Effect of peculiar velocities on the gravitational potential in cosmological models with perfect fluids

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We consider a universe filled with perfect fluid with the constant equation of state p = ωε and ω = const. In the theory of scalar perturbations, we study the effect of peculiar velocities on the gravitational potential. For radiation with ω = 1/3, we obtain the expression for the gravitational potential in the integral form. Numerical calculation clearly demonstrates the modulation of the gravitational potential by acoustic oscillations due to the presence of peculiar velocities. We also show that peculiar velocities affect the gravitational potential in the case of the frustrated network of cosmic strings with ω = −1/3.

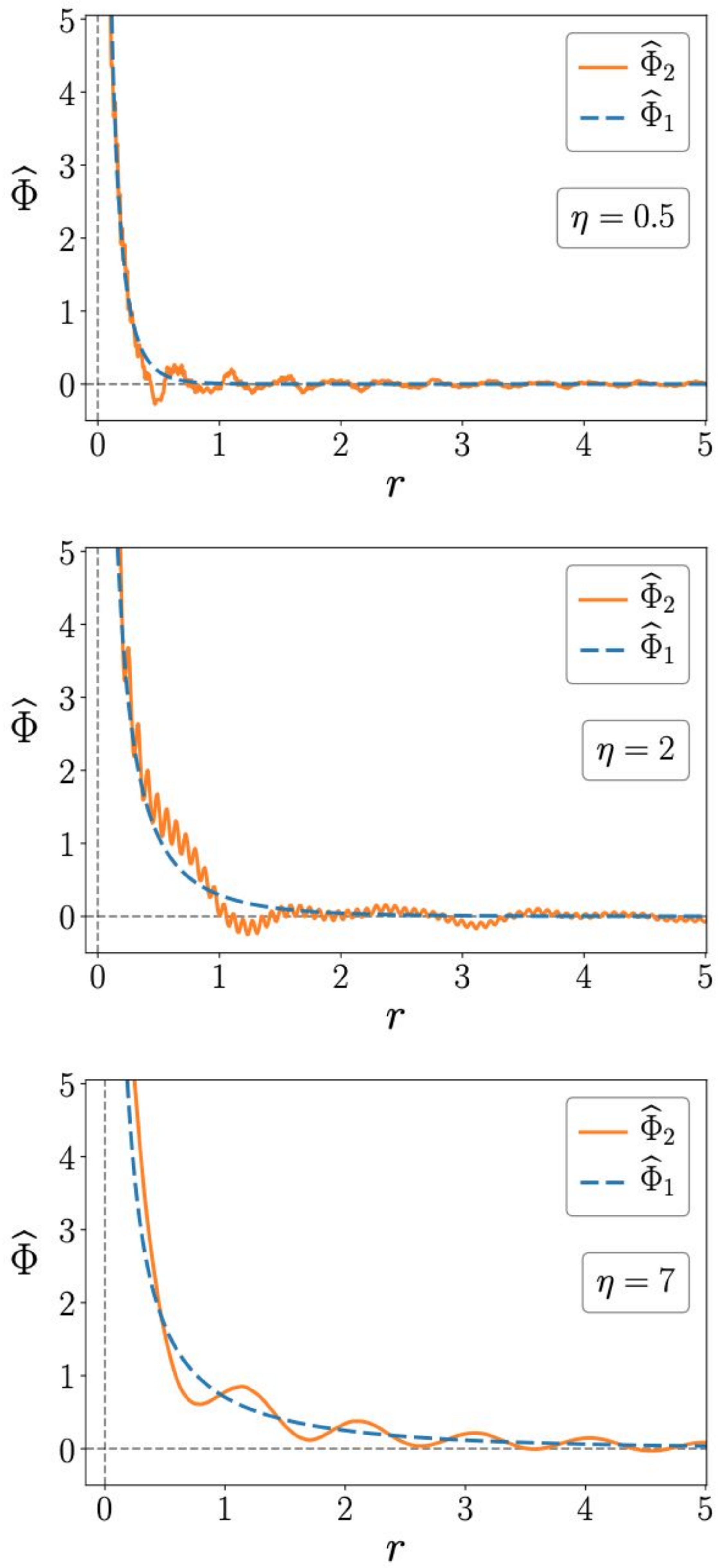

Figure 1: Gravitational potentials in the case of radiation for different values of the parameter η. The dashed blue line corresponds to pure Yukawa potential and the solid orange line takes into account the effect of peculiar velocity.

Starting from an arbitrary value of ω, we then concentrated on relativistic fluid with ω = 1/3. Here, peculiar velocities undergo acoustic oscillations. In the momentum space, we have obtained the formulas for the gravitational potentials both in the presence and absence of peculiar velocities.

To get the exact form of potentials in the position space, we have assumed that the matter fluctuation is a localized inhomogeneity in the form of the delta function. If we neglect peculiar velocities, then the gravitational potential has the form of the Yukawa potential.

Since the Fourier integral for the velocity-dependent potential can be calculated only numerically, we have depicted the results graphically in figures 1 and 2.. These figures clearly demonstrate **the modulation of the gravitational potential by acoustic oscillations due to the presence of peculiar velocities.**

To illustrate the effect of the peculiar velocities on the gravitational potential, we also considered the case of the frustrated network of cosmic strings with ω = −1/3 (figures 3 and 4). In this exceptional case, acoustic oscillations are absent. Nevertheless, the difference between the figures demonstrates the effect of peculiar velocity.

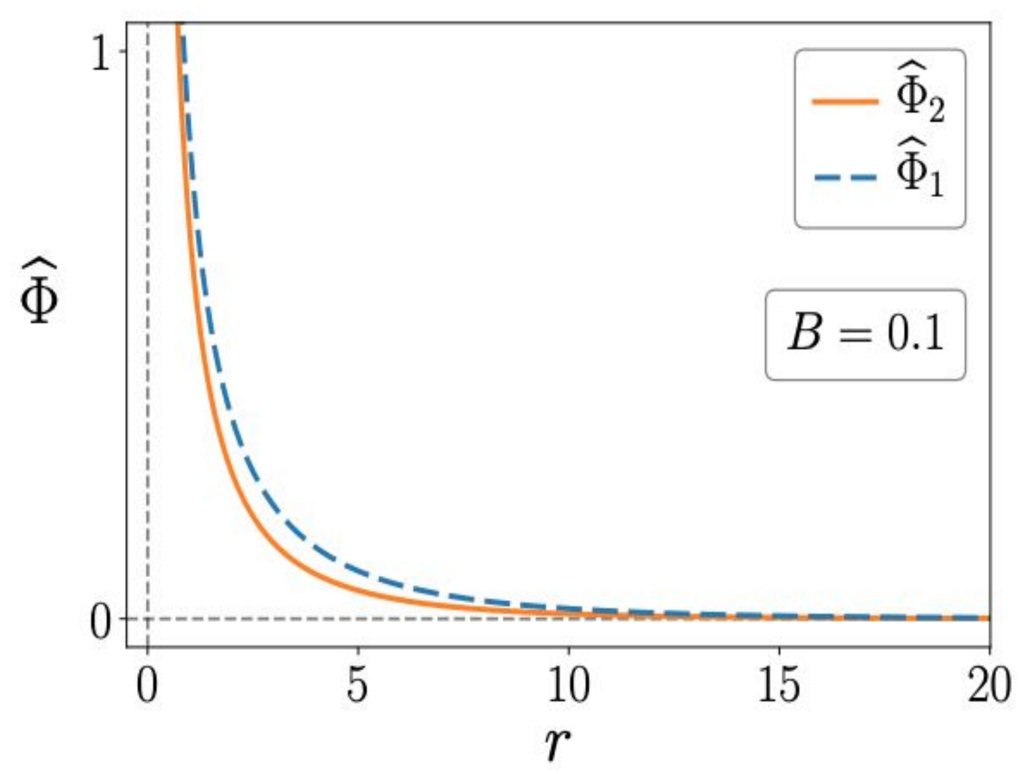

Figure 3: Gravitational potentials in the case of the frustrated network of cosmic strings for parameter $B = 0.1$. The dashed blue line corresponds to pure Yukawa potential, and the solid orange line takes into account the effect of peculiar velocity.

Initial model

For the considered model, the background Friedmann equation is

$$\frac{3\mathcal{H}^2}{a^2}=\frac{3H^2}{c^2}=\kappa\bar{\varepsilon}\,,$$

the perturbed (because the background matter is perturbed by inhomogeneities of perfect fluid) metrics in conformal Newtonian gauge looks like

$$ds^2=a^2(\eta)[(1+2\Phi)d\eta^2-(1-2\Phi)d\mathbf{r}^2]\,,$$

and the perturbed Einstein equations read

$$\Delta\Phi-3\frac{a'}{a}\left(\Phi'+\frac{a'}{a}\Phi\right)=\frac{1}{2}\kappa a^2\delta\varepsilon,$$

$$\Phi'+\frac{a'}{a}\Phi=-\frac{1}{2}\kappa a^2(\bar{\varepsilon}+\bar{p})v\,,$$

$$\Phi''+3\frac{a'}{a}\Phi'+\left(2\frac{a''}{a}-\frac{a'^2}{a^2}\right)\Phi=\frac{1}{2}\kappa a^2\delta p\,,$$

where v(η, **r**) is the peculiar velocity potential. The energy density fluctuation can be expressed as follows

$$\delta\varepsilon=\frac{\delta A}{a^{3(1+\omega)}}+3(1+\omega)\,\bar{\varepsilon}\,\Phi\,,$$

where Φ is singled out.

The perturbed Einstein equation, rewritten in momentum space with the help of the Fourier transform

$$F(\mathbf{r})=(2\pi)^{-3/2}\int_{\mathbb{R}^3}d\mathbf{k}e^{i\mathbf{kr}}\tilde{F}(\mathbf{k})\,.$$

The general solution is

$$\tilde{\Phi}(\eta)=C_1\eta^{\nu}J_{-\nu}(u_{\mathrm{s}}k\eta)+C_2\eta^{\nu}J_{\nu}(u_{\mathrm{s}}k\eta)\,,\qquad u_{\mathrm{s}}\neq 0\,,$$

where Jv are Bessel functions and

$$\nu=-\frac{5+3\omega}{2(1+3\omega)}\,.$$

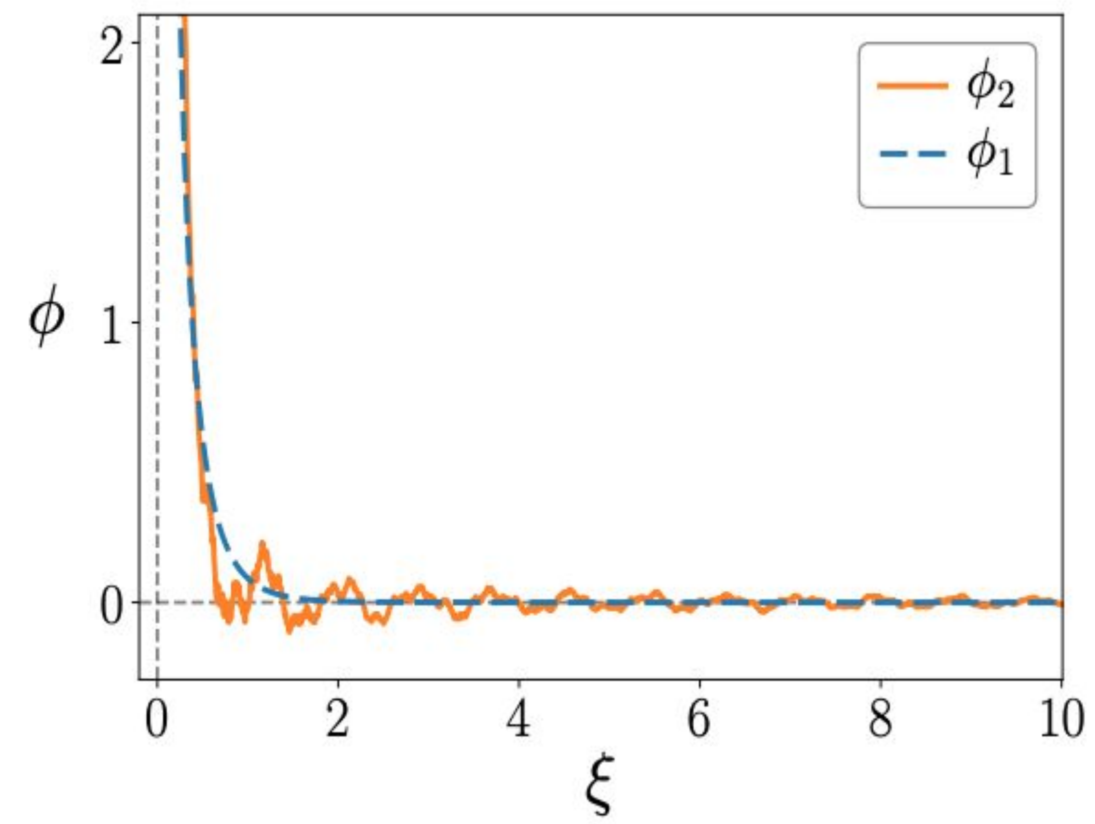

Figure 2: Gravitational potentials (41) $\phi(\xi)$ where $\xi = r/\eta$. Dashed blue and solid orange lines have the same meaning as in Figure (1).

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Relativistic perfect fluid

If to neglect the contribution of peculiar velocity, then the matter density fluctuation is

$$\frac{\delta\tilde{A}_{\mathrm{r}}}{a^4}=-\frac{2}{\kappa a^2}\left(k^2+\frac{a^2}{\lambda_{\mathrm{r}}^2}\right)\tilde{\Phi}\equiv-\frac{2}{\kappa a^2}f_1(k)\tilde{\Phi}\,.$$

On the other hand, the peculiar velocity contribution leads to the appearance of an additional k-dependent term:

$$\frac{\delta\tilde{A}_{\mathrm{r}}}{a^4}=-\frac{2}{\kappa a^2}\left[k^2-\frac{3}{\eta^2}\frac{(u_{\mathrm{s}}k\eta)^2\sin(u_{\mathrm{s}}k\eta)}{u_{\mathrm{s}}k\eta\,\cos(u_{\mathrm{s}}k\eta)-\sin(u_{\mathrm{s}}k\eta)}\right]\tilde{\Phi}\equiv-\frac{2}{\kappa a^2}f_2(k)\tilde{\Phi}\,,$$

The gravitational potentials in the position space:

$$\Phi_i(\mathbf{r})=-\frac{1}{(2\pi)^{3/2}}\frac{\kappa}{2a^2}\int_{\mathbb{R}^3}d\mathbf{k}e^{i\mathbf{kr}}\frac{\delta\tilde{A}_{\mathrm{r}}}{f_i(k)}=-\frac{G_N}{c^2}\frac{M}{ar}\frac{2}{\pi}\int_0^{\infty}dk\,\frac{k\sin(kr)}{f_i(k)}\,,\quad i=1,2.$$

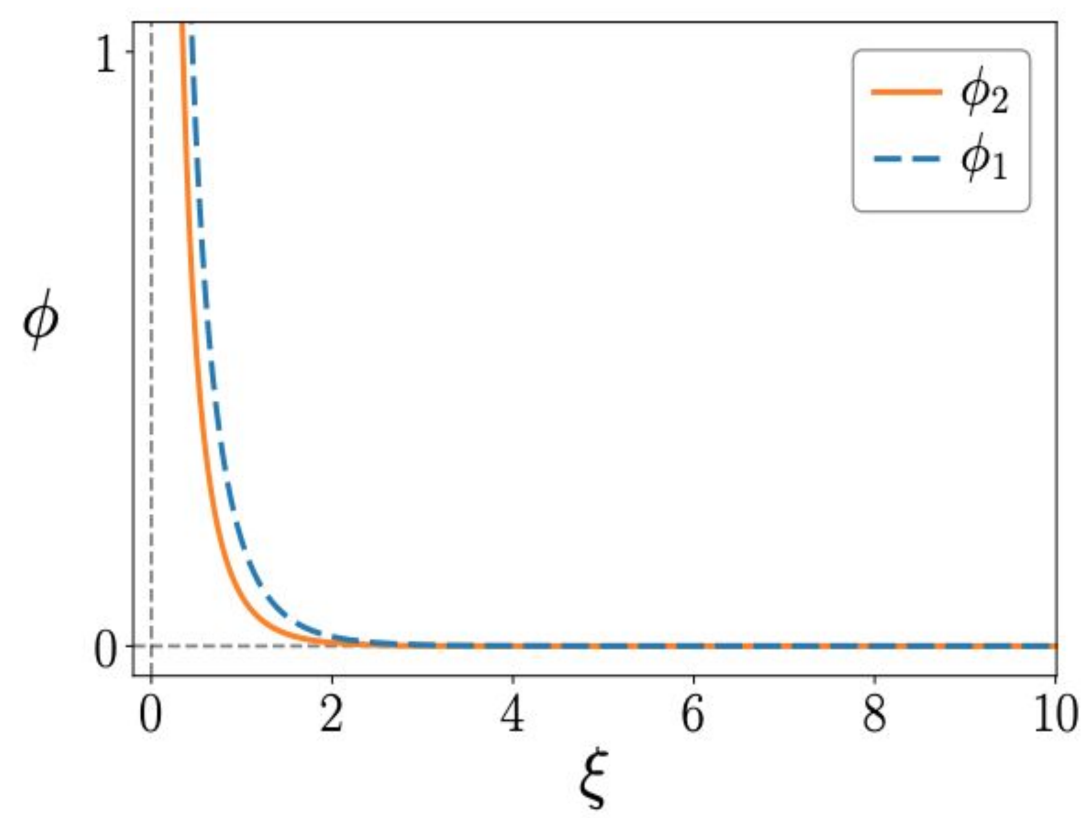

Figure 4: Gravitational potentials (55) $\phi(\xi)$ where $\xi = Br$. The dashed blue and the solid orange lines have the same meaning as in Figure (3).