Bounds on graviton mass from galaxy clusters

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Abstract

In quantum field theory graviton is the elementary particle that mediates the gravitational force and is expected to be massless. But if the graviton has a small but nonzero mass, gravity would have a finite rather than infinite range, characterized by the graviton's so-called Compton wavelength λ_g. In the last few years, there has been a resurgence of interest in obtaining observational bounds on the graviton mass, following the detection of GW, because of the versatility of massive graviton theories in resolving multiple problems in cosmology and fundamental physics. Here, we are presenting summary of how we achieved new strong bounds on graviton mass. We calculated the gravitational acceleration in Yukawa-like fall off potential and formulated as a function of Hubble parameter, Compton wavelength and Galaxy cluster mass, including catalogs from 2500 sq. degree SPT-SZ survey, the Planck all-sky SZ catalog, and a redMaPPer selected catalog from 10,000 sq. degree of SDSS-DR8 data. After performing χ^2 analysis, we obtained 90% c.l. upper limits, which were $m_g < 4.73 \times 10^{-30}$ eV, 3.0×10^{-30} eV, and 1.27×10^{-30} eV for SPT, Planck and SDSS. These limits are about five times more stringent than the previous best bound from galaxy clusters. For Chandra X-ray sample, we computed temperature and gas density profiles of 12 relaxed galaxy clusters using parameters given in Table 2 and 3 of [3]. Upper bound on graviton mass was determined by evaluating total dynamical mass from the hydrostatic equilibrium equation in Yukawa gravity and comparing it with the corresponding mass in Newtonian gravity. The best limit is obtained for Abell 2390, corresponding to $\lambda_g > 3.58 \times 10^{19}$ km or $m_g < 3.46 \times 10^{-29}$ eV. This is the first proof of principles demonstration of setting a limit on the graviton mass using a sample of related galaxy clusters with x-ray measurements and can be easily applied to upcoming x-ray surveys such as eRosita.

Methodology - For Stacked galaxy clusters

- Newtonian gravitational acceleration follows the inverse square law. But, to determine the Yukawa acceleration we need to obtain the gradient of Yukawa potential, which is $V(r) = \exp(-r/\lambda_g)(GM/r)$

$$a_y = \frac{GM_\Delta}{R_\Delta} exp(-R_\Delta/_g)(\frac{1}{R_\Delta} + \frac{1}{\lambda_g}) \quad (1)$$

where R_Δ is the distance from the core of cluster at which the density of galaxy cluster becomes Δ times the critical density ρ_c. The critical density is given by $\rho_c = \frac{3H^2(z)}{8\pi G}$, where $H(z)$ is the Hubble parameter at redshift z.

- The mass of the galaxy cluster can be evaluated from the density of the galaxy cluster within a radial distance of R_Δ [2]:

$$M_\Delta = \Delta \times \rho_c \times \frac{4\pi}{3} R_\Delta^3 \quad (2)$$

- For our analysis, we need to write down the equations for both these accelerations in terms of observables and eliminate unknowns such as the galaxy cluster radius. Therefore, rewriting eqs. 1, 2 and critical density expression, we get

$$a_n(z, M_\Delta) = (GM_\Delta)^{1/3} \left(\frac{H^2(z)\Delta}{2}\right)^{2/3} \quad (3)$$

$$a_y(z, M_\Delta, \lambda_g) = (GM_\Delta)^{2/3} \left(\frac{H^2(z)\Delta}{2}\right)^{1/3} \exp\left[-\frac{1}{\lambda_g}\left(\frac{2M_\Delta G}{H^2(z)\Delta}\right)^{1/3}\right]\left[\frac{1}{\lambda_g} + \left(\frac{H^2(z)\Delta}{2M_\Delta G}\right)^{1/3}\right]$$

- To mimics the behavior of H(z) in a flat ΛCDM cosmology and to determine value at any input redshift, we have fit the data to a non-linear function,

$$H(z) = A\sqrt{(B(1+z)^3 + C)} \quad (4)$$

where we have kept A fixed at 70 km/sec/Mpc and the unknowns B and C in Eq. 4 can be obtained using the least squares fitting technique as seen in Fig. 1. For fitting, we have used 31 measurements obtained from the cosmic chronometric technique within the redshift range of $0.07 < z < 1.965$

- Once we have the mass for a given cluster, to quantify the deviations between Newtonian and Yukawa gravity, we construct a χ^2 functional given by:

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{a_n - a_y}{\sigma_a}\right)^2, \quad (5)$$

where σ_a is the error in acceleration given by eq. 6; N is the total number of clusters and is defined in [2] as follows:

$$\sigma_a = \frac{a_n}{3}\sqrt{\left(\frac{\sigma_{M_\Delta}}{M_\Delta}\right)^2 + 16\left(\frac{\sigma_H}{H(z)}\right)^2} \quad (6)$$

Analysis & Results

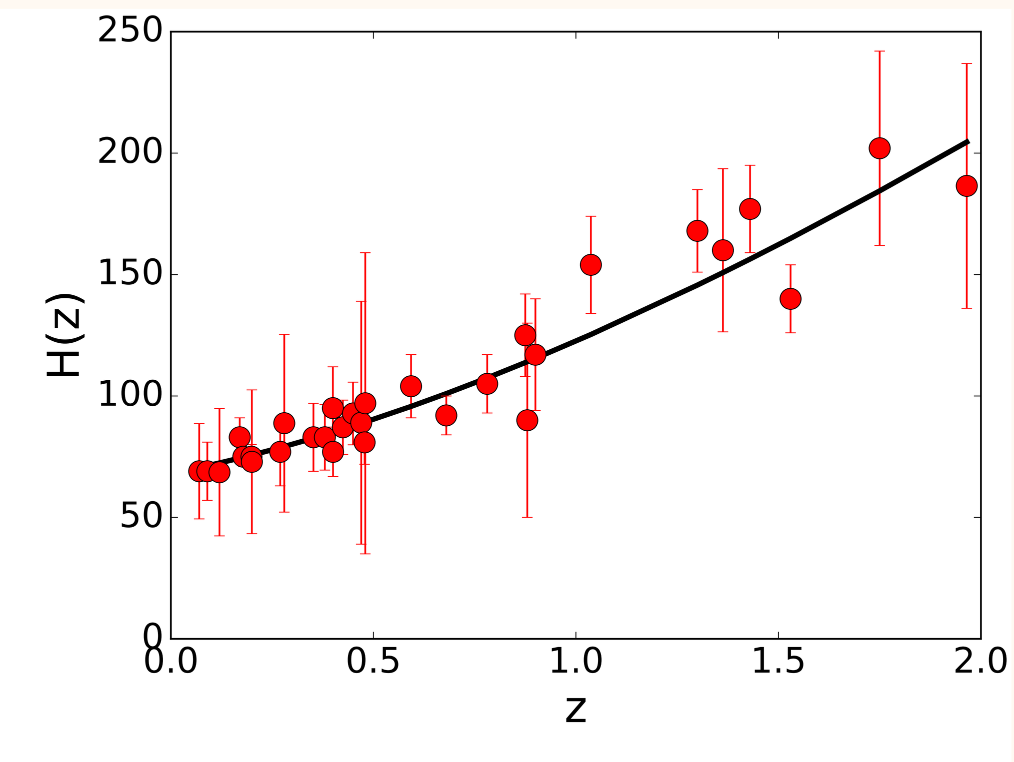

Figure 1: $H(z)$ as a function of redshift, z. The red points along with error bars denote the 31 measurements of H(z), which are fitted against Eq. 4 using least-squares fitting, giving us $B = 0.3 \pm 0.025$ and $C = 0.65 \pm 0.078$.

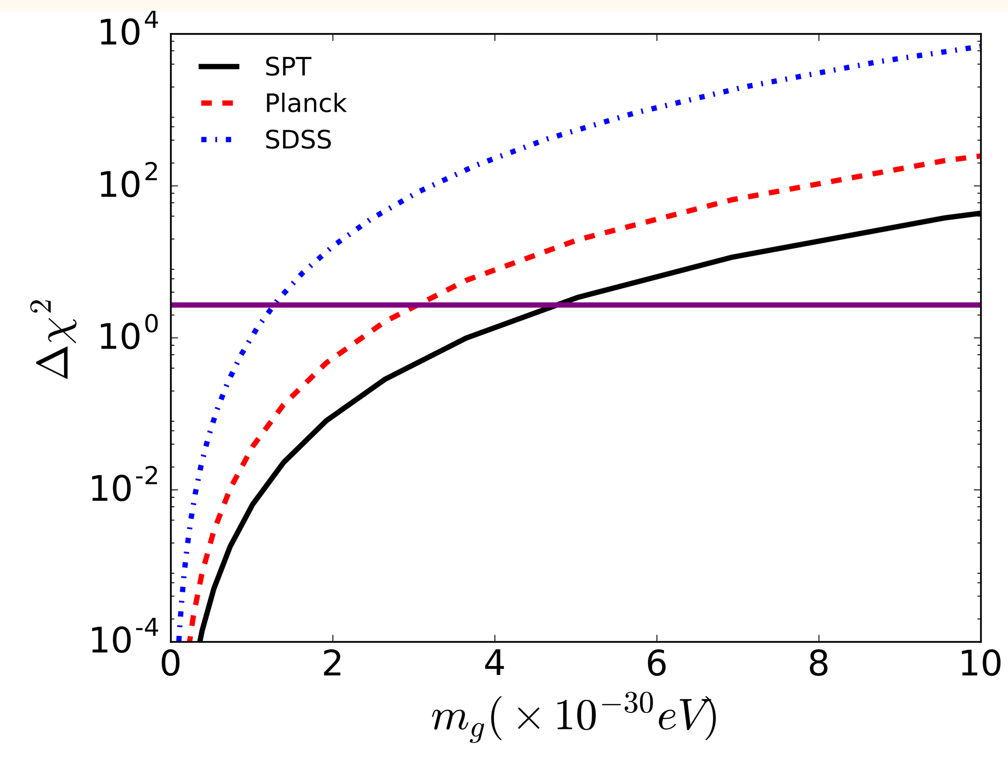

Figure 2: $\Delta\chi^2$ as a function of graviton mass using stacked cluster catalogs from SPT, Planck and SDSS DR8 selected using redMaPPer. The solid magenta line at $\Delta\chi^2 = 2.71$ gives us the 90% c.l. upper limit on the graviton mass. These upper limits correspond to $m_g < 4.73 \times 10^{-30}$ eV, 3.0×10^{-30} eV, and 1.27×10^{-30} eV for SPT, Planck and SDSS respectively and are about five times more stringent than the corresponding limits in Ref. [2].

Table 1: Tabular summary of our 90% c.l. (upper) limits on the graviton mass (m_g) and (lower) limits on the Compton wavelength (λ_g) for SPT, Planck and SDSS catalogs.

Catalog Name	Clusters	Type	$m_g <$ (eV)	$\lambda_g >$ (km)
SPT	516	SZ	4.73×10^{-30}	2.62×10^{20}
Planck	907	SZ	3.0×10^{-30}	4.12×10^{20}
SDSS	26111	Optical	1.27×10^{-30}	9.76×10^{20}

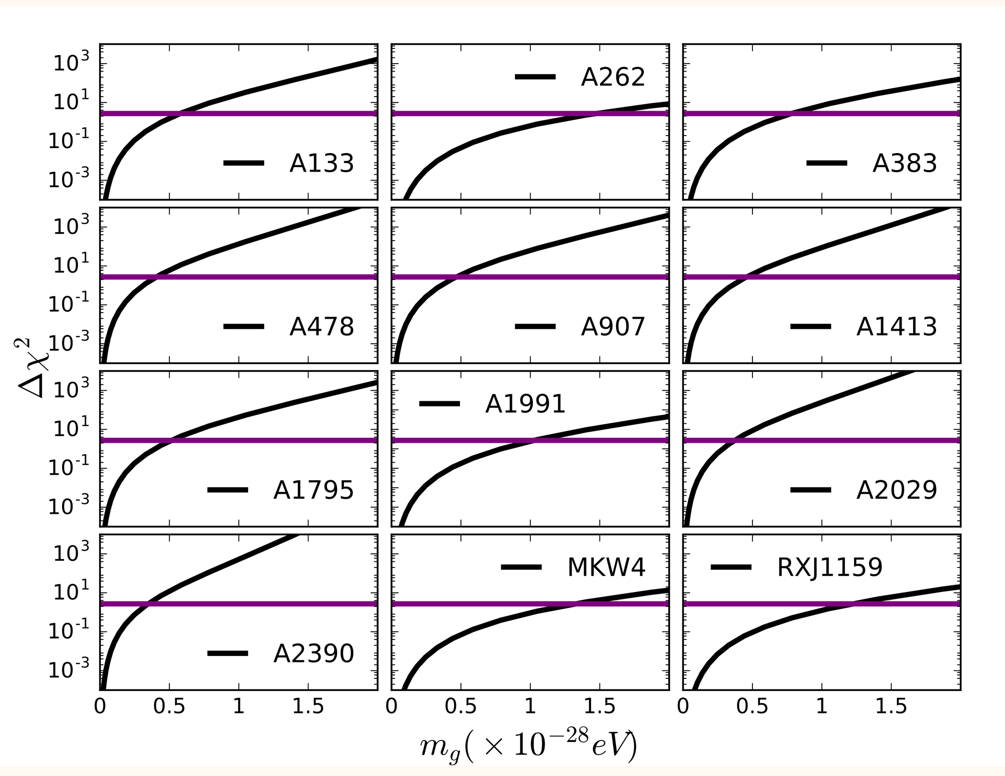

Figure 3: $\Delta\chi^2$ as a function of graviton mass for each of the 12 clusters used for the analysis in each of the sub-panels. The magenta line in each sub-panel corresponds to the ordinate of 2.71, from which the corresponding 90% c.l. on graviton mass can be determined. Summary of these limits for each cluster can be found in Tab. 2.

Table 2: 90% confidence level upper (lower) limit on graviton mass (Compton wavelength) for each of the 12 galaxy clusters used in our analysis. The letter 'A' in the prefix of some of the clusters is an acronym for Abell. The best limit is for Abell 2390 or A2390 ($m_g < 3.46 \times 10^{-29}$ eV or $\lambda_g > 3.58 \times 10^{19}$ km)

Cluster Name	$m_g <$ (eV)	$\lambda_g >$ (km)
A 133	5.76×10^{-29}	2.15×10^{19}
A 262	1.47×10^{-28}	8.44×10^{18}
A 383	7.80×10^{-29}	1.59×10^{19}
A 478	4.04×10^{-29}	3.06×10^{19}
A 907	4.65×10^{-29}	2.66×10^{19}
A 1413	4.57×10^{-29}	2.71×10^{19}
A 1795	5.12×10^{-29}	2.42×10^{19}
A 1991	1.02×10^{-28}	1.21×10^{19}
A 2029	3.70×10^{-29}	3.34×10^{19}
A 2390	3.46×10^{-29}	3.58×10^{19}
MKW 4	1.32×10^{-28}	9.38×10^{18}
RX J1159+5531	1.21×10^{-28}	1.02×10^{19}

Methodology - For Chandra X-ray Cluster Sample

- [3] presented ρ and temperature profiles for a total of 13 nearby relaxed galaxy clusters using measurements from the archival or pointed observations with the Chandra X-ray satellite. To reconstruct gas and total mass estimates consider a gas in hydrostatic equilibrium. Gas pressure can be related to the density, assuming an ideal gas equation of state $P = \rho K_b T/\mu m_p G$, where m_p is the mass of the proton, μ is the mean molecular weight of the cluster in a.m.u. and is approximately equal to 0.6 [1]. Putting all this together, also for Yukawa potential (using 1), we get

$$M_{tot}^N(r) = -\frac{k_b T r}{G\mu m_p}\left(\frac{d\ln\rho_{gas}}{d\ln r} + \frac{d\ln T}{d\ln r}\right), \quad (7)$$

$$M_{tot}^{yuk}(r) = -\exp(r/\lambda_g)\frac{k_b T r}{G\mu m_p}\left(\frac{d\ln\rho_{gas}}{d\ln r} + \frac{d\ln T}{d\ln r}\right)\frac{r\lambda_g}{\lambda_g + r}, \quad (8)$$

- Temperature and density profiles were directly modeled from [3] which were fit to the observed data.

$$n_e(r)n_p(r) = \frac{(r/r_c)^{-\alpha'}}{(1+\frac{r^2}{r_c^2})^{3\beta-\alpha'/2}}\frac{n_0^2}{(1+\frac{r^\gamma}{r_s^\gamma})^{\epsilon/\gamma}} + \frac{n_{02}^2}{(1+\frac{r^2}{r_{c'}^2})^{3\beta'}}. \quad (9)$$

$$\rho_g \approx 1.624 m_p \sqrt{n_p(r)n_e(r)}, \quad (10)$$

The physical interpretations of the empirical constants r_c,α', β, r_s, γ, n_0, n_{02}, β' for the twelve galaxy clusters are discussed in [3] and can be found in Table 2 therein.

$$T(r) = T_0 \frac{(x_0 + T_{min}/T_0)}{x_0 + 1}\frac{(r/r_t)^{-a'}}{[1+(r/r_t)^b]^{c'/b}}, \quad (11)$$

where $x_0 = \left(\frac{r}{r_{cool}}\right)^{a_{cool}}$. The physical meanings of the eight free parameters $a', b, c', T_{min}, r_t, T_0, r_{cool}$, and a_{cool} and their corresponding values for the 12 clusters can be found in [3]

- To get the corresponding limit on the graviton mass, we compare the dynamical masses in Newtonian and Yukawa gravity and calculated the χ^2 differences between the two using the expression

$$\chi^2 = \sum_{i=1}^{N}\left(\frac{M_{tot}^{yuk}(r) - M_{tot}^N(r)}{\sigma_{M_{tot}^N}}\right)^2, \quad (12)$$

where $\sigma_{M_{tot}^N}$ is the error in M_{tot}^N. For each cluster, χ^2 was evaluated at these points for which the errors in temperature and radii were available, allowing us to do error propagation.

- To evaluate the error in the mass, we used eq. 13 where σ_T and σ_r denote the errors in the measurement of temperature and radius.

$$\sigma_{M_{tot}^N} = \left[\left(\frac{\partial M_{tot}^N}{\partial T}\right)^2\sigma_T^2 + \left(\frac{\partial M_{tot}^N}{\partial r}\right)^2\sigma_r^2\right]^{1/2}, \quad (13)$$

We used the errors in distance and temperature provided to us by [3].

Conclusions & Discussions

- Our limits are about five times more stringent than the corresponding ones from [2]. The most stringent limits are for the SDSS sample, because of the larger number of clusters (26,111).
- Among the ongoing Stage-III dark energy experiments, the Dark Energy Survey is expected to discover 100,000 clusters covering 5,000 square degrees and the expected sensitivity using the same method is approximately 8×10^{-31} eV.
- The best limit is obtained for Abell 2390, corresponding to $\lambda_g > 3.58 \times 10^{19}$ km or $m_g < 3.46 \times 10^{-29}$ eV.
- Newly released X-ray missions such as eRosita and upcoming like Athena should be able to improve upon the limits set in this paper.
- For more information, please scan the QR code:

References

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