polarized gammas from helical RBE configuration generate polarized positrons
- E166 - “The value of K in the present experiment was small, about 0.17, because of practical limitations to the current in the (pulsed) undulator.” [Alexander2009]

Summary

- System proposed to measure low emittance beams (<100 nm-rad)
- Non-destructive
- Single shot
- Comprised of high energy and low energy diagnostics
  - Calorimeter and multi-filter
  - Bent crystal spectrometer
  - Covering 4 keV through >MeV
- Based on existing systems (E200 and BNL ATF)
- Easily extend system to diagnose resonant betatron excitation