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Energy-double simulations of LCLS-II beams and the study of ion motion induced energy spread growth

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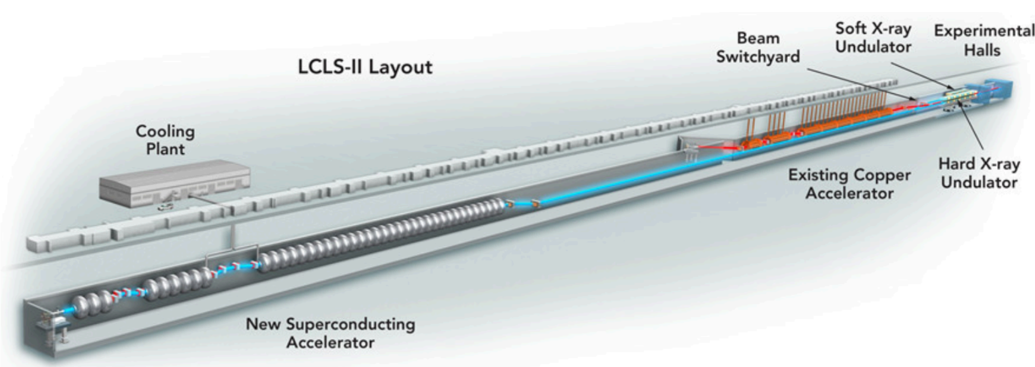
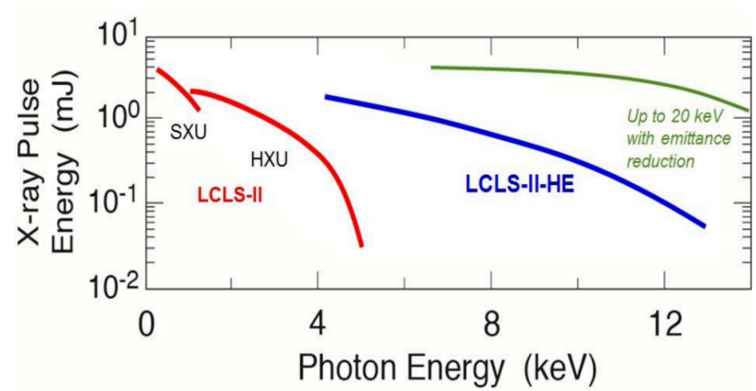
Motivation

- LCLS-II
  - High repetition frequency (super conducting, ~MHz)
  - Energy: 8 GeV, Current: 2kA
  - plasma acceleration: 8 GeV→ 16 GeV to reach >20 keV radiation

$$\lambda_r = \frac{\lambda_u}{2\gamma^2}(1+K^2)$$





https://portal.slac.stanford.edu/sites/lcls_public/lcls_ii/

Main Challenges

- X-FEL have stringent requirements to the beam quality: ε_N, I and σ_E/E

➢ I: can be conserved

➢ ε_N: can be conserved using plasma to matching in and out; ion motion induced growth[1]

➢ σ_E/E: (1) slice energy spread in an infinitely thin slice: transverse non-uniformity of the E_z field − may be caused by residual electrons inside the wake or the ion motion

(2) slice energy spread in the cooperation length@~60nm: the local chirp of the E_z and the slice energy spread in an infinitely thin slice, <**$\rho \sim 3\times10^{-4}$** **(**16 GeV beam, λ_r=0.062 nm**)**

(3) projected energy spread: the profile of the E_z, can increase the bandwidth of the radiation

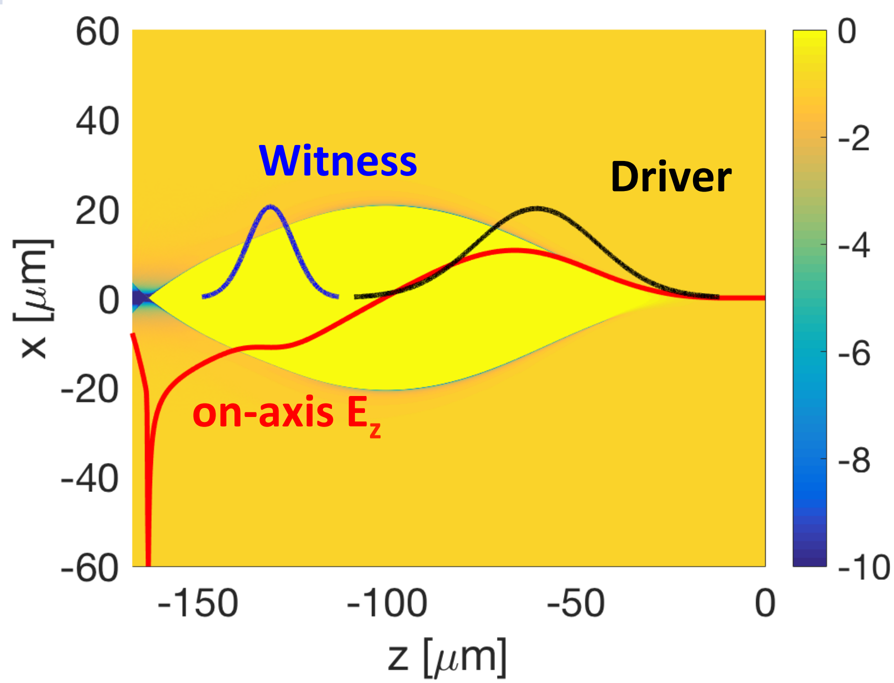
[1]Weiming An, et al., PRL 118, 244801 (2017)

Main Parameters

	I [kA]	σ_z [μm]	ε_n [μm]	σ_r [μm]	Q [pC]	E_b [GeV]	σ_{Eb} [keV]
Driver	2	16	1.2	0.52	269	8	80
	I [kA]	σ_z [μm]	ε_n [μm]	σ_r [μm]	Q [pC]	E_b [GeV]	σ_{Eb} [keV]
Witness	2	6	0.4	0.3	103	8	80
	n_p [cm^{-3}]	k_p^{-1} [μm]					
Plasma	7e16	20					

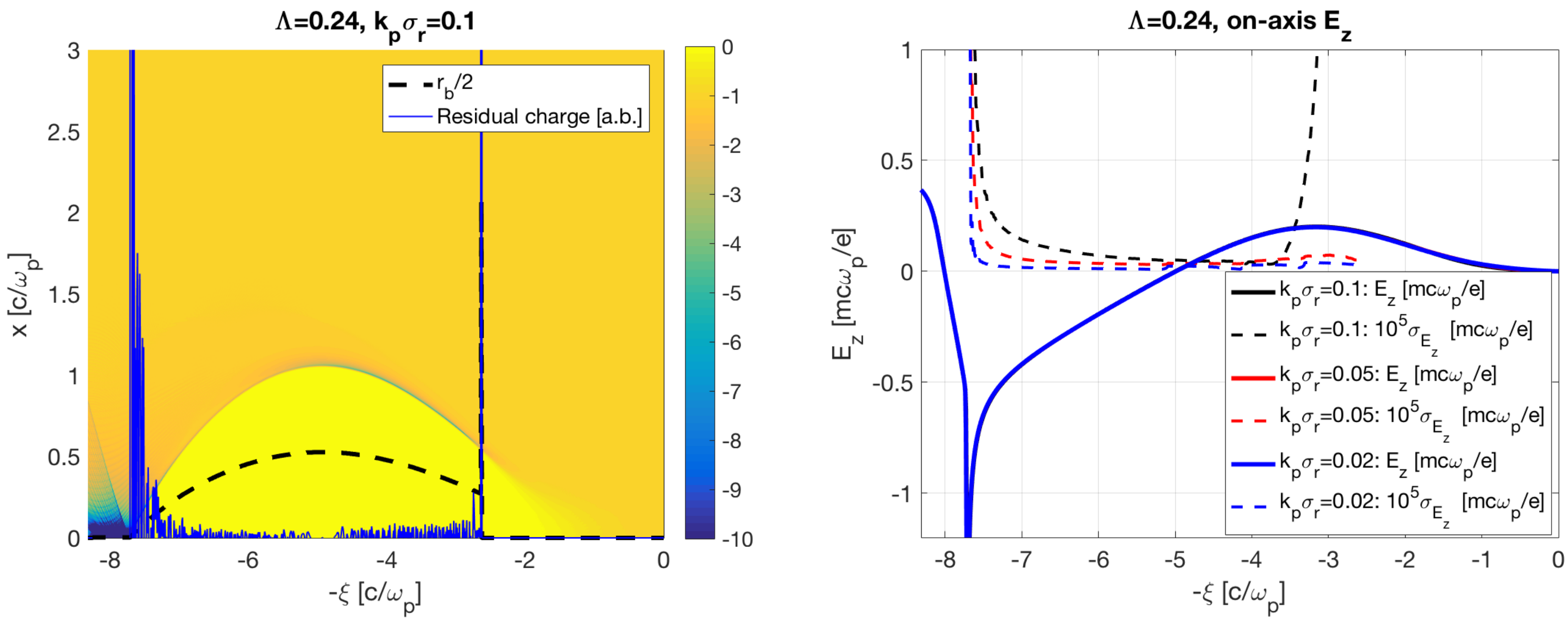
QuickPIC simulation setup:

- Box: 167 μm × 167 μm × 167 μm
- Grid Size: 40 nm × 40 nm × 163 nm
- Grid numbers: $2^{12} \times 2^{12} \times 2^{10}$



The slice energy spread in an infinitely thin slice - transverse non-uniformity of the E_z

Because there are some residual charge inside the wake, the E_z field is not perfectly uniform in the transverse direction.



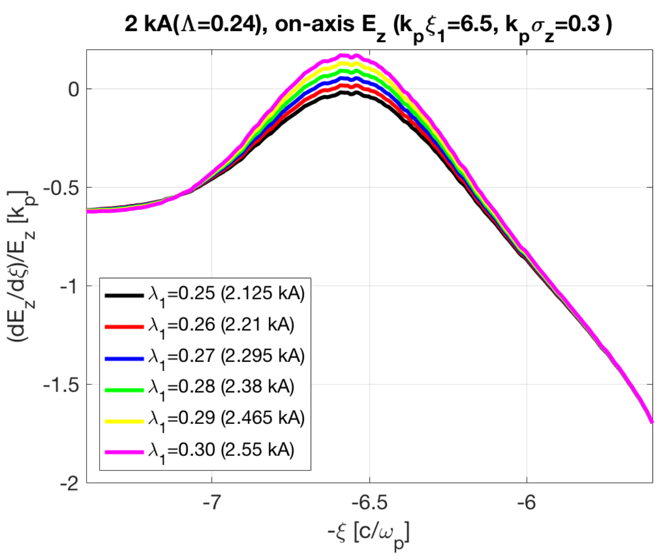
We can see the relative non-uniformity of E_z is less than **10^{-5}** in most of the wake, thus the growth of the energy spread in an infinitely thin slice due to this effect can be neglected.

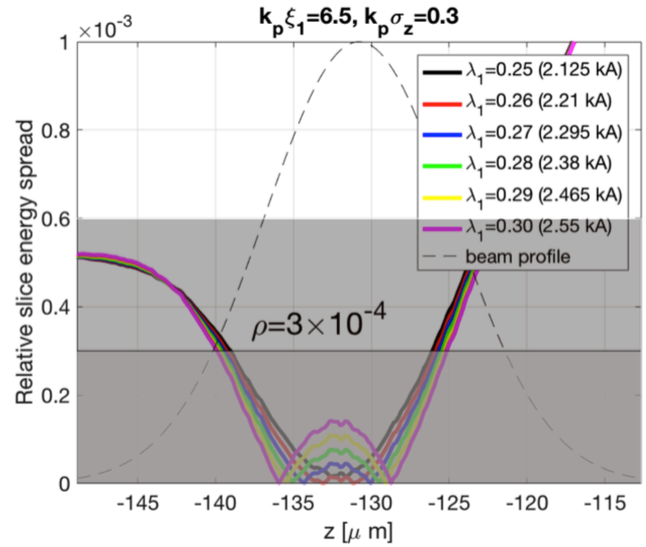
The slice energy spread in a cooperation length – beam loading

- The cooperation length $L_C \approx \dfrac{\lambda_r}{2\sqrt{\pi}\rho}$

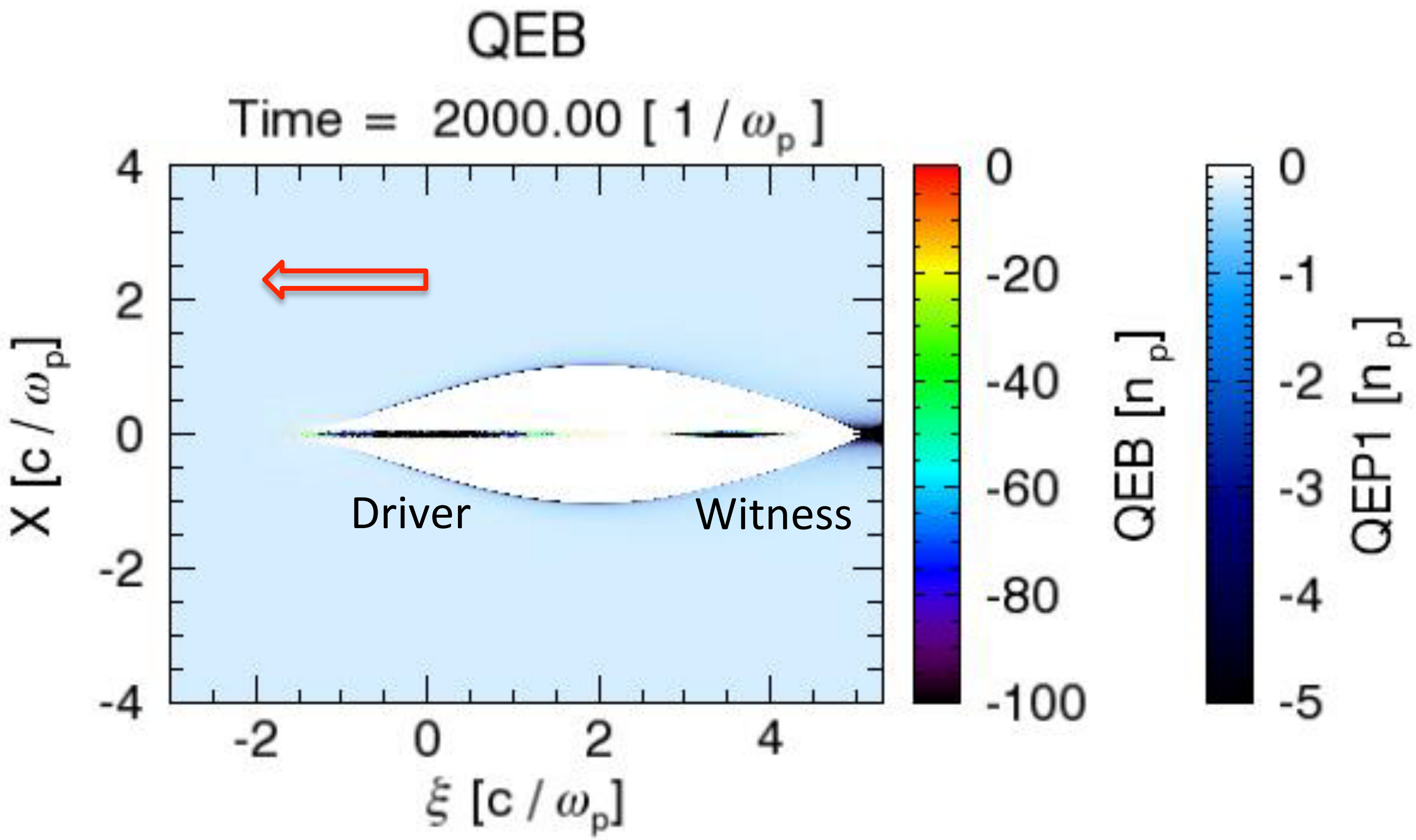
➢ 16 GeV case, λ_r=0.062 nm, $\rho\sim3\times10^{-4}$ → $L_C\approx$60 nm ($k_pL_C\sim3\times10^{-3}$).

- If we assume the E_z field is smooth on the scale of L_C, the relative variation of E_z over L_C can be estimated as
$$\frac{\Delta E_z}{E_z} \approx \left|\frac{\mathrm{d}E_z/\mathrm{d}\xi}{E_z}\right| L_c$$



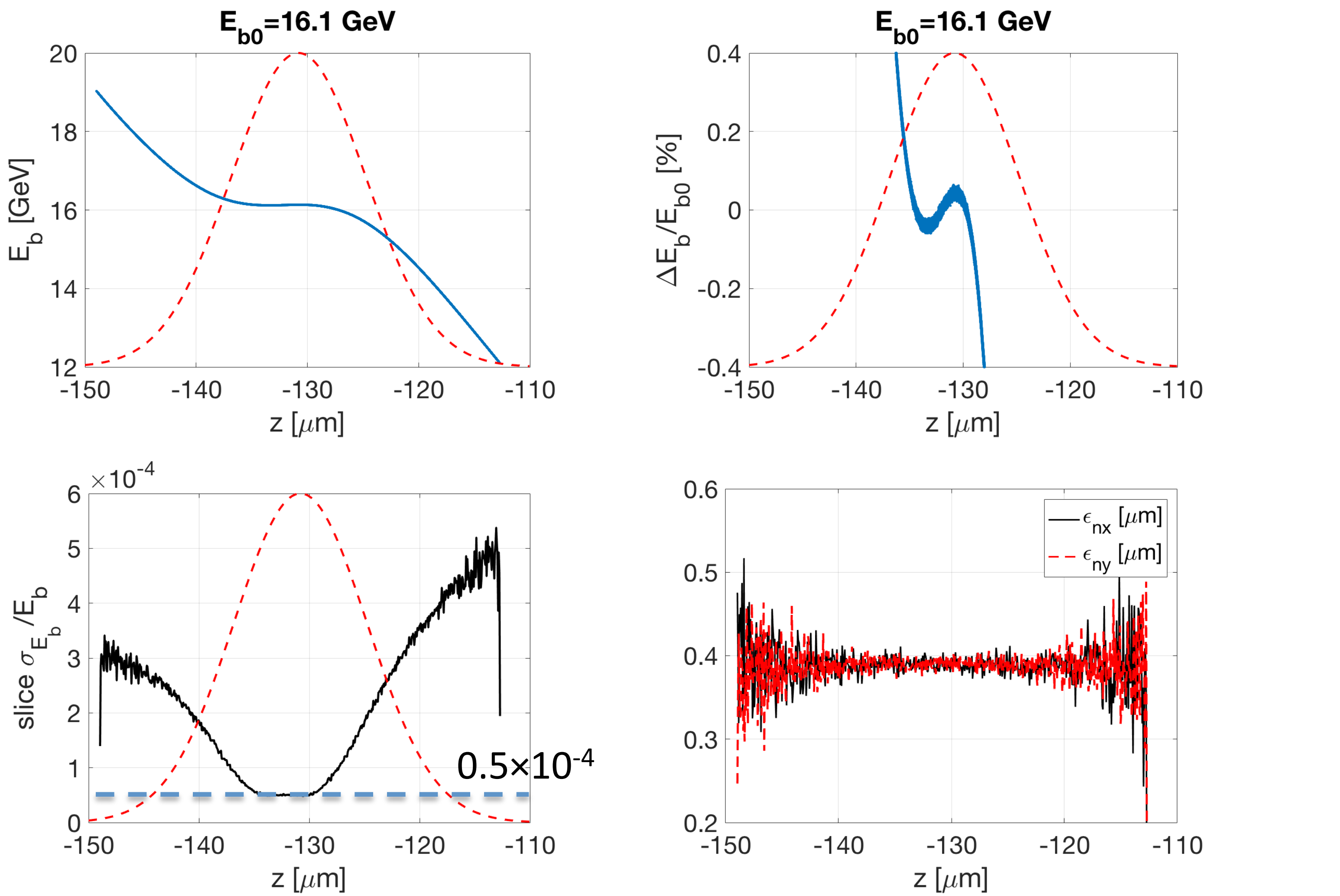


QuickPIC simulation Results



The values of ‘Time’ are not correct after Time=6000. Please ignore these values.

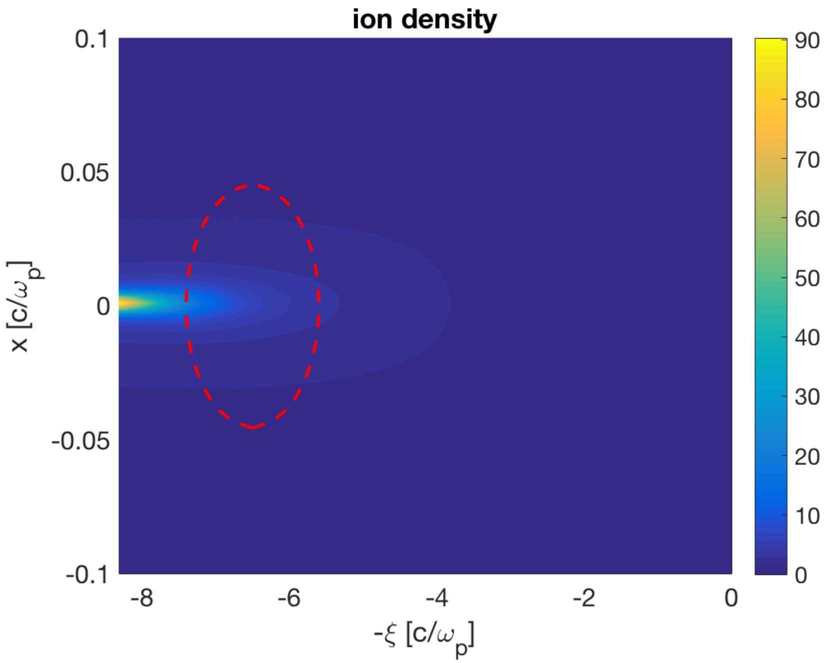
QuickPIC Simulation results at z=1.45 m



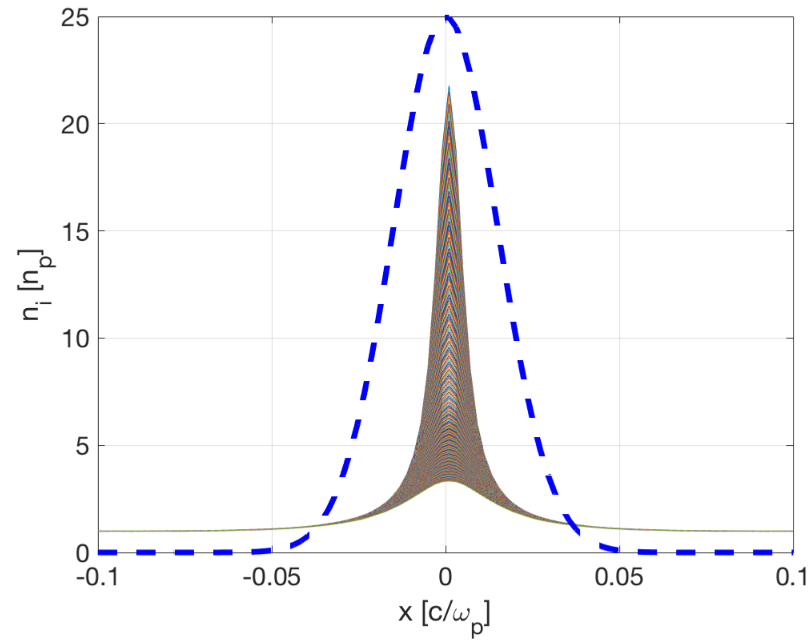
Ion motion induced by the beams

QuickPIC one-step simulation

- $n_d \approx 350\ n_p$ (8GeV, ε_N=1.2μm, σ_r=0.5μm)
- $n_w \approx 1000\ n_p$ (8GeV, ε_N=0.4μm, σ_r=0.3μm)



Red line: 3-sigma boundary of the witness beam



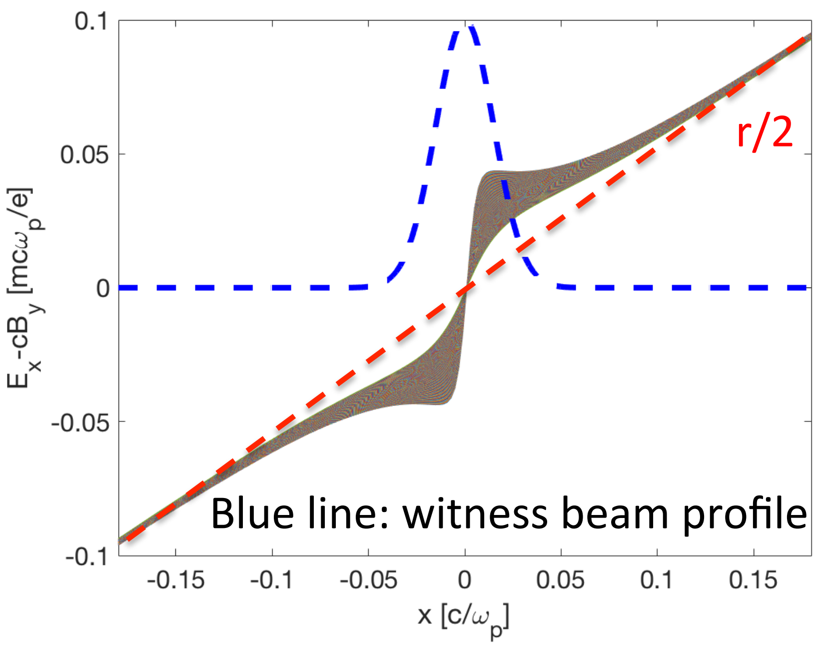
Blue line: witness beam profile

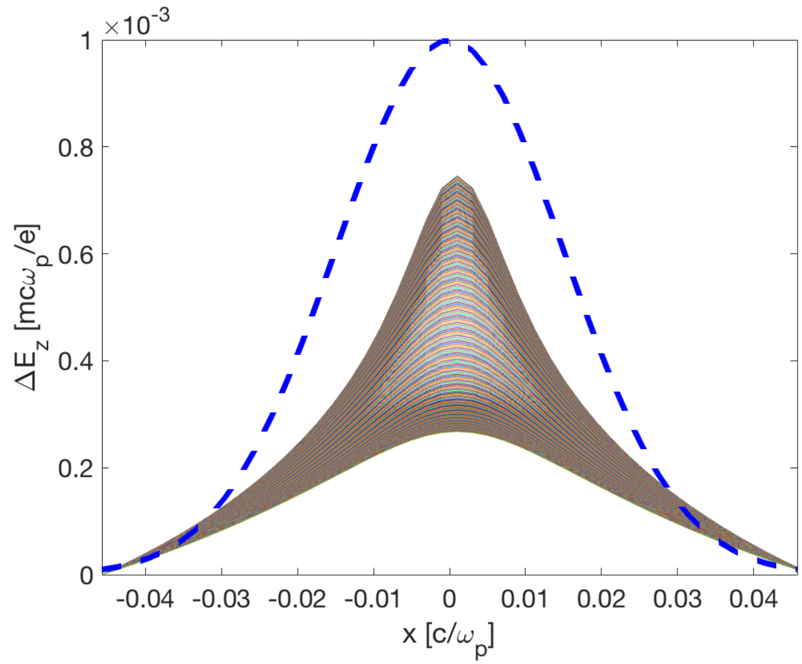
Ion motion induced by the beams

Panofsky-Wenzel theorem

$$E_r - cB_\theta = \frac{1}{2\pi r}\int_0^r r' dr' n_i(r')$$

$$\frac{\partial E_z}{\partial r} = \frac{\partial}{\partial \xi}(E_r - cB_\theta)$$





Weiming An, et al., PRL 118, 244801 (2017)

Quantify the slice energy spread growth

(1) Assume the transverse distribution of the beam is fixed, thus the distribution of E_z is fixed.

(2) Axial-symmetry geometry.

Energy Gain: $\gamma = \int_{z_i}^{z_f} dz' E_z\left(x(z')\right) = \int_{z_i}^{z_f} dz' E_z\left((\gamma/2)^{-1/4}\epsilon_N^{1/2}\cos\frac{z'}{\sqrt{2\gamma}}\right)$

Gaussian Distribution: $E_z(r) = E_{z0} + \Delta E_z \exp\left(-\frac{r^2}{2\sigma_o^2}\right)$

$$
\begin{aligned}
\Delta\gamma &\approx \int_z^{z+2\pi\sqrt{2\gamma}} dz' \left(E_{z0} + \Delta E_z \exp\left[-\frac{\epsilon_N \cos^2(z'/\sqrt{2\gamma})}{\sqrt{2\gamma}\sigma_o^2}\right]\right) \\
&\approx 2\pi\sqrt{2\gamma}\left[E_{z0} + \Delta E_z \exp\left(-\frac{\epsilon_N}{2\sqrt{2\gamma}\sigma_o^2}\right) I\left(0, \frac{\epsilon_N}{2\sqrt{2\gamma}\sigma_o^2}\right)\right] \\
&= 2\pi\sqrt{2\gamma}\left[E_{z0} + \Delta E_z \exp\left(-\frac{\hat{x}^2}{4\sigma_o^2}\right) I\left(0, \frac{\hat{x}^2}{4\sigma_o^2}\right)\right]
\end{aligned}
$$

Quantify the slice energy spread growth

Gaussian beam distribution: $f(x, p_x) = \frac{1}{2\pi\sigma_x\sigma_{p_x}} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{p_x^2}{2\sigma_{p_x}^2}\right)$

then: $f(\hat{x}) = \frac{\hat{x}}{\sigma_x^2} \exp\left(-\frac{\hat{x}^2}{2\sigma_x^2}\right)$

We can get:

$$\Delta\bar{\gamma} = \int_0^{+\infty} \Delta\gamma(\hat{x}) f(\hat{x}) d\hat{x}$$

$$\approx 2\pi\sqrt{2\gamma}\left[E_{z0} + \frac{\Delta E_z}{\sqrt{1+\kappa^2}}\right]$$

where $\kappa = \sigma_x/\sigma_o$

$$\sigma_{\Delta\gamma} = \sqrt{\int_0^{+\infty} \left[\Delta\gamma(\hat{x}) - \Delta\bar{\gamma}\right]^2 f(\hat{x}) d\hat{x}}$$

$$= 2\pi\sqrt{2\gamma}\Delta E_z \sqrt{\frac{(2/\pi)E[(1+\kappa^{-2})^{-2}] - 1}{(1+\kappa^2)}}$$

Quantify the slice energy spread growth

If $E_{z0}/\Delta E_z \gg 1$ which is valid in most cases, we can get

$$\frac{\sigma_{\Delta\gamma}}{\Delta\bar{\gamma}} \approx \frac{\Delta E_z}{E_{z0}} \sqrt{\frac{\frac{2}{\pi}E[(1+\kappa^{-2})^{-2}] - 1}{1+\kappa^2}}$$

where $\kappa = \sigma_x/\sigma_o$

Next we discuss the induced relative energy spread in two opposite limits. If $\kappa \ll 1$,

$$\frac{\sigma_{\Delta\gamma}}{\Delta\bar{\gamma}} \approx \frac{\kappa^2}{2} \frac{1}{1+E_{z0}/\Delta E_z} \approx \frac{\Delta E_z}{E_{z0}} \frac{\kappa^2}{2} \tag{14}$$

If $\kappa \gg 1$,

$$\frac{\sigma_{\Delta\gamma}}{\Delta\bar{\gamma}} \approx \frac{2}{\pi} \frac{\Delta E_z}{E_{z0}} \frac{\log(\kappa)}{\kappa} \tag{15}$$

Quantify the slice energy spread growth

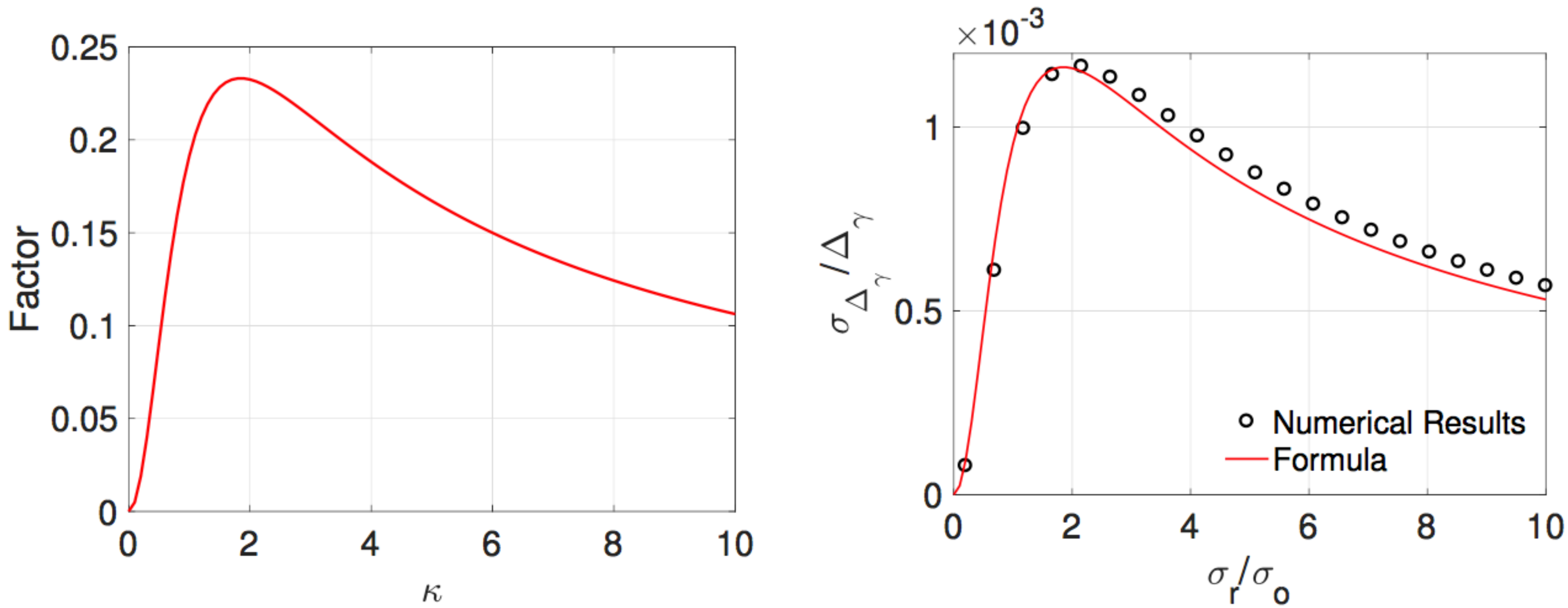
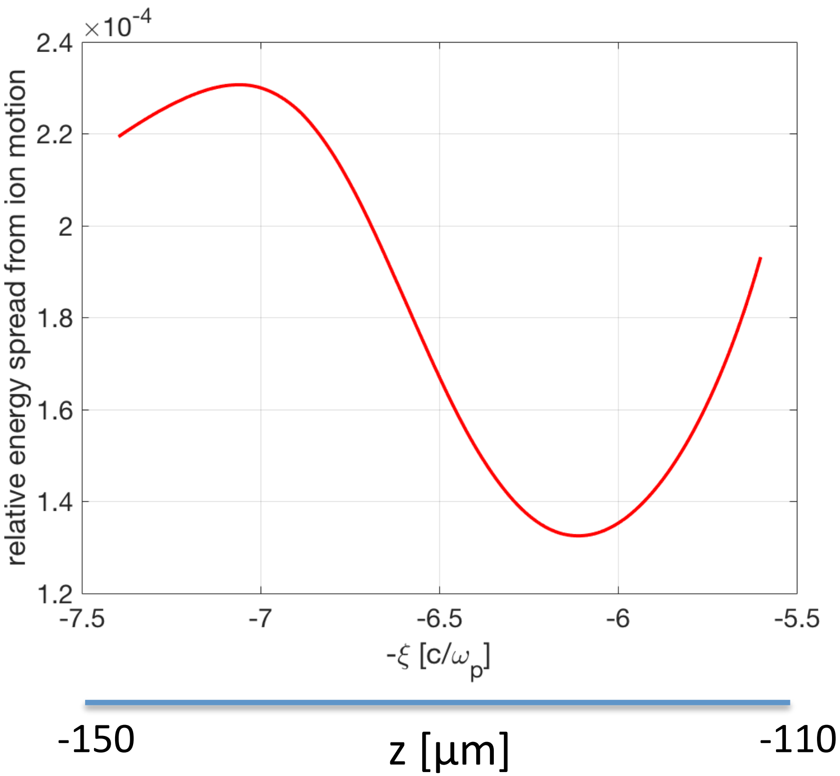
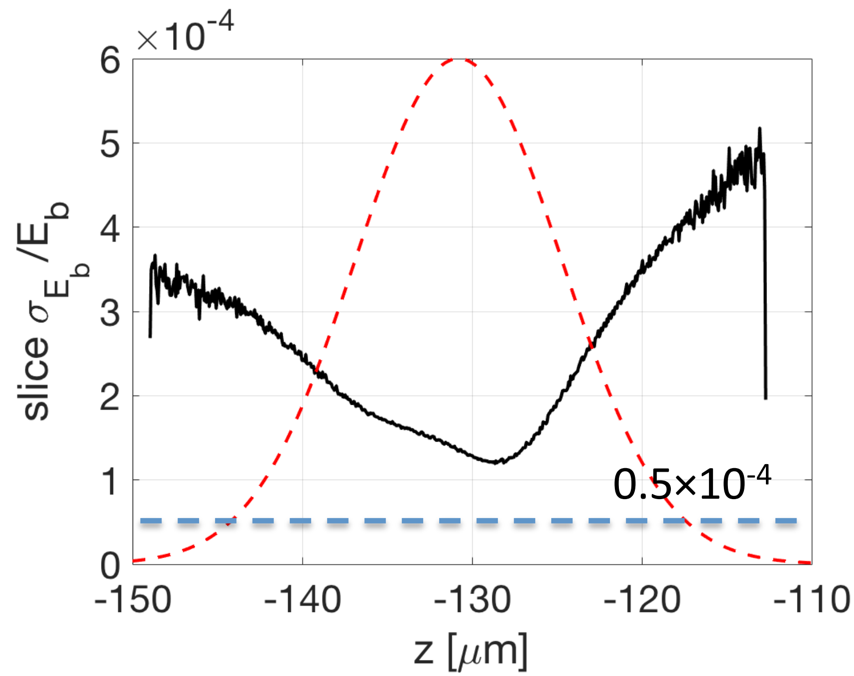


Figure 1: Left: The dependence of the factor on κ. Right: the comparison of the slice energy spread from numerical calculations (black circles) and the formula (red line). Parameters: $E_i = 8$ GeV, $E_{z0} = 2, \Delta E_z = 0.01$ and the acceleration distance is $L = 8000$. Because the spot size of the beam decreases as the energy grows, there is small difference between the numerical results and the formula.

Predication from QuickPIC one-step simulation



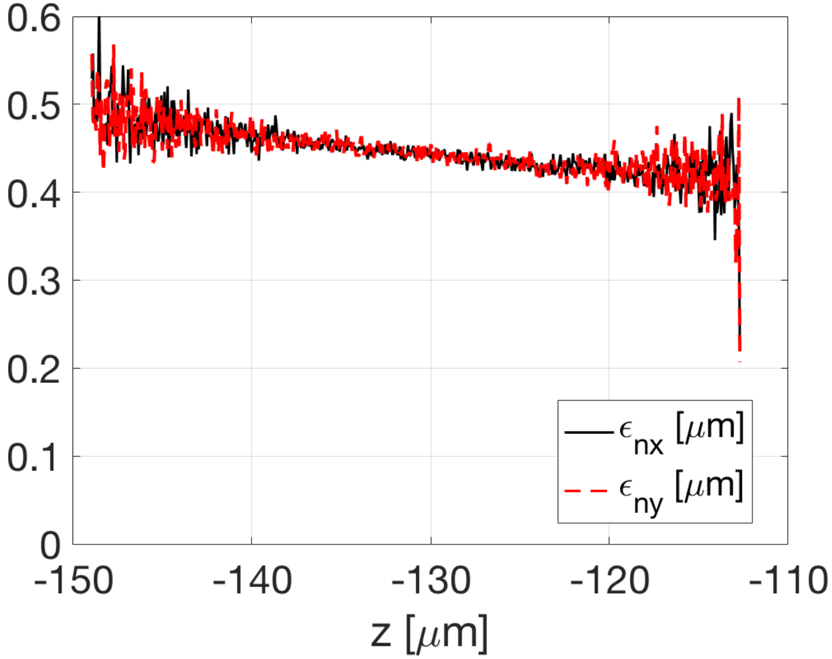
QuickPIC long simulation at z=1.45 m (E_b=16.1 GeV)

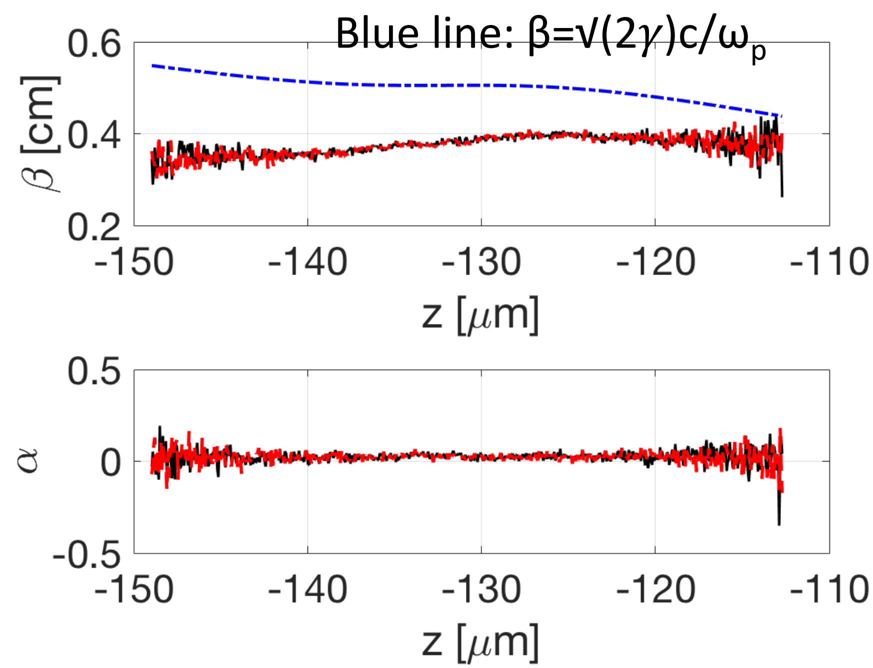


Agreement is good for the region whose slice energy spread is dominated by the ion motion.

QuickPIC long simulation at z=1.45 m (E_b=16.1 GeV)

Ion motion also cause the emittance growth and the distortion of the β-function.



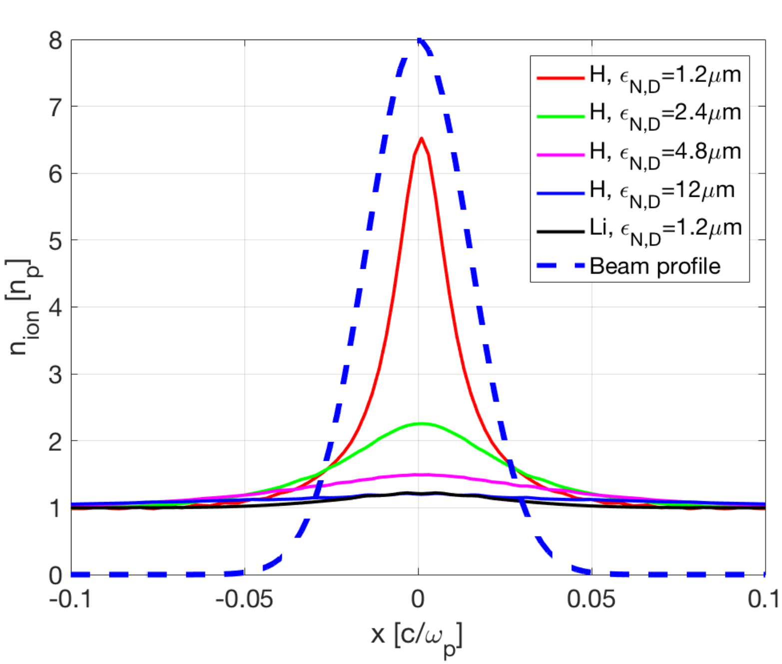


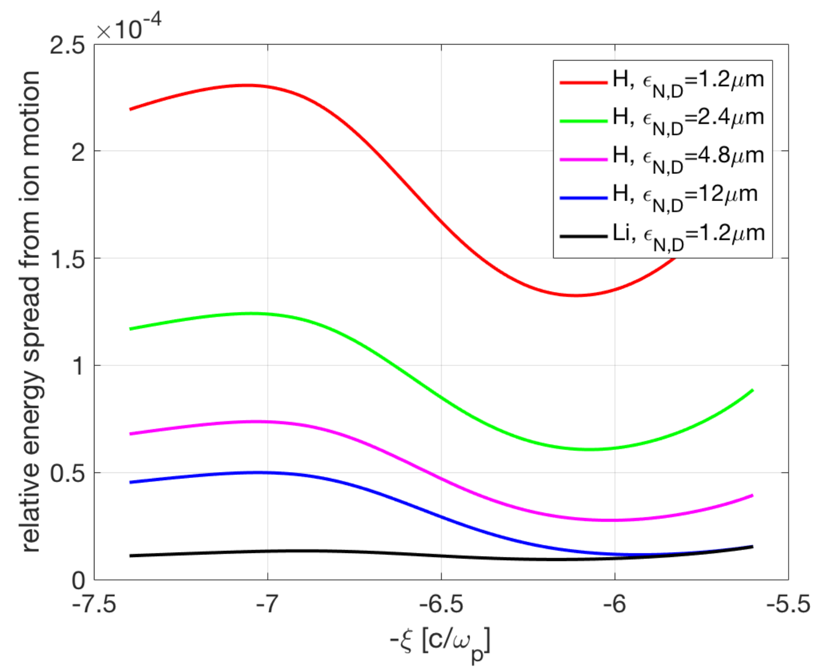
Solutions

- Spoil the emittance of the driver
- Use Lithium instead of Hydrogen

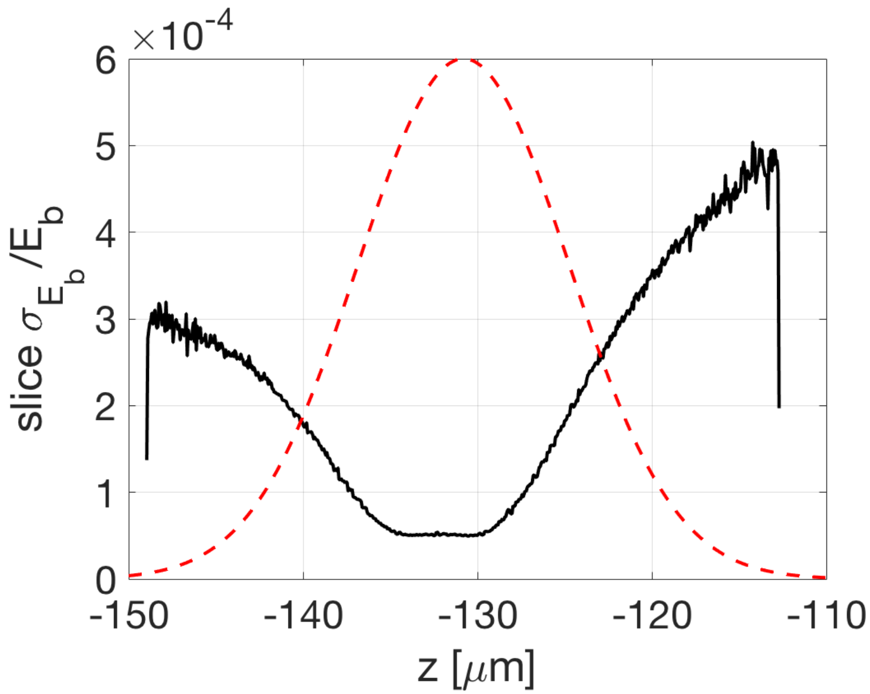
$$\hat{E}_r = \frac{\Lambda}{\hat{\sigma}_r}\frac{1-\mathrm{Exp}\left(-\hat{r}^2/2\hat{\sigma}_r^2\right)}{\hat{r}} \sim \frac{\Lambda}{2\hat{\sigma}_r}$$

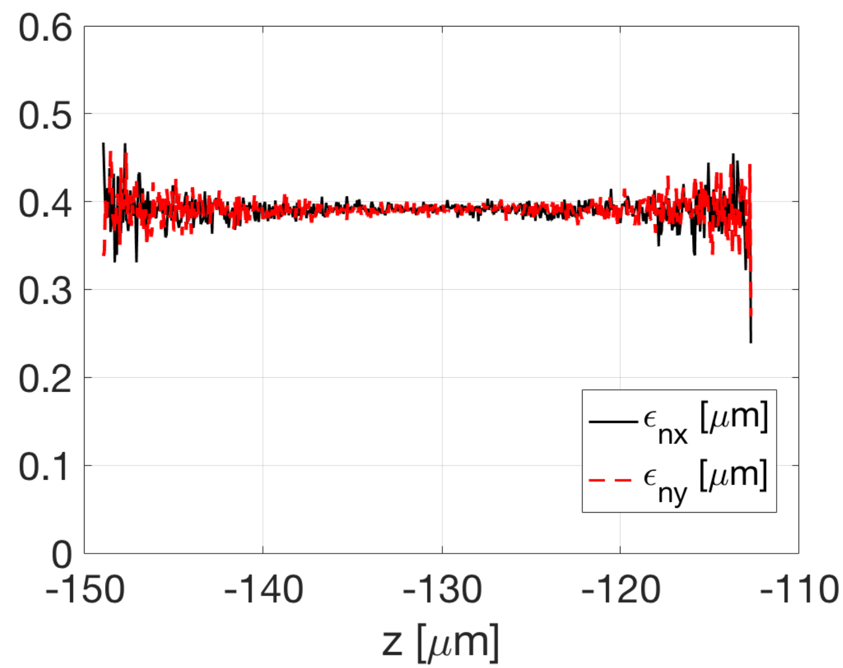
0.2 TV/m, not strong enough to ionize the 2nd electron of Lithium





QuickPIC long simulations with Lithium





Summary

- We simulate the energy-double of the LCLS-II beams in plasma wakefield acceleration.
- Possible growth of the emittance and the energy spread and the distortion of the β-function induced by the ion motion are studied.
- Solutions to suppress the ion motion for our parameters are proposed.

- Thanks!