Non-parametric Density Estimators for the Measurements of Ionization Cooling

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Machine Learning Applications for Particle Accelerators

ILLINOIS INSTITUTE OF TECHNOLOGY

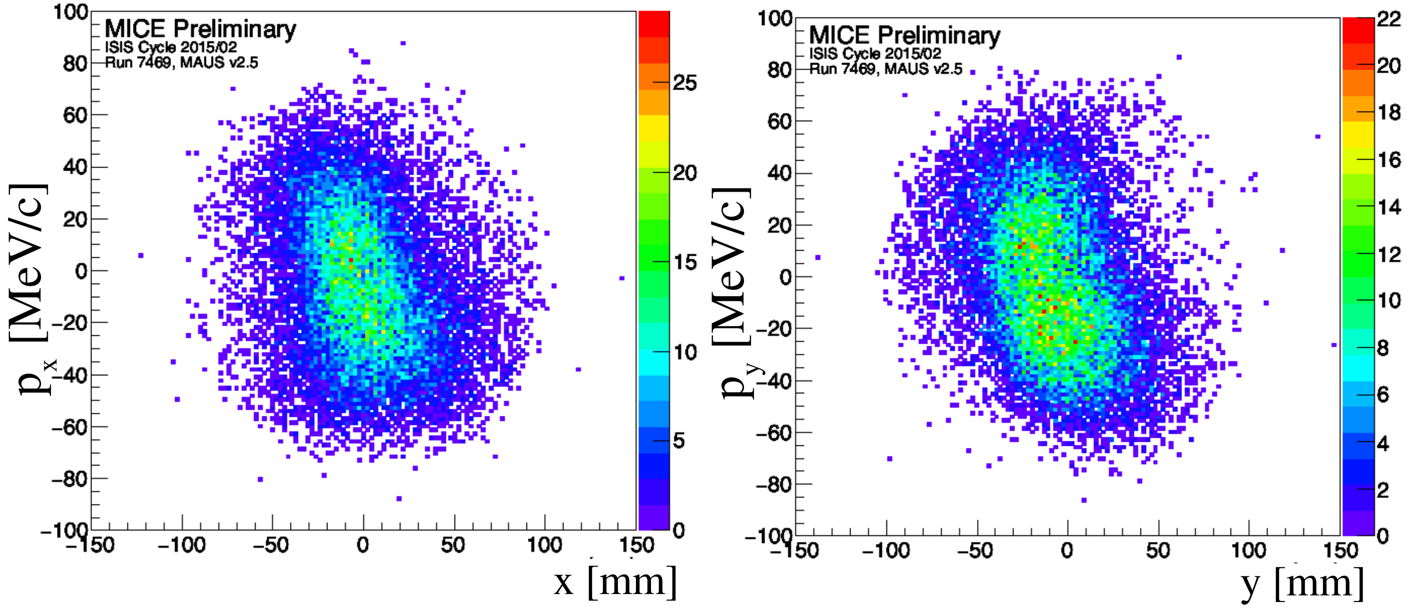
MICE

Outline

- Motivation
- Muon Ionization Cooling
- Muon Ionization Cooling Experiment (MICE)
- Non-parametric Density Estimators
- Simulation Results
- Discussion, on-going efforts
- Conclusion

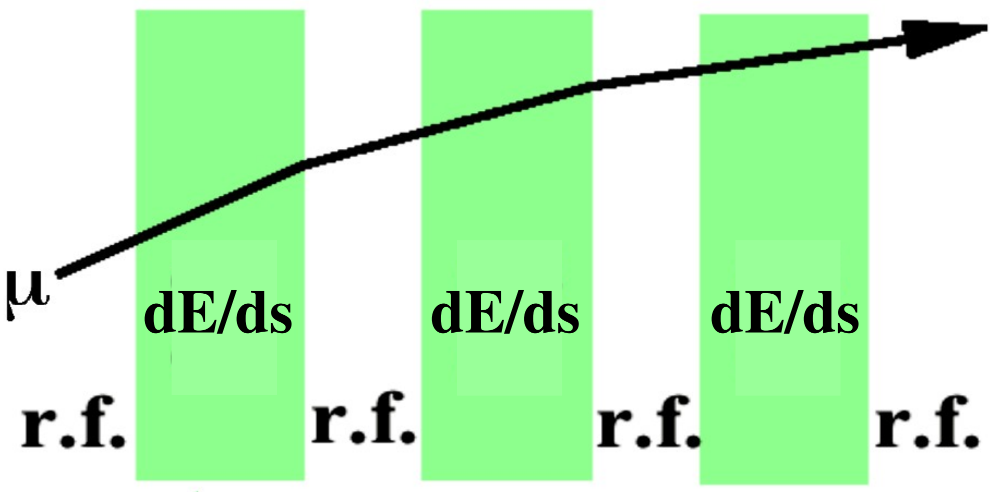
Motivation

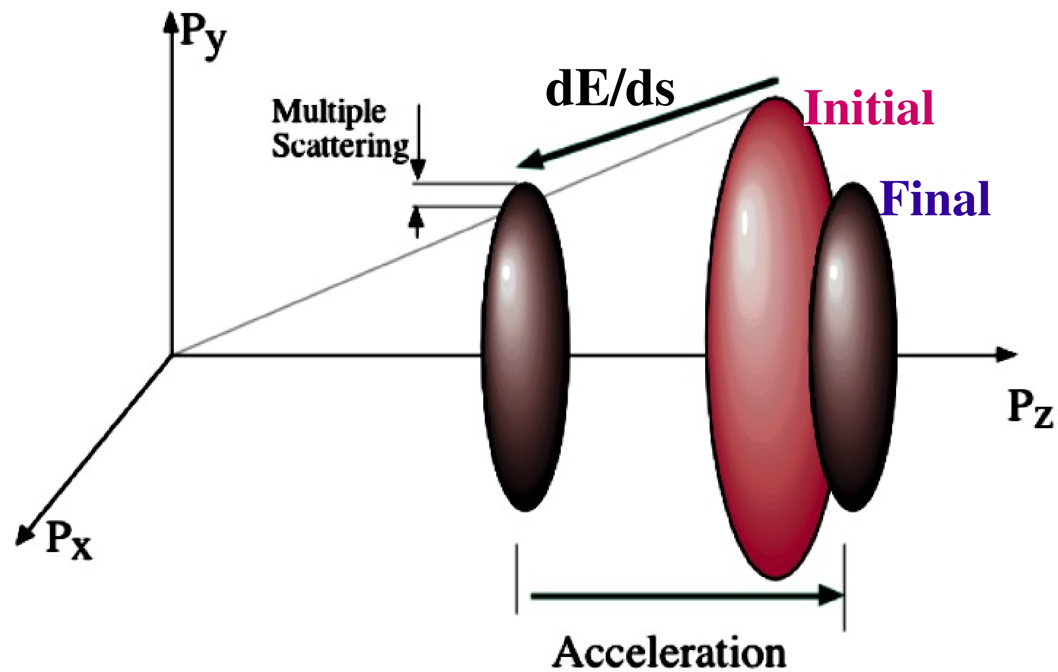
Phase Space of Measured Muon Beam



- Purpose:
  - Muon Ionization Cooling Experiment (MICE): measure muon ionization cooling, **RMS emittance** reduction to high precision
- Challenge:
  - **RMS emittance** assumes **Gaussian**
  - Real-life particle beam not entirely **Gaussian** (chromatic and non-linear effects)
- Solution:
  - **Non-parametric density estimators**:
    - Predict beam distribution with fewer assumptions

Muon Ionization Cooling



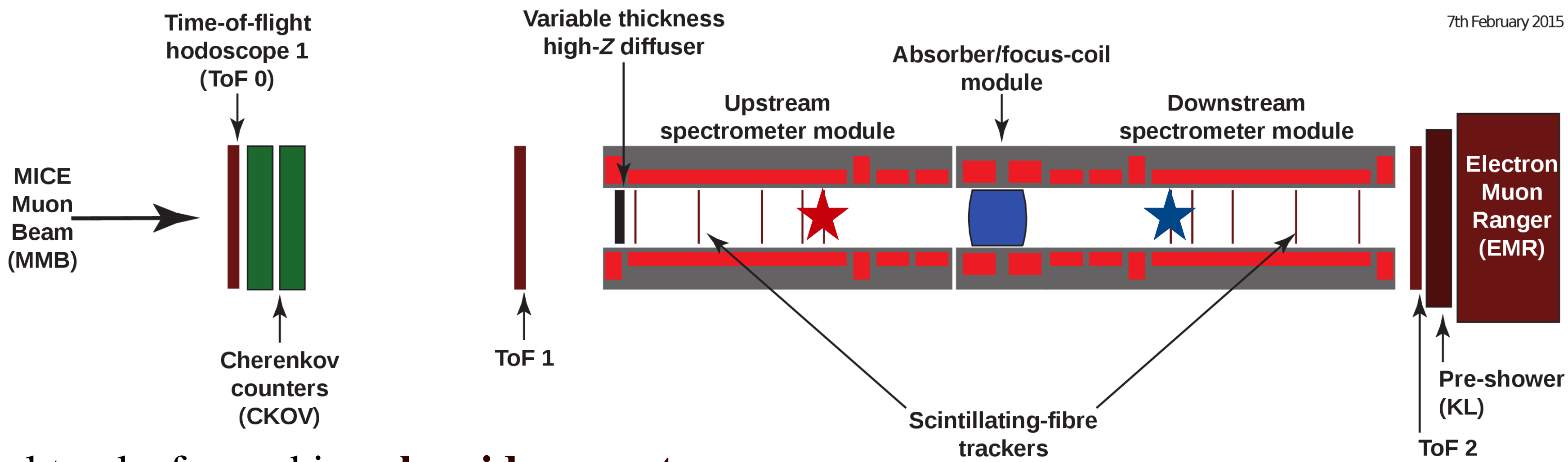


- **Cooling by ionization energy loss**
- **Heating by multiple (Coulomb) scattering**

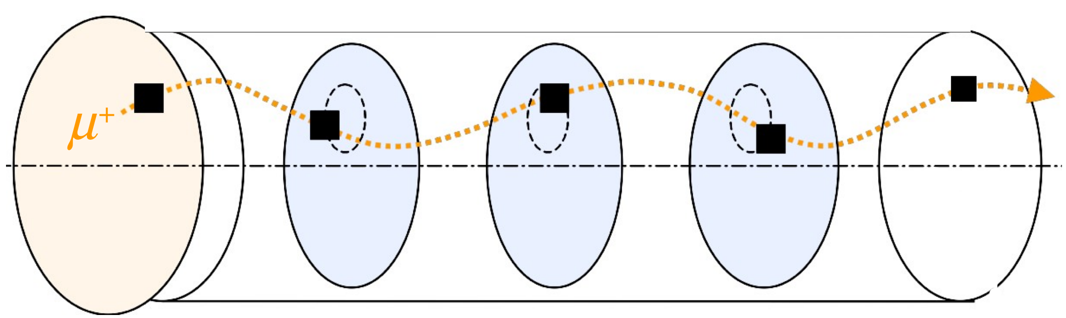
- Measures of muon beam cooling:
  - ★ Reduction: **phase-space volume**, **RMS emittance**
  - ★ Increase: **phase-space density**

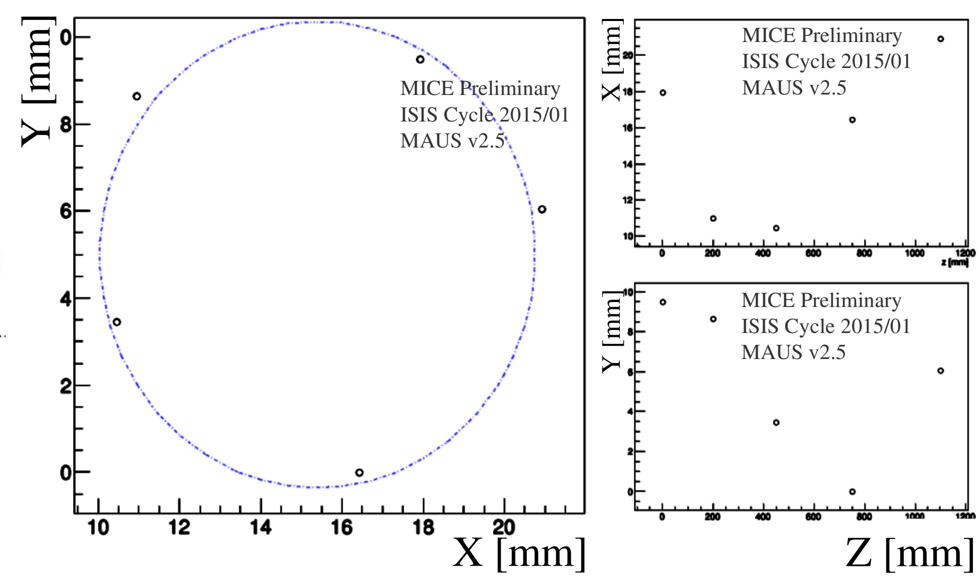
Muon Ionization Cooling Experiment

- Particle ID with Time-of-flight, Cherenkov counters, calorimetry (μ beam contaminated with e, π)
- Muons measured one by one:
  - ★ Density, volume, emittance before (upstream reference plane, **US RP** ★) and after (downstream reference plane, **DS RP** ★) an absorber



- Helical tracks formed in **solenoid magnets**:
  - ★ Phase-space coordinates reconstructed in **trackers**

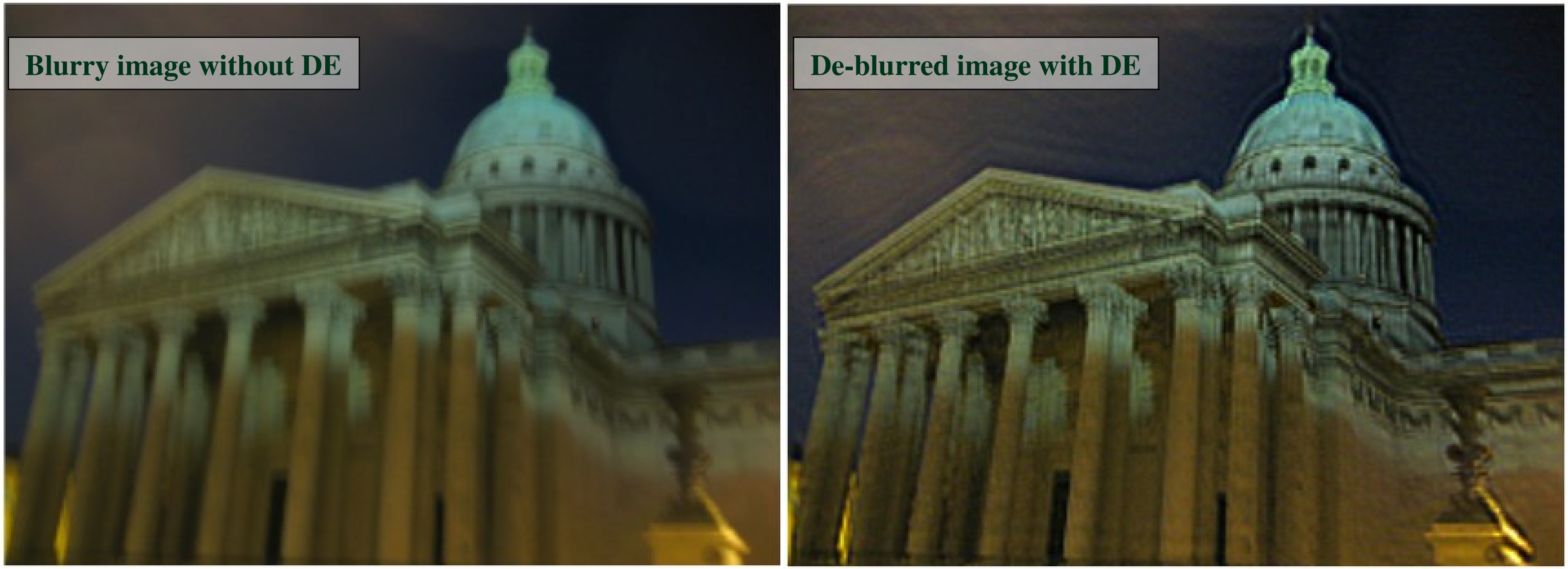




Non-parametric Density Estimators (DE)

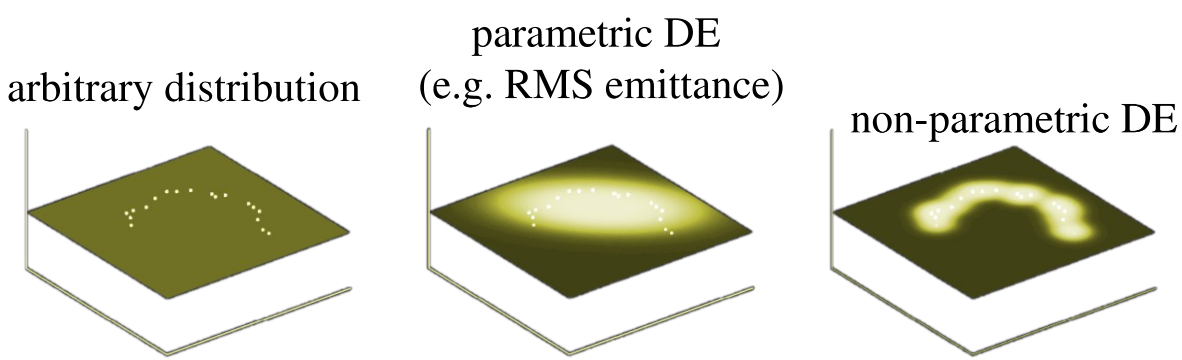
- **Unsupervised** learning (no known output, exploratory in nature):
  - ★ Predicts/estimates the **density** (probability density function, PDF)

Image Processing



D. Krishnan et al., "Blind Deconvolution Using a Normalized Sparsity Measure", DOI: 10.1109/CVPR.2011.5995521

Particle Beam Distributions

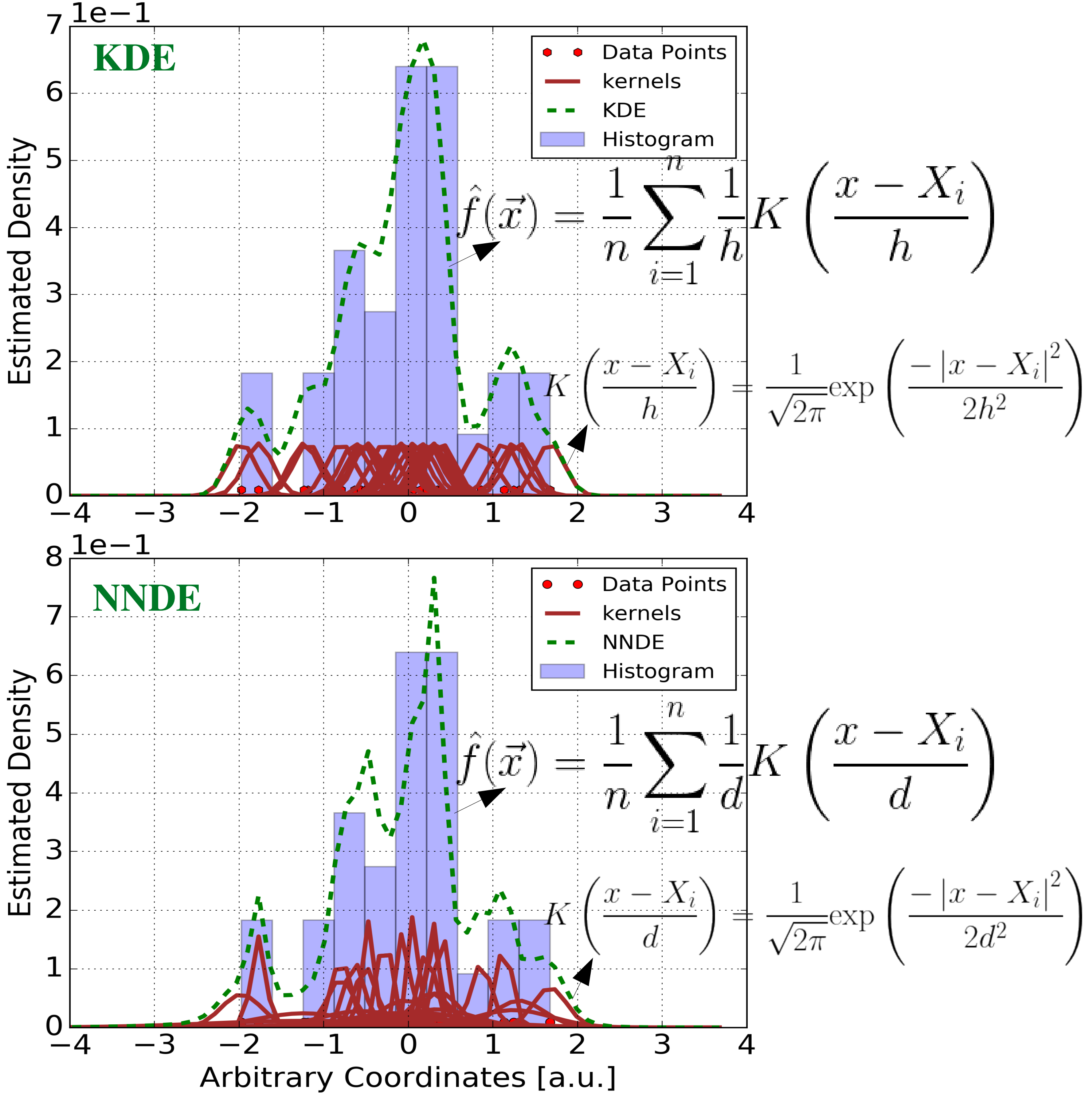


M. Rousson et al., "Efficient Kernel Density Estimation of Shape and Intensity Priors for Level Set Segmentation", DOI:10.1007/978-0-387-68343-0_13

- **Non-parametric**: does not assume functional form
- Data points "speak for themselves"
- Leads to a more precise measure of the **density** in MICE

Non-parametric DE cont.

- Oldest example:
  - **Histogram**
  - Count **data points** within bins of certain widths
- Other examples:
  - kernel density estimation (**KDE**), kth nearest neighbor (**NNDE**)
  - Sum **Gaussian kernels** (smooth weight functions) of certain width centered on **each data point**



h and *d* (distance from *x* to its k^{th} nearest neighbor): widths of KDE and NNDE kernels

X_i: ith data point

n: sample size

Choice of Kernel Width

- Optimal kernel width:
  - Minimizes mean integrated squared error (MISE):
    - Measure of uncertainty in DE

$$\mathrm{MISE} = \int \left\{ \hat{f}(\vec{x}) - f(\vec{x}) \right\}^2 dx$$

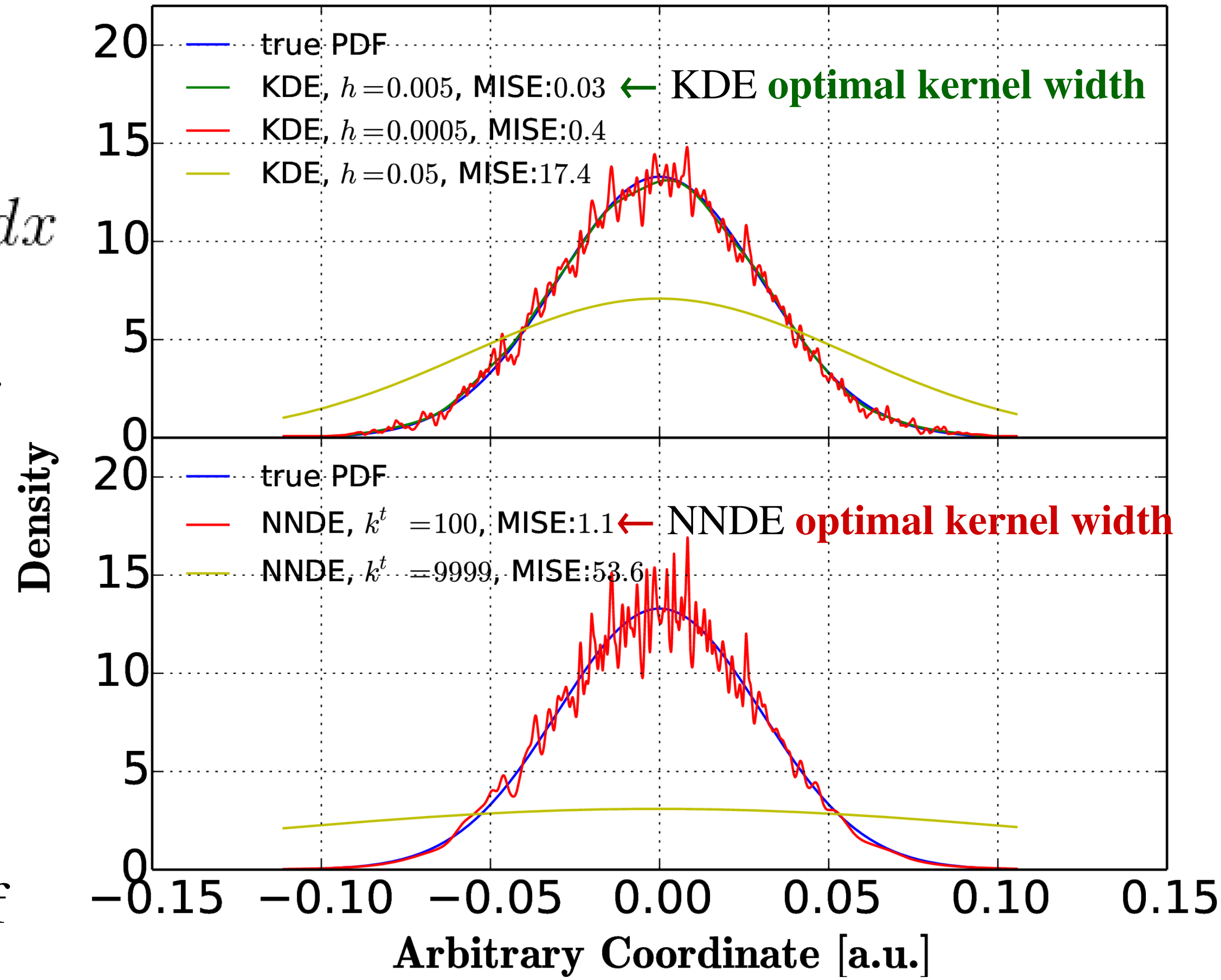
- **True:** Gaussian
- KDE/NNDE: predicted density of the simulated x-distribution in MICE
- Minimized MISE with KDE **optimal kernel width, *h***:

$$h = \Sigma n^{\frac{1}{d+4}}$$

- Noisier prediction with NNDE **optimal kernel width, *k*** (order of the nearest neighbor)

$$k = \sqrt{n}$$

- Verdict: **KDE with optimal *h***

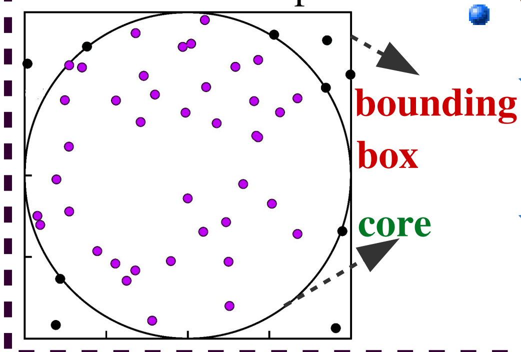


n: sample size (*10,000*)
Σ: covariance matrix
d: dimensionality (*1*)

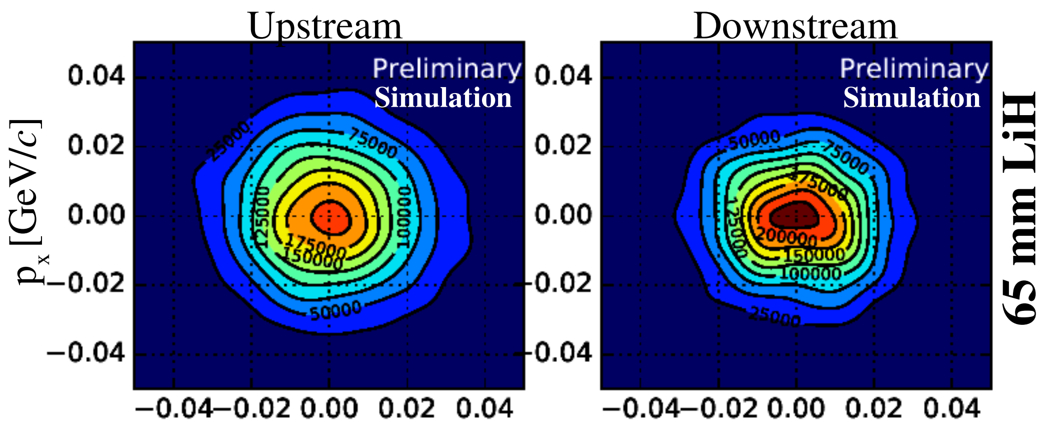
KDE Density and Volume – MICE Baseline

- **Density**:
  - Predict density using kernels centered at each muon in 4D phase space
  - Extract (cluster/classify) the **core** contour (9th percentile)

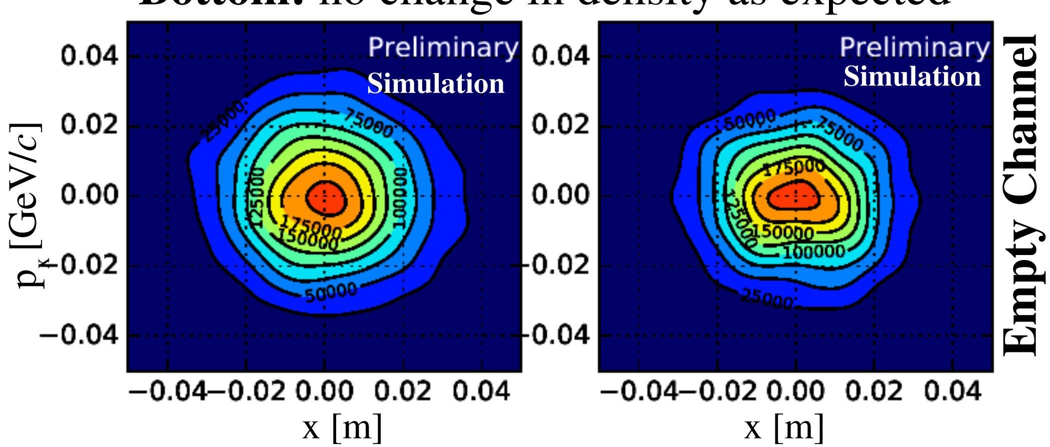
2D volume example

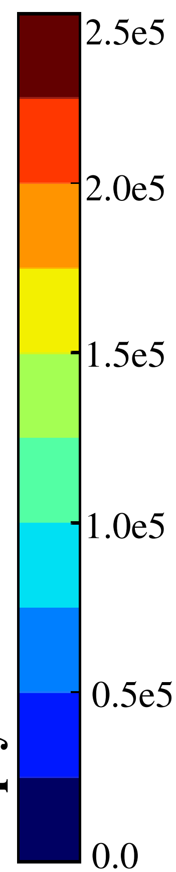


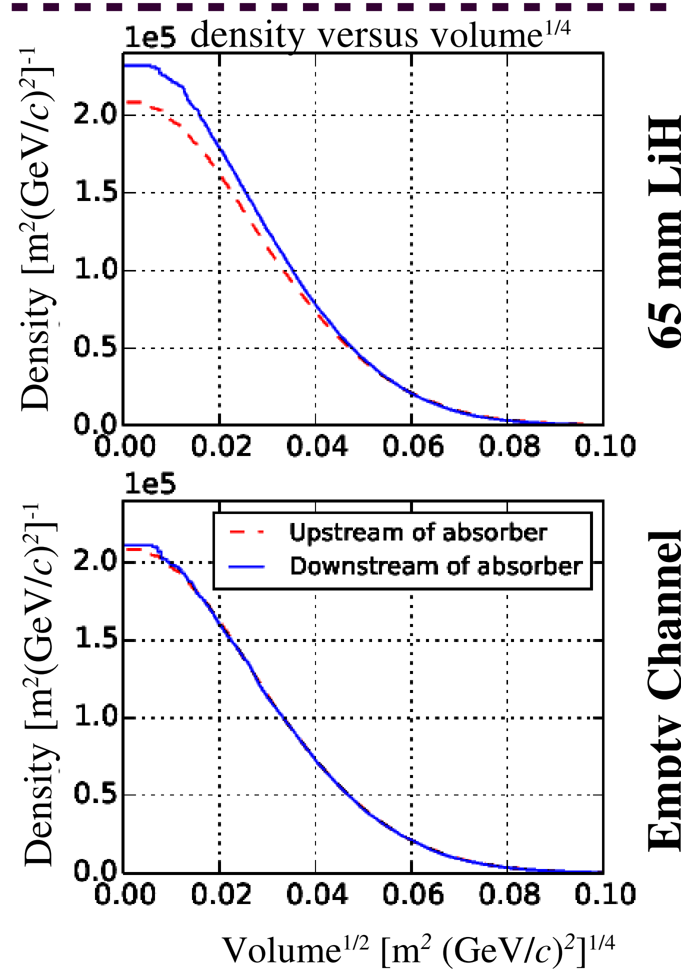
- **Volume**:
  - Generate MC (Monte Carlo) points inside the **box** bounding the **core**
  - Compute **core** volume as a fraction of **MC points inside** the **box**



Top: density increase in beam **core**
Bottom: no change in density as expected







Experimental Parameters in MICE

- Muon ionization cooling model:
  - Cooling process as the change in **normalized RMS emittance**

Cooling by ionization energy loss

$$\frac{d\varepsilon_\perp}{ds} \simeq -\frac{\varepsilon_\perp}{\beta^2 E_\mu}\left\langle\frac{dE}{ds}\right\rangle + \frac{\beta_\perp (13.6\mathrm{MeV/c})^2}{2\beta^3 E_\mu m_\mu X_0}$$

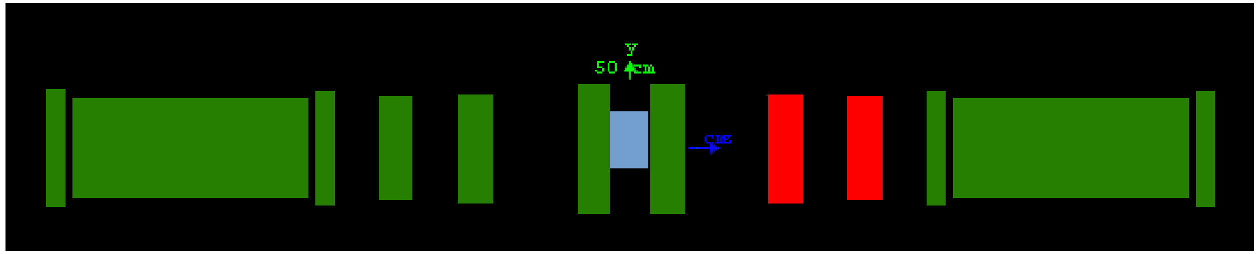
Heating by multiple (Coulomb) scattering

  - Defines:
    - **Minimum achievable (equilibrium) emittance**:
      - Achieved with balance of the cooling and heating rates

$$\varepsilon_\perp \simeq \frac{\beta_\perp (13.6\mathrm{MeV/c})^2}{2\beta m_\mu X_0}\left\langle\frac{dE}{ds}\right\rangle^{-1}$$

      - Input emittance >> **equilibrium emittance**
      - Strong focusing at absorber
      - Low-Z absorber material

Simulation Process

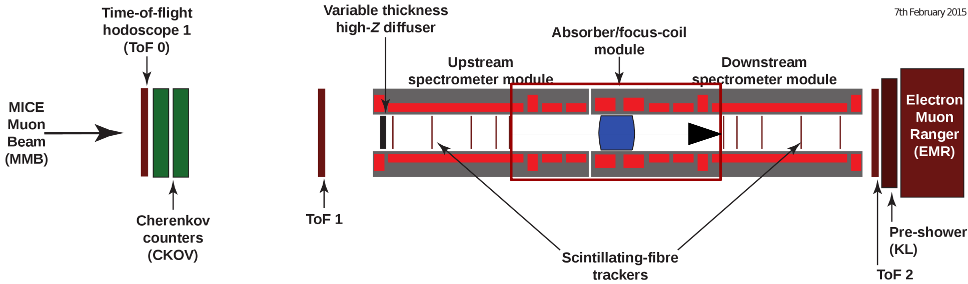


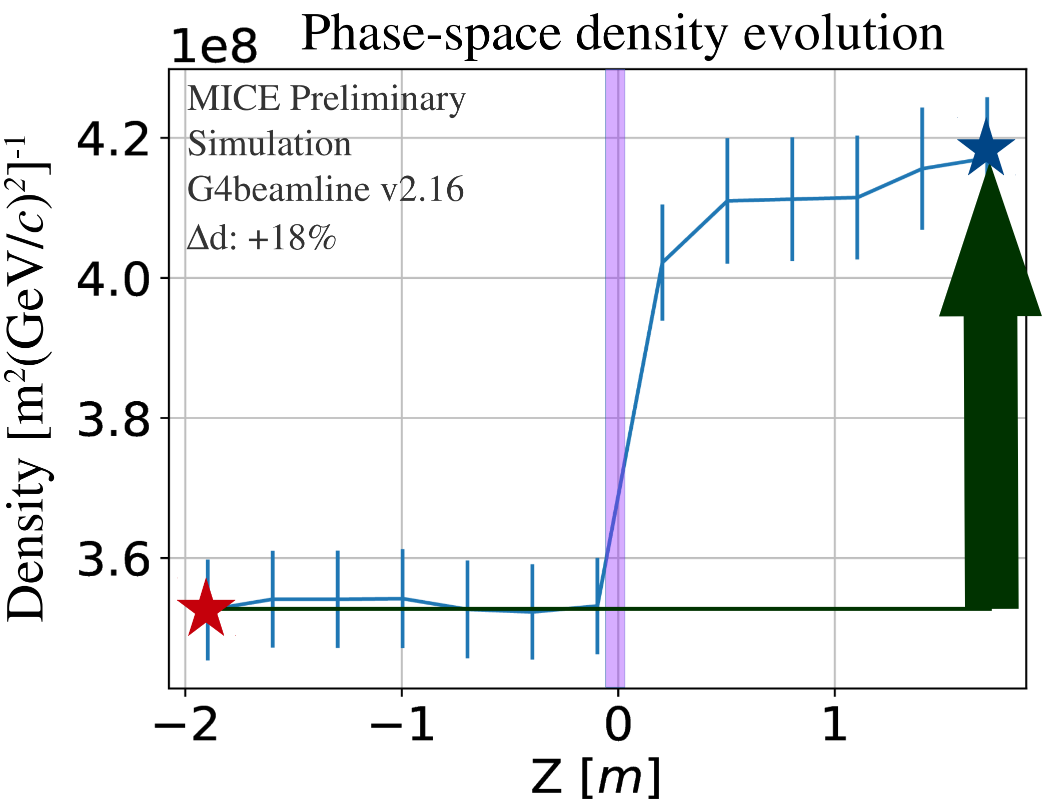
- **Input data**: transverse coordinates (feature space) from virtual detector hits
- **KDE** applied to **input data** at each virtual detector:
  - **Predicts** density of each muon
  - **Clusters/classifies** core muons based on muon densities
  - **Extracts** muon densities within the core
  - Computes volume of the core contour

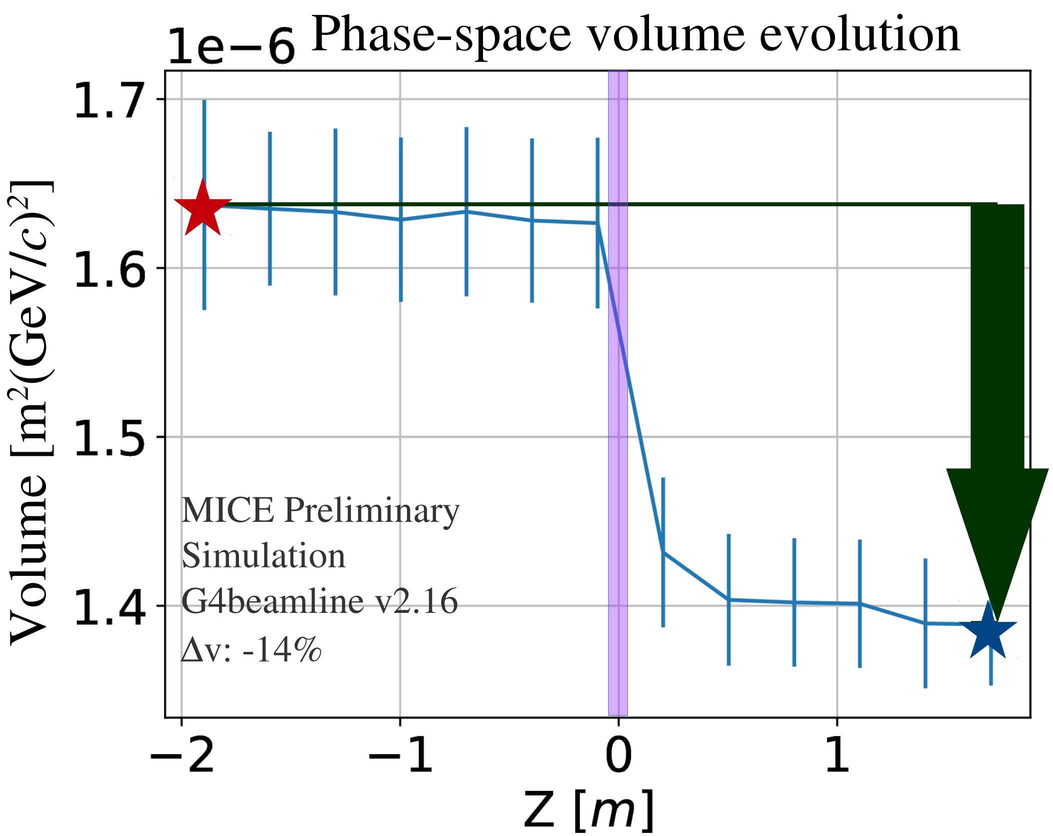
Parameters	Values
Transmission [%]	84
Solenoid magnet setting	Downstream match coils **off**
Absorber type	65 mm LiH
Input $\varepsilon_\perp$ [π mm]	6, 10
Equilibrium ε [π mm]	~5
$\beta_\perp$ [mm]	731
p_{ref} [MeV/c]	140

Simulation Results

- Beam setting:
  - ★ Input transverse emittance: 6 π mm
  - ★ Momentum: 140 MeV/c
- Tracked the evolution of the 9th percentile contour (~1σ of the distribution in 4D) across the LiH **absorber** from **US RP★** to **DS RP★**:
  - ★ **Density** ↑, **Volume** ↓

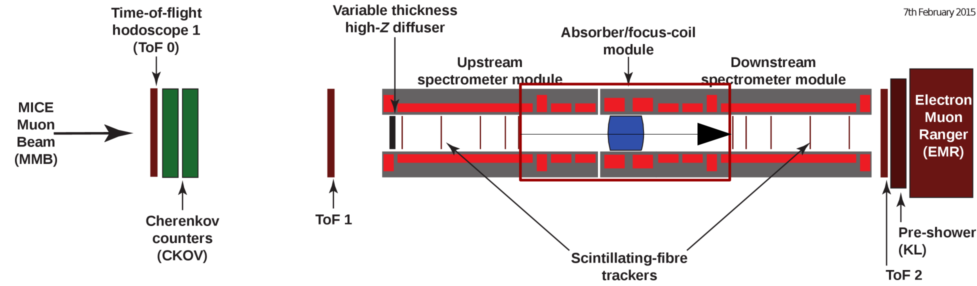


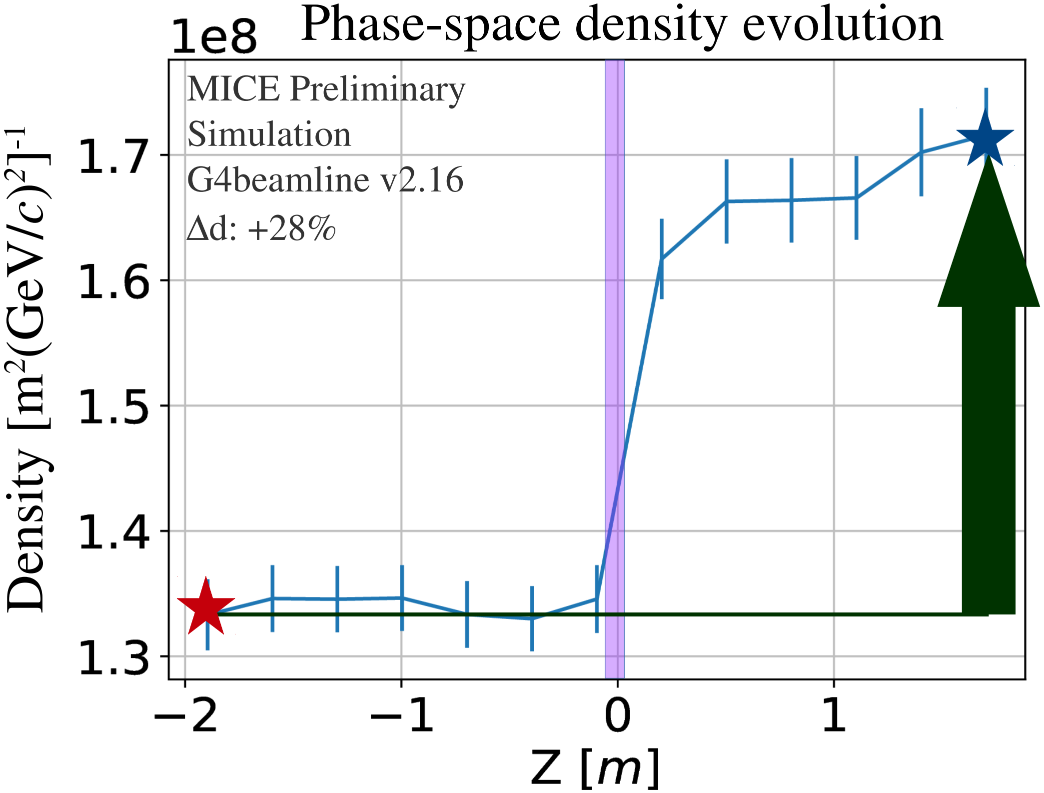


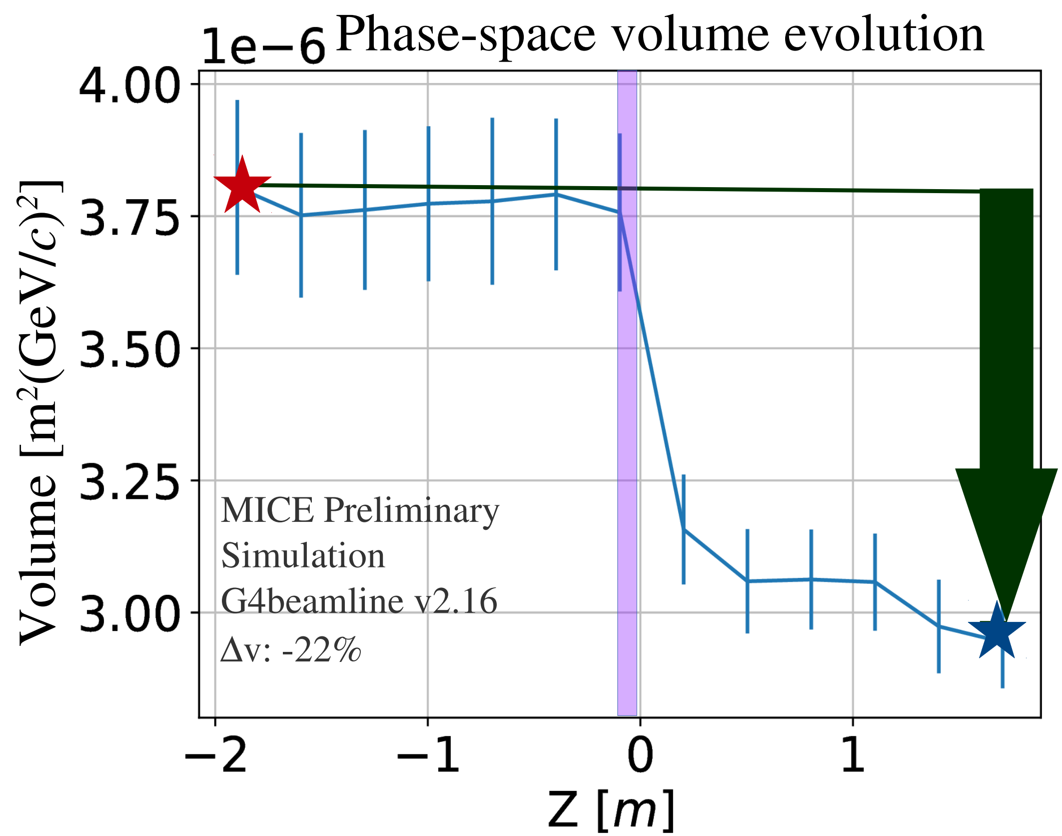


Simulation Results cont.

- Beam setting:
  - ★ Input transverse emittance: 10 π mm
  - ★ Momentum: 140 MeV/c
- Tracked the evolution of the 9th percentile contour (~1σ of the distribution in 4D) across the LiH **absorber** from **US RP★** to **DS RP★**:
  - ★ **Density** ↑, **Volume** ↓



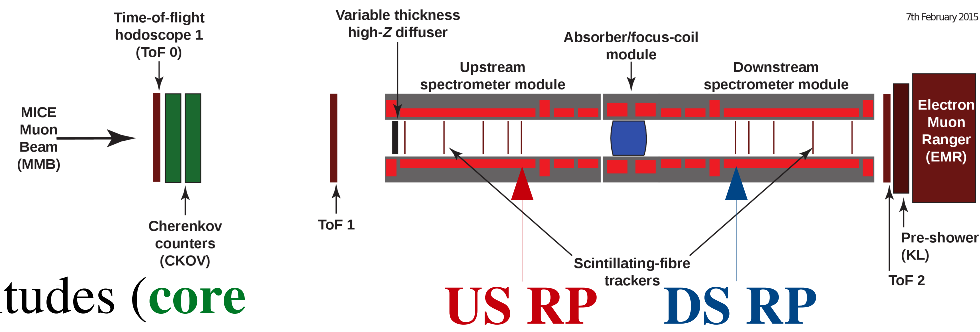


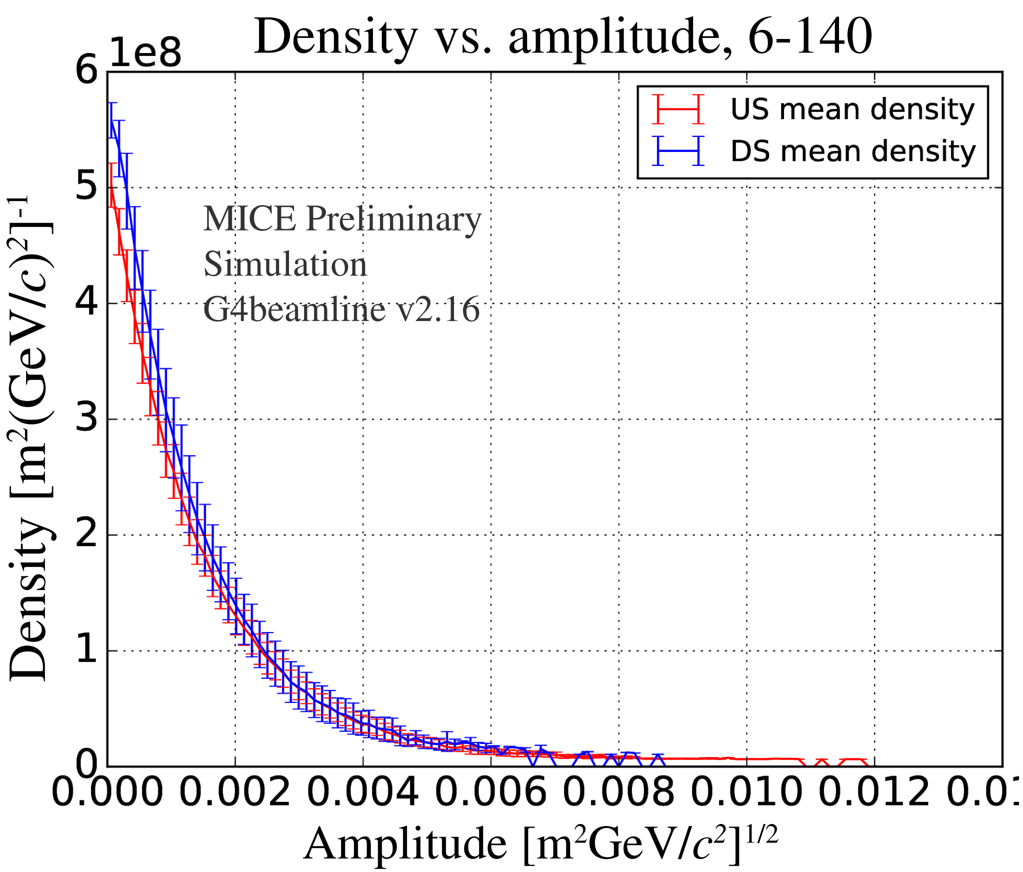


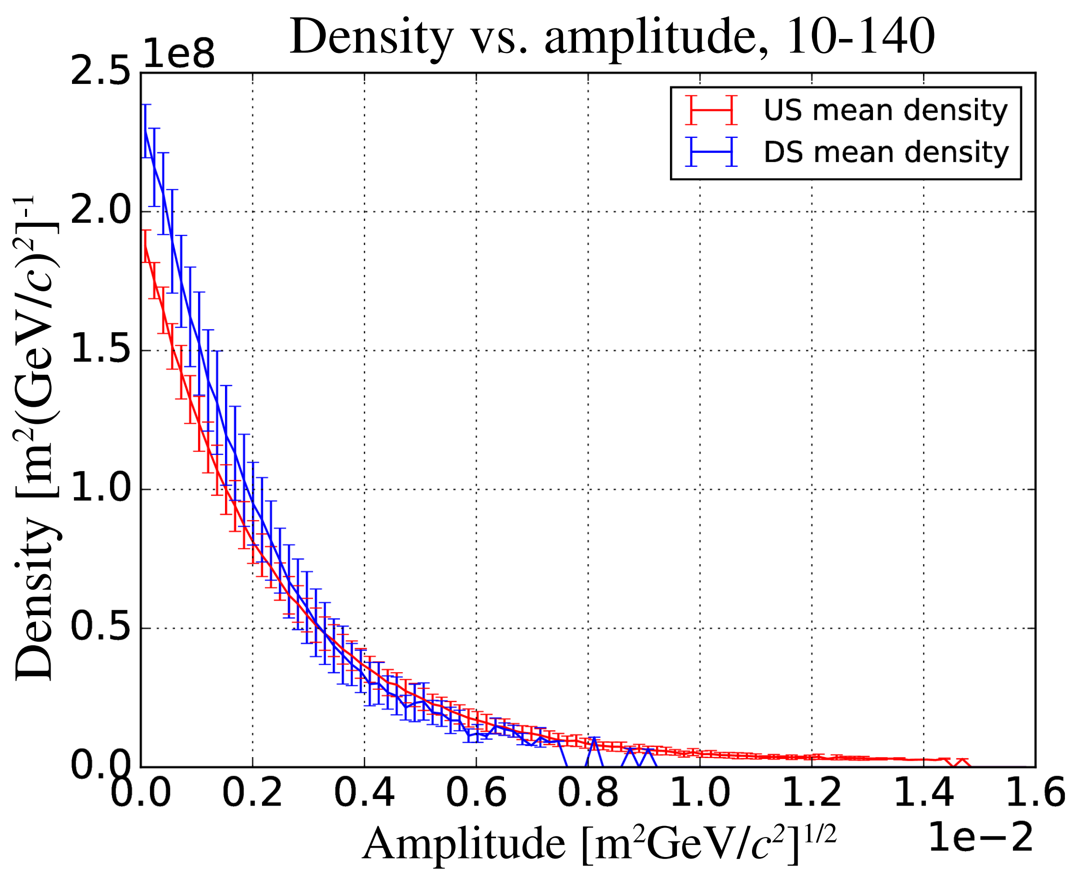
- Beam setting:
  - ★ Input transverse emittance: 6, 10 mm
  - ★ Momentum: 140 MeV/c
- Transverse amplitude:

$$A_{\perp} = \epsilon_{\perp} \vec{x}^{T} \Sigma \vec{x}$$

- Density ↑ from **US RP** to **DS RP** at smaller amplitudes (**core** density increase)







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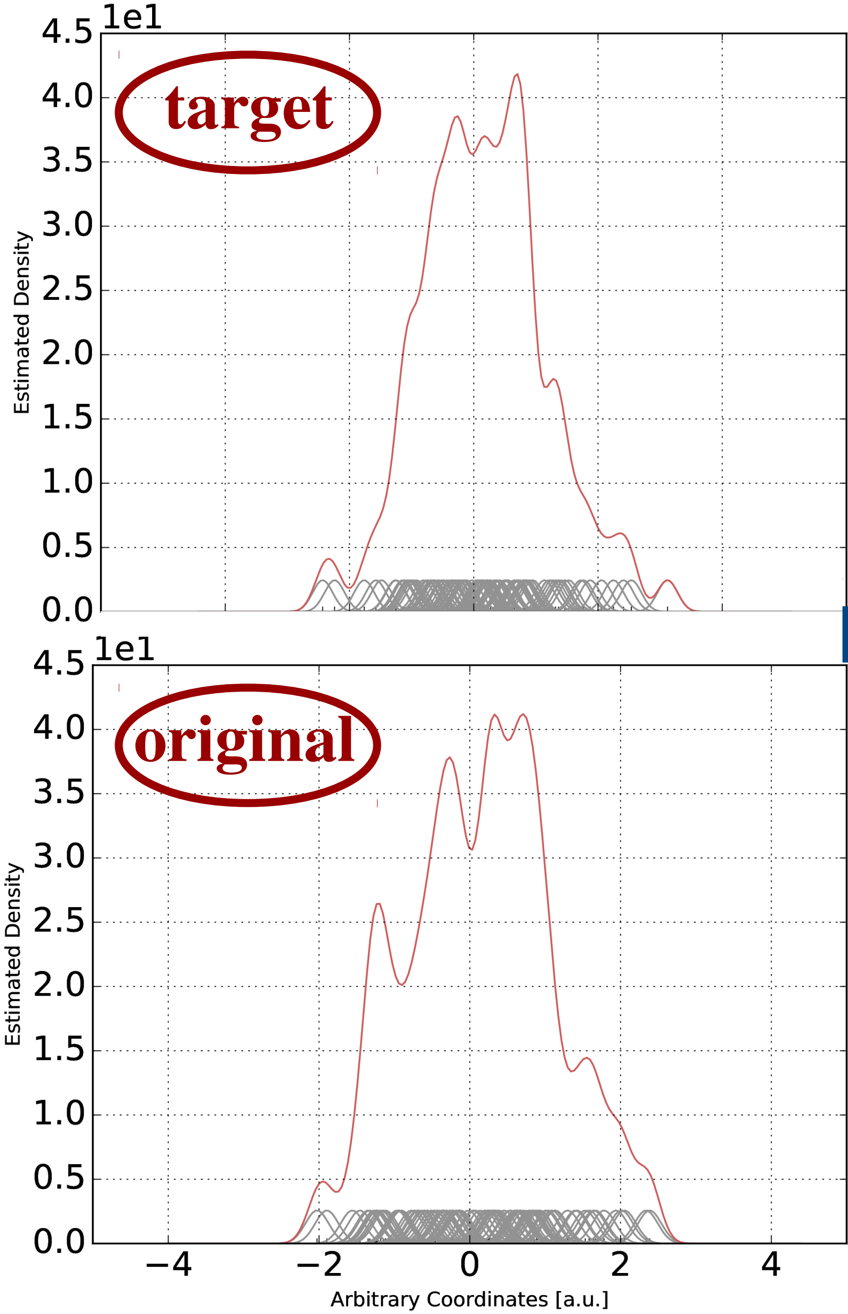
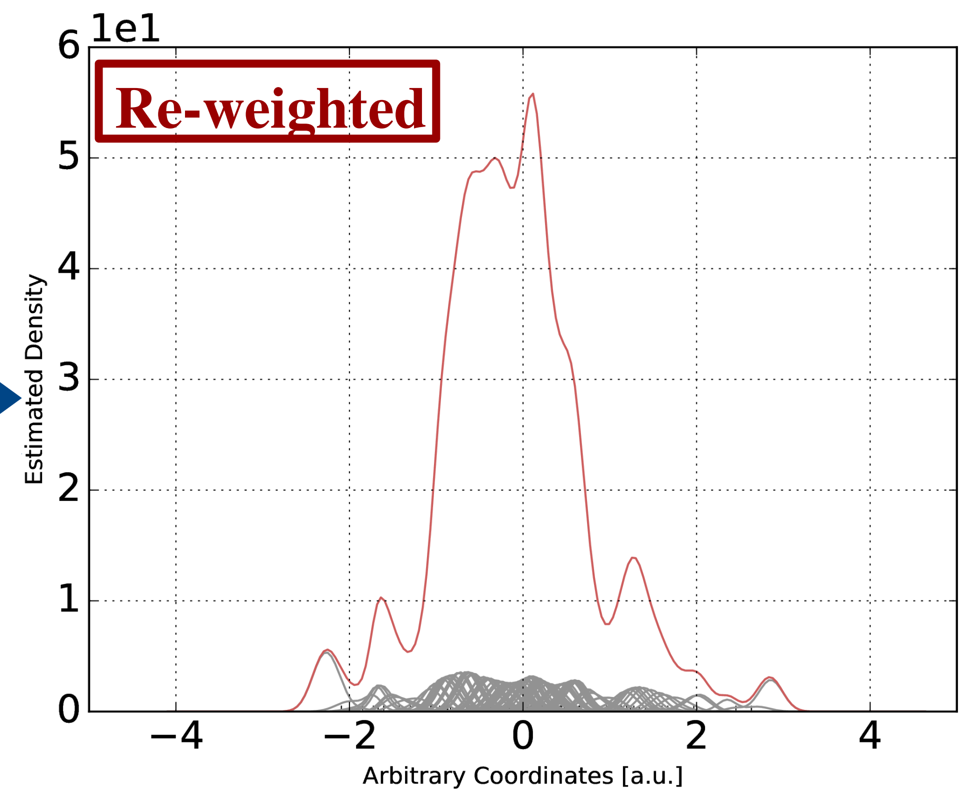
- Muon Ionization Cooling Experiment (MICE)
- Non-parametric Density Estimators
- Simulation Results
- **Discussion, on-going efforts**
- Conclusion

Re-weighting



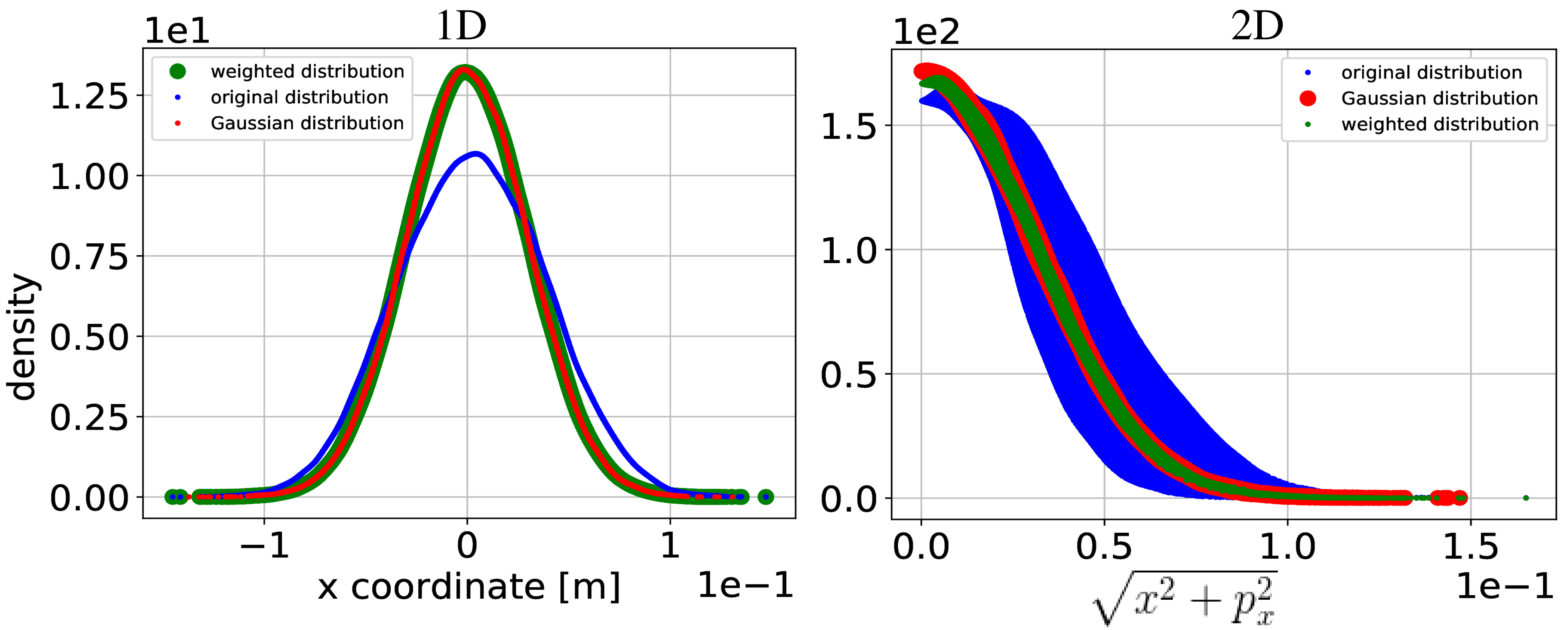
- Make a non-Gaussian beam distribution, Gaussian:
  - ★ Re-weight the distribution



- Example (for illustration purposes only):
  - ★ Re-weight **original** to match **target**:
  - ★ Re-weight the kernels (instead of re-weighting the coordinates or the density)

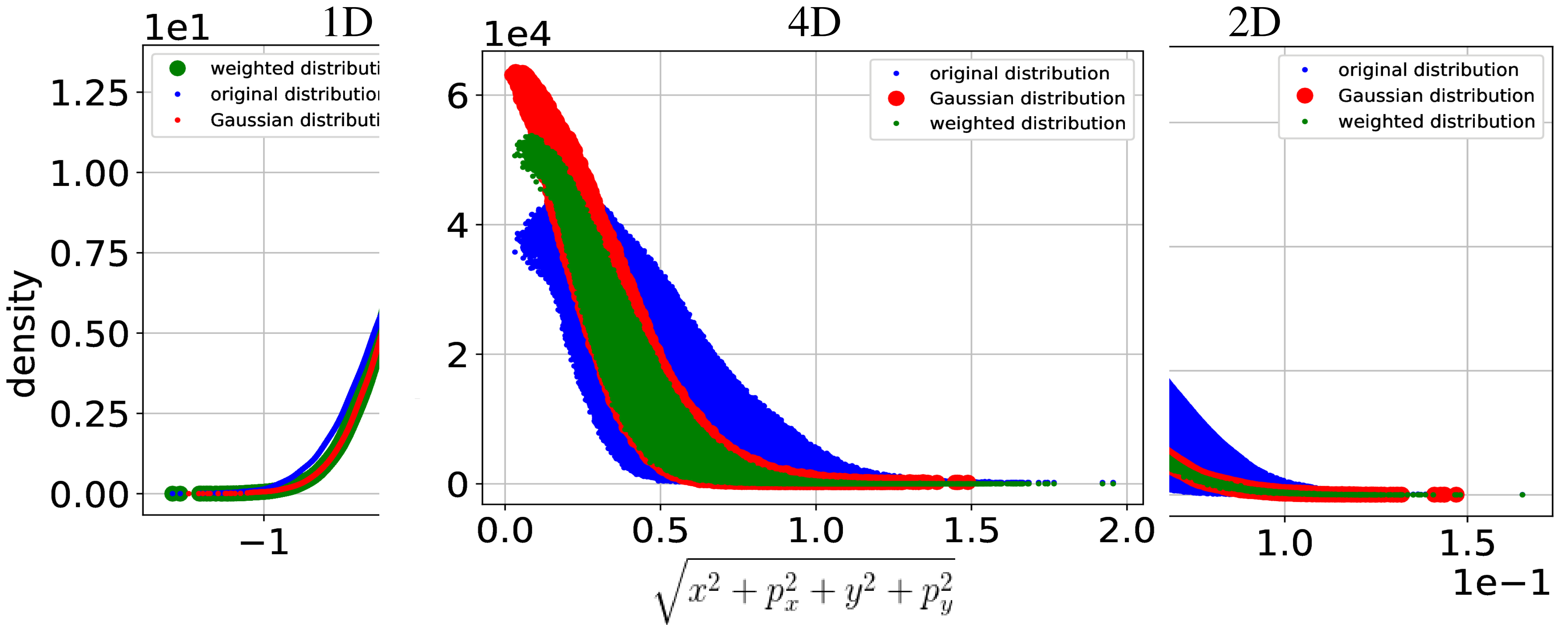
Re-weighting with KDE – Test

- **Original distribution**: phase-space coordinates at the entrance to MICE. **Target distribution**: Gaussian. **Weighted distribution**: distribution with re-weighted kernels



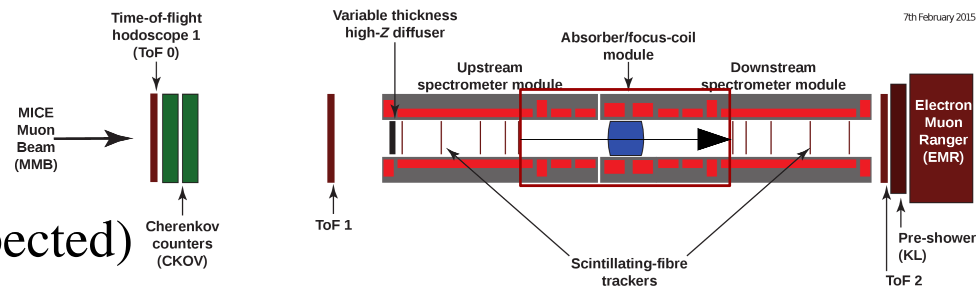
Re-weighting with KDE – Test

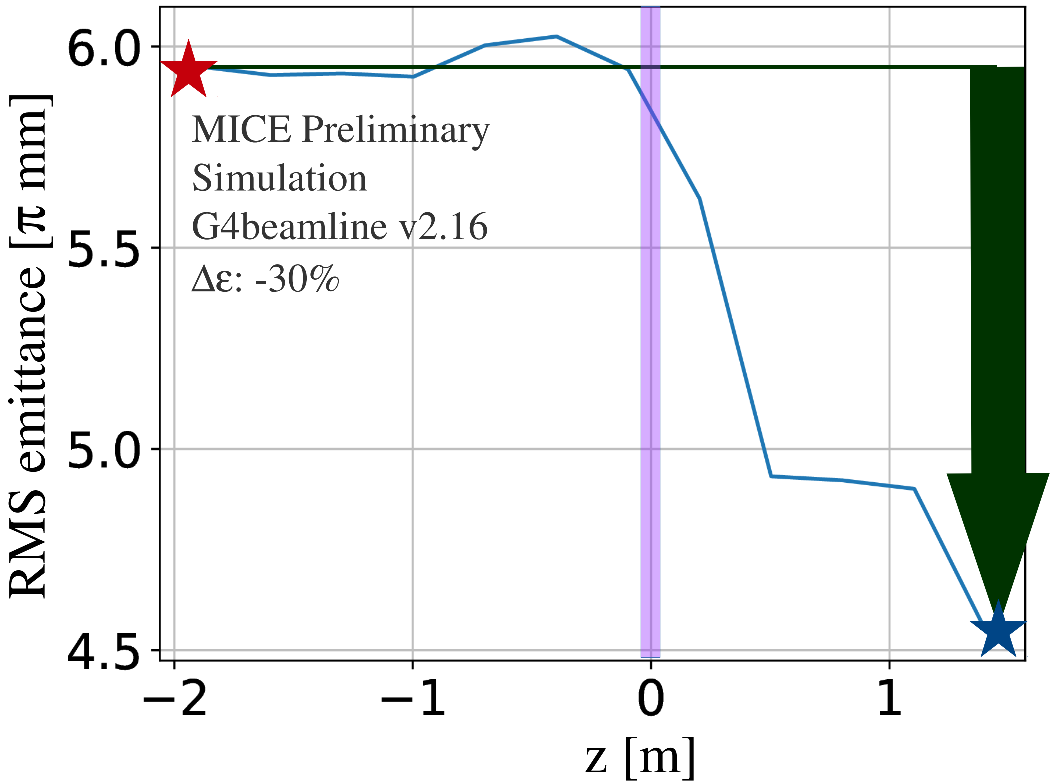
- **Original distribution**: phase-space coordinates at the entrance to MICE. **Target distribution**: Gaussian. **Weighted distribution**: distribution with re-weighted kernels
- Core under-estimation: exploring supervised re-weighting (boosted decision trees)

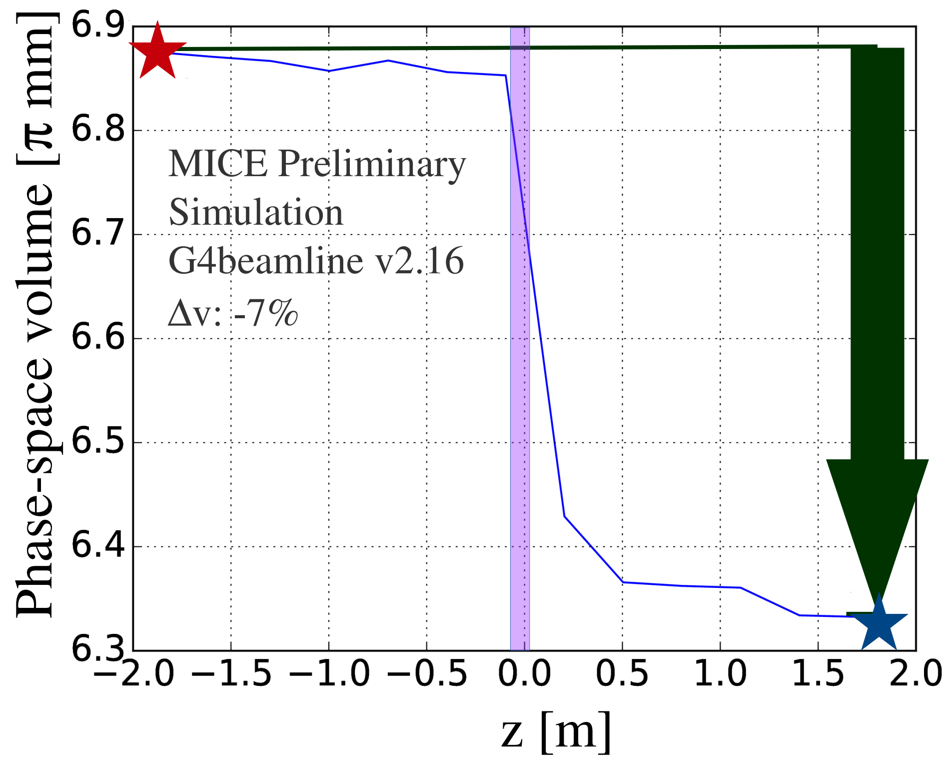


Emittance vs. KDE Volume

- Beam setting:
  - Input transverse emittance: 6 π mm
  - Momentum: 140 MeV/c
- RMS emittance affected by transmission loss:
  - Apparent emittance reduction (**-30%** Δε not expected)
- KDE volume in units of emittance:
  - Unaffected by transmission loss (yields expected **-7%** Δv)
- Future extension to **supervised** learning: expected cooling performance as output data







Conclusion

- KDE based measurements:
  - Provide a detailed diagnostics of the muon beam traversing a material
  - Proven to be robust against beam loss
- Re-weighter routine:
  - Removes correlations in the beam
  - Further investigation in MC and data in progress
- Future supervised learning:
  - Expected cooling performance as output data
  - Supervised re-weighting techniques (e.g. boosted decision trees)
- MICE has gathered great amount of data:
  - Application of KDE to data on-going

Thank You!

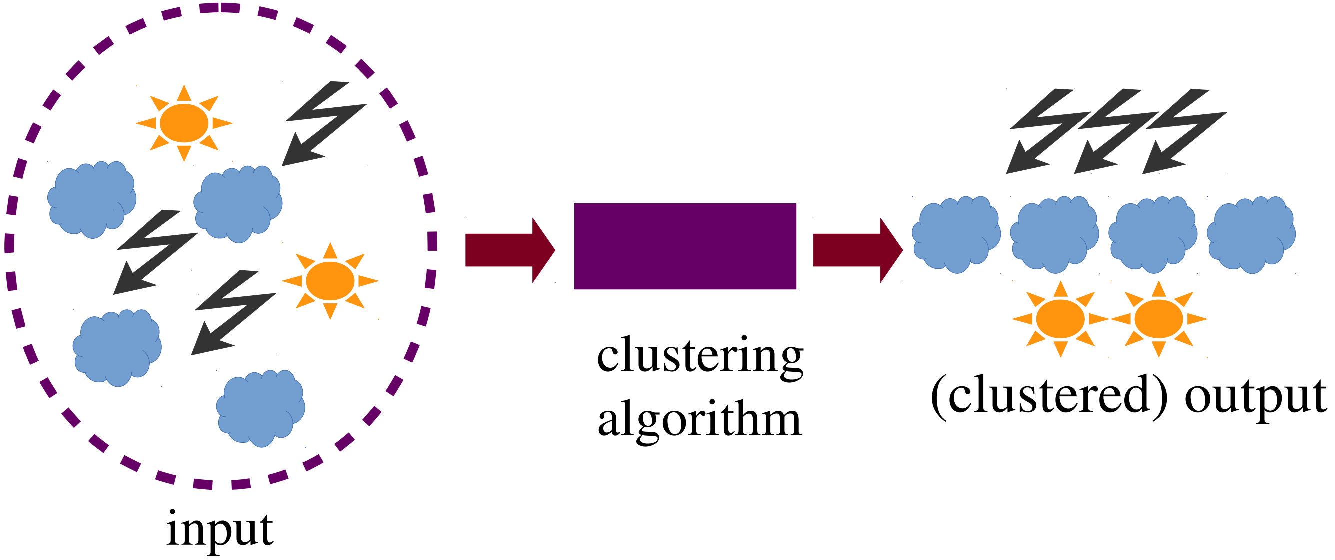
References

1. T. A. Mohayai, et al., “Novel Application of Density Estimation Techniques in Muon Ionization Cooling Experiment”, arXiv:1710.04780 (2017)
2. T. A. Mohayai, et al., “Novel Implementation Of Non-Parametric Density Estimation in MICE”, IPAC’17, IPAC-2017-WEPAB135 (2017)
3. T. A. Mohayai, “Measurements Of Beam Cooling In Muon Ionization Cooling Experiment”, University of Mississippi invited colloquium talk (2017)
4. T. A. Mohayai, “Novel Application of Kernel Density Estimation in MICE”, MICE-Note-506 (2017)
5. T. A. Mohayai, et al., “Measurements of Beam Cooling in the Muon Ionization Cooling Experiment”, APS April Meeting'17 (2017)
6. T. A. Mohayai, et al., “Simulated Measurements of Beam Cooling in Muon Ionization Cooling Experiment”, Proc. NA-PAC’16, NA-PAC-2016-WEPOA36 (2016)
7. T. A. Mohayai, et al., “Simulated Measurements of Cooling in Muon Ionization Cooling Experiment”, Proc. IPAC’16, IPAC-2016-TUPMY011 (2016)

Additional Slides

Machine Learning Aspect of Density Estimators

- **Supervised** learning:
  - Train algorithm to produce desired output from input (input, output known)
- **Unsupervised** learning:
  - Known input, unknown output, exploratory in nature
  - Non-parametric density estimation: clustering/classification, density prediction/estimation, re-distribution/re-weighting

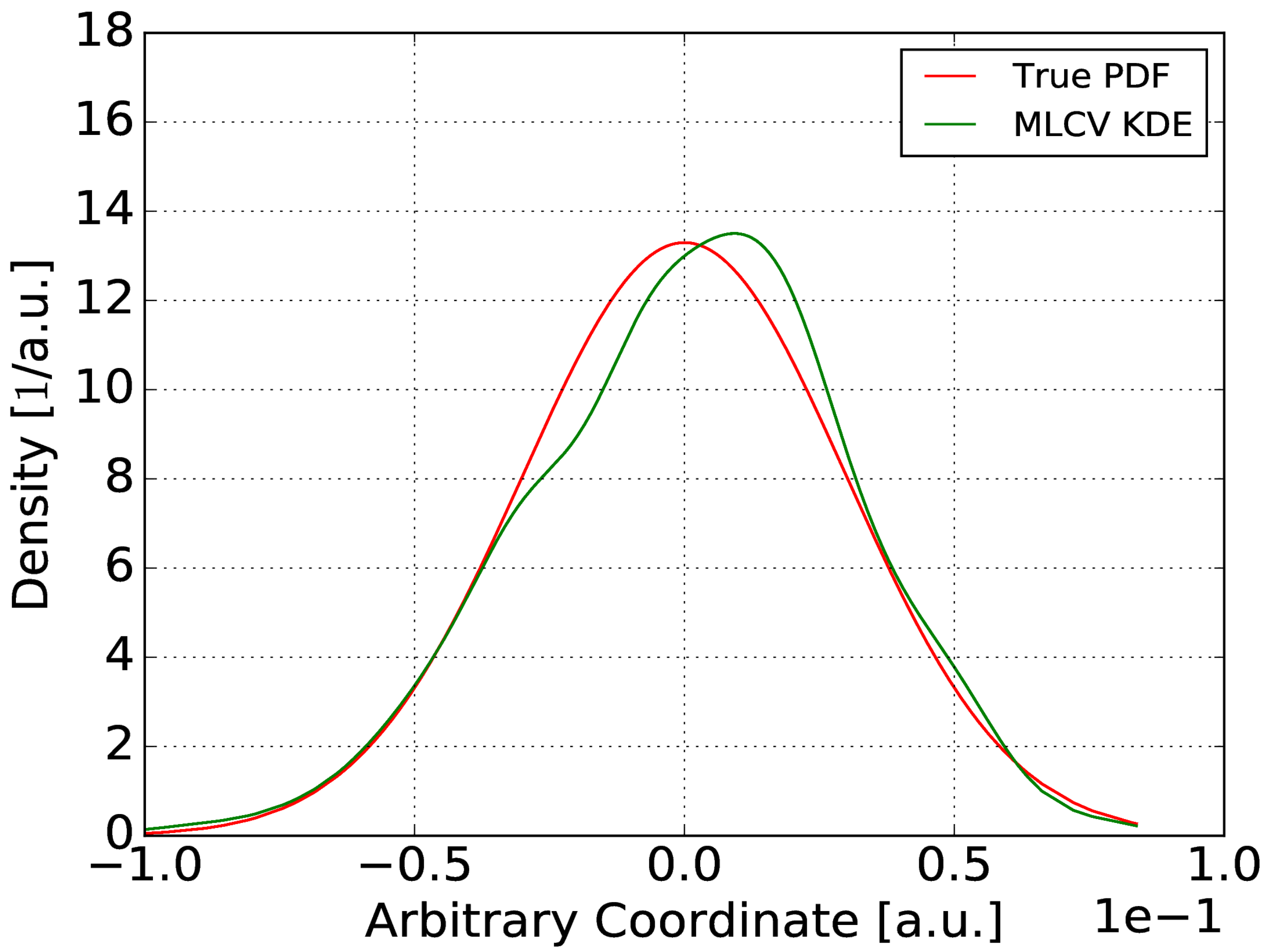


Choice of Kernel Width, Semi-supervised Approach

- Estimate density of an independent sub-sample (one data point left out of original sample) using KDE **optimal kernel width**
- Repeat n times and take the sum

$$\mathrm{CV}(h) = \frac{1}{n}\sum_{j=1}^{n-1}\log\hat{f}_j(x)$$

- **Cross validated kernel width**: maximizes CV(h) (MLCV)

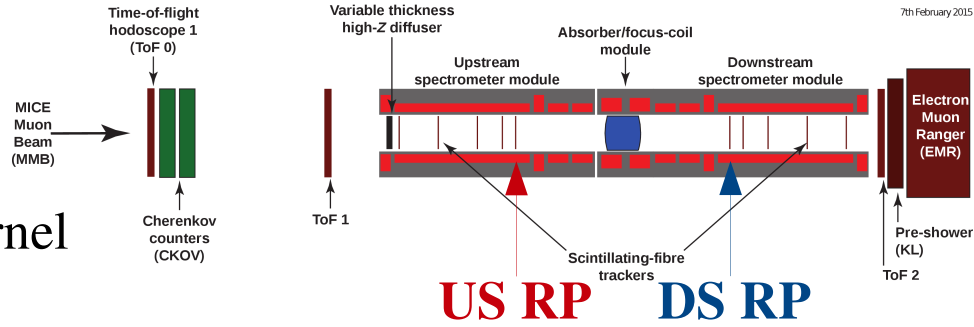


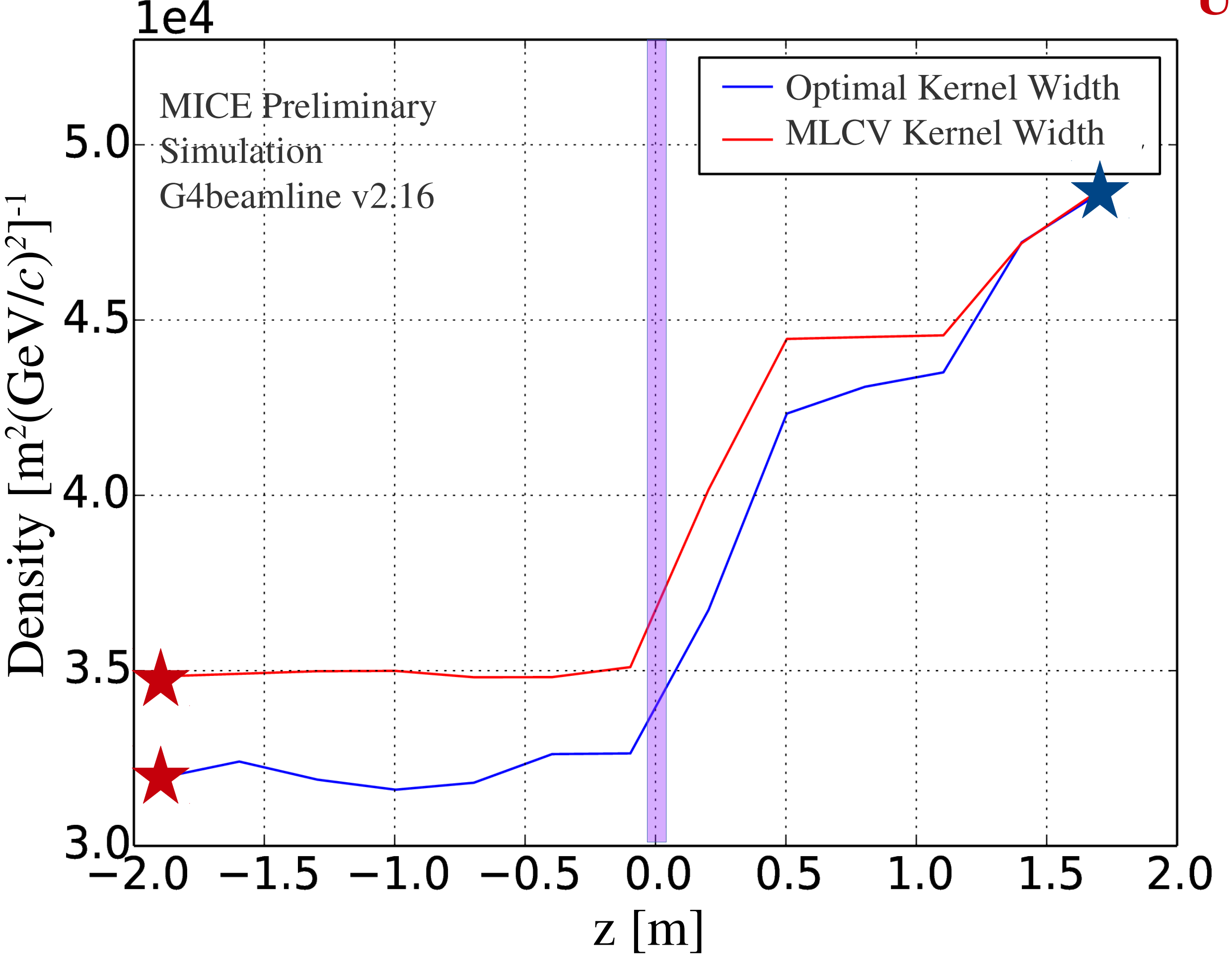
n: sample size (*10,000*)

f_j: estimated density for each subsample of size *n-1*

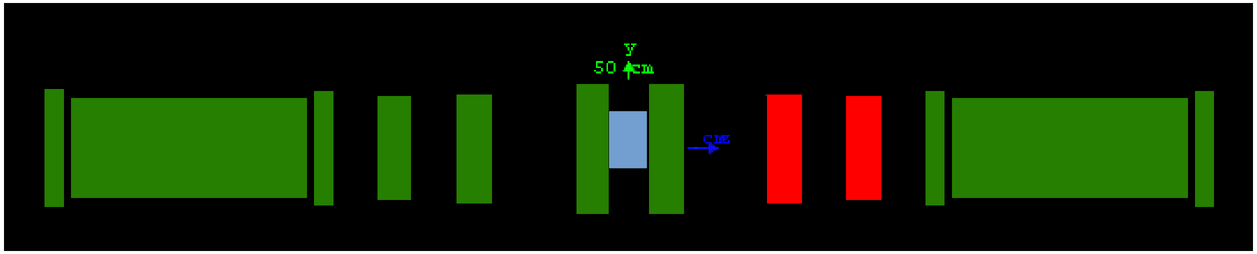
MLCV Kernel Width, Performance in Simulation

- Beam setting:
  - Input transverse emittance: 6 mm
  - Momentum: 140 MeV/c
- Density ↑ from **US RP** to **DS RP**
- **Agreement** between KDE optimal and MLCV kernel width





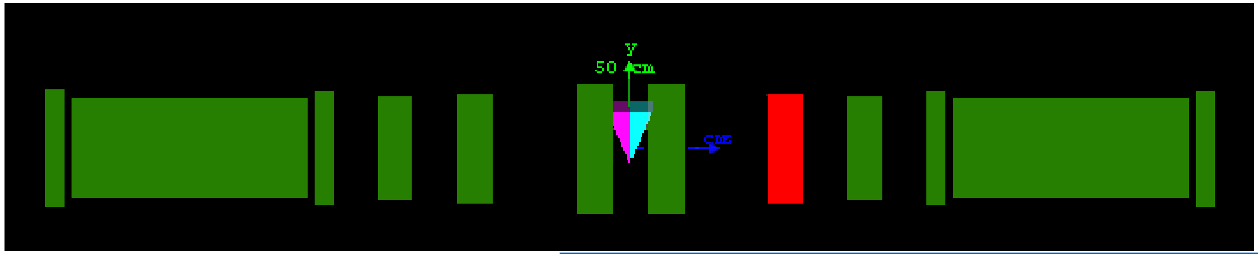
MICE G4beamline Simulation Parameters – LiH



Coil Name	Values [A]
US-ECE	205
US-M2	171
US-M1	211
FCU	58
DS-ECE	205
DS-M2	**0**
DS-M1	**0**
FCD	58

Parameters	Values
Transmission [%]	84
Cooling channel setting	Downstream match coils **off**
Absorber type	65 mm Flat LiH
Input $\varepsilon_\perp$ [π mm]	6
$\beta_\perp$ [mm]	731
p_{ref} [MeV/c]	140

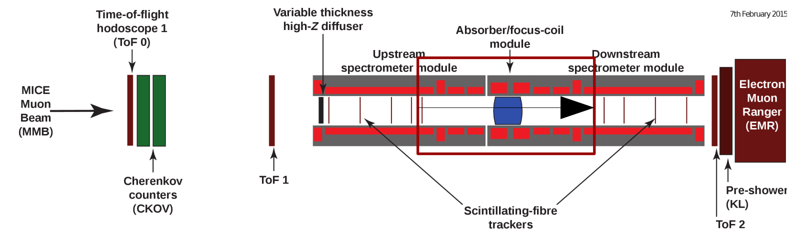
MICE G4beamline Simulation Parameters – Wedge



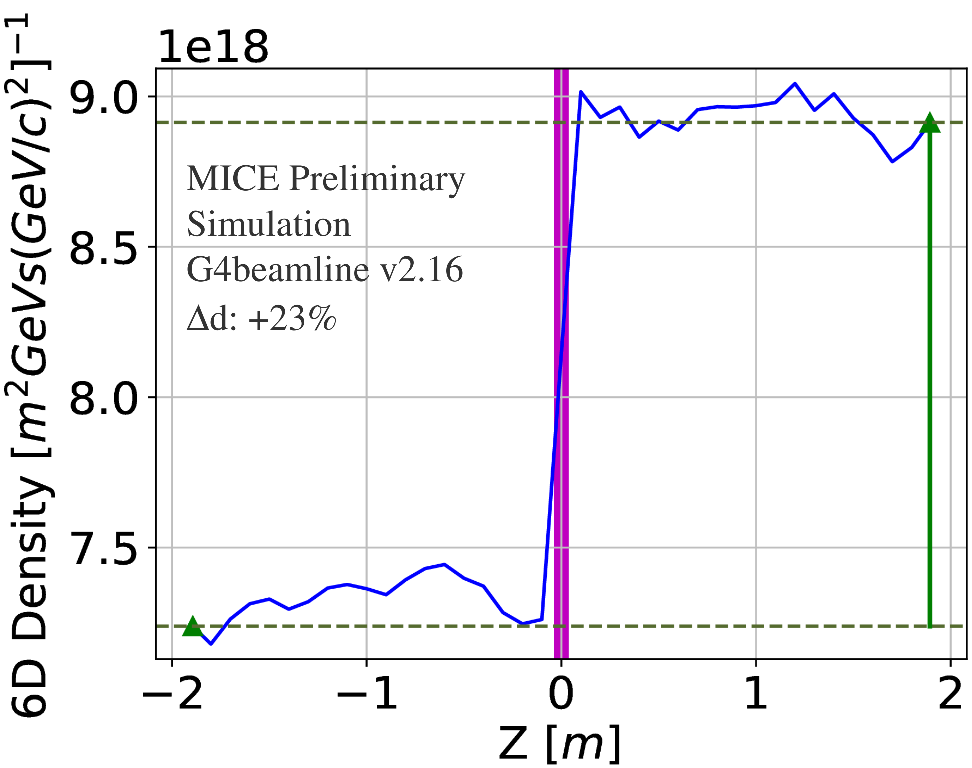
Coil Name	Values [A]
US-ECE	205
US-M2	168
US-M1	191
FCU	129
DS-ECE	-144
DS-M2	-195
DS-M1	**0**
FCD	-129

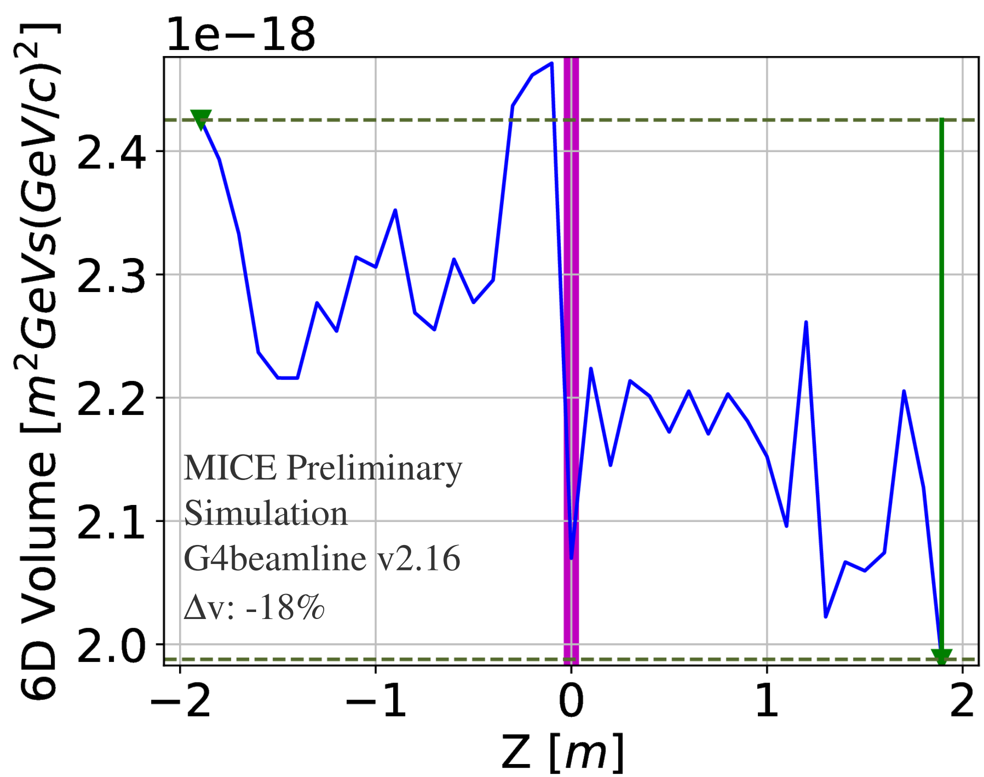
Parameters	Values
Transmission [%]	77
Cooling channel setting	DS match coil 1 **off**
Absorber Type	Polyethlene Wedge
Wedge angle [°]	45
Wedge on-axis length [mm]	52
Dy [mm]	300
Input $\varepsilon_\perp$ [π mm]	6
$\beta_\perp$ [mm]	363
p_{ref} [MeV/c]	140

Wedge Simulation – 6D Phase-space

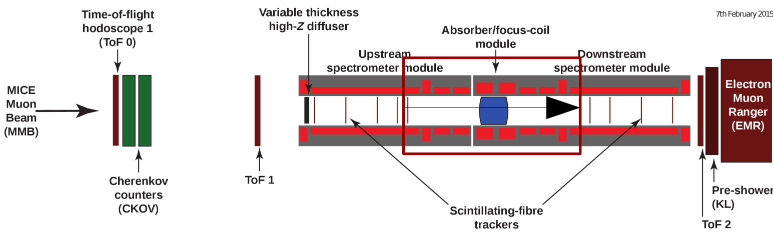


- Beam setting:
  - Input transverse emittance: 6 mm
  - Total momentum: 140 MeV/c
  - Vertical dispersion: 300 mm
- Tracked the evolution of the 2nd percentile contour (~1σ of the distribution in 6D) across the **absorber**:
  - **Density** ↑, **Volume** ↓: 6D cooling

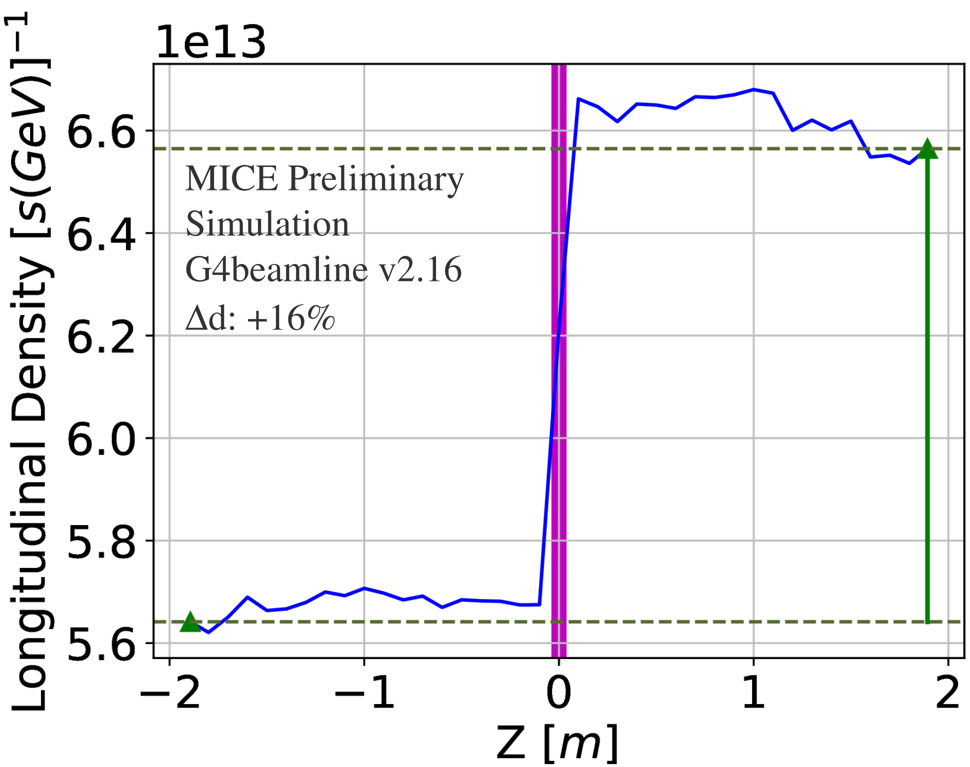


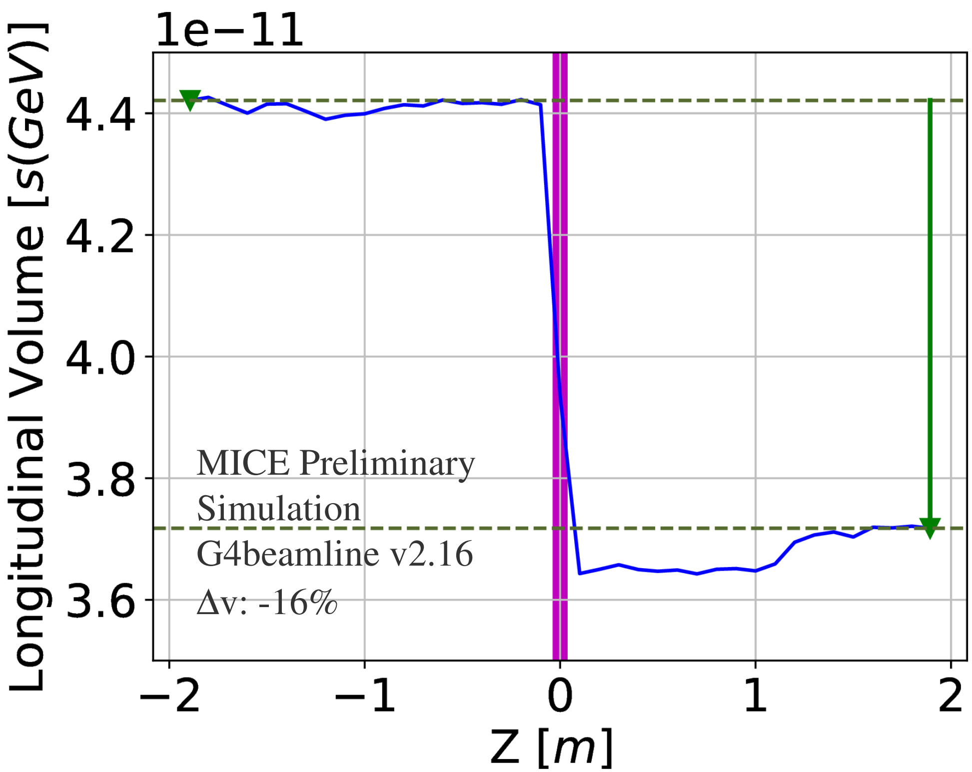


Wedge Simulation – Longitudinal Phase-space

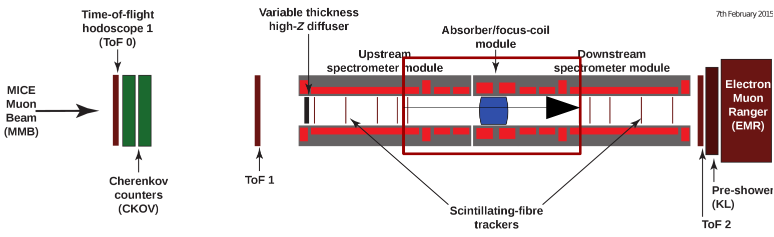


- Beam setting:
  - Input transverse emittance: 6 mm
  - Total momentum: 140 MeV/c
  - Vertical dispersion: 300 mm
- Tracked the evolution of the 24th percentile contour (~1σ of the distribution in 2D) across the **wedge absorber**:
  - **Density** ↑, **Volume** ↓: longitudinal cooling





Wedge Simulation – Transverse Phase-space



- Beam setting:
  - ★ Input transverse emittance: 6 mm
  - ★ Total momentum: 140 MeV/c
  - ★ Vertical dispersion: 300 mm
- Tracked the evolution of the 9th percentile contour (~1σ of the distribution in 4D) across the **wedge absorber**